# Investigation into computer modelling, simulation and control of the stirling engine.

Stella-Sawicki, M. A

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# INVESTIGATION INTO COMPUTER MODELLING, SIMULATION AND CONTROL OF THE STIRLING ENGINE

A thesis submitted to the Department of Electrical Engineering and Electronics, KING'S COLLEGE, University of London, in fulfilment of the requirements for the degree of DOCTOR of PHILOSOPHY

by

MAREK Andrzej STELLA-SAWICKI,
M. Sc. (Eng.), C. Eng., M. I. E. E.

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Author: Marek Andrzej Stella-Sawicki, M.Sc.(Eng), C.Eng., M.I.E.E.

Title : Investigation into Computer Modelling, Simulation and Control of the Stirling Engine.

## ABSTRACT

This thesis presents the results of an investigation into the computer modelling, simulation and control of the Stirling Engine.

A theoretical computer model based on practical engine data is introduced using SYSTRAN ( a digital/analog simulator program). This basic engine is initially proportionally controlled, but, after derivation of control system requirements and implementation of Error Derivative Compensation, experimental evidence confirms the effectiveness and sense of the method applied.

Classical control features include Bode graphs, Root-locus, etc., which are used throughout in the form of relevant computer programs to evaluate various system parameters, i.e. stability, gain margins, phase margins, and phase margin frequency. The analysis used in a Derivative Error Control system is extended further in order to achieve an optimal control scheme. This system is shown to possess better controlability and "speed accuracy" whilst the transient response remains satisfactory.

Suggestions are also made as to recommendations for further research work.

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## LIST OF COMPUTER PROGRAMS

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## A C K N O W L E D G E M E N T S

I wish to record my deep appreciation of the valuable assistance given by a large number of people throughout the period of this research and in particular the following:

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# I. AN INTRODUCTION INTO THE STIRLING CYCLE MACHINE PHILOSOPHY

A.D. 1816 . . . . . . . . N° 4081.

## Steam Engine and Saving Fuel.

LETTERS PATENT to Robert Stirling, of Edinburgh, Clerk, for his invented "IMPROVEMENTS FOR DIMINISHING THE CONSUMPTION OF FUEL, AND IN PARTICULAR AN ENGINE CAPABLE OF BEING APPLIED TO THE MOVING MACHINERY ON A PRINCIPLE ENTIRELY NEW." 6 months.

Dated 16th November 1816.

*(No Specification enrolled.)*

LONDON:
Printed by GEORGE EDWARD EYRE and WILLIAM SPOTTISWOODE,
Printers to the Queen's most Excellent Majesty. 1857.

Fig.1. Letter Patent to Robert Stirling, Anno Domini 1816

## (I.I) NOMENCLATURE AND HISTORICAL BACKGROUND

The Stirling engine was developed by a clergyman, The Rev. Robert Stirling of Kilmarnock (Scotland) in the early nineteenth century, about 1816 and was one of the first successful hot-air engines. However there were several more or less successful constructions before this, for example: a hot-gas engine built in 1807 by Sir George Cayley. The Stirling-cycle machine is a device which operates on a closed regenerative thermodynamic cycle with cyclic compression and expansion of the working medium at different levels of temperature, and it was described in a "LETTER PATENT" (see Fig. 1) Anno Domini 1816, no: 4081 under the original title : "STEAM ENGINE AND SAVING FUEL..." issued to Robert Stirling of Edinburgh, a clerk, for his invented "Improvements for diminishing the consumption of fuel and particular an engine capable of being applied to the moving machinery on a principle entirely new".
The Stirling engine (see Fig. 2) and the regenerative engines invented later by a Swedish Engineer John Ericsson, were used extensively throughout all the nineteenth century because of their higher reliability, better efficiency and good safety conditions when compared with dangerous steam engines which claimed many lives of early inventors, experimenters and of course users - mostly because of steam-boiler explosions. From the beginning there were many other names used for Stirling Cycle Machines due to absence of general nomenclature and clarity in the subject.
For example: Hot-air/Hot-gas engines, HEINRICI/ROBINSON/RANKINE - NAPIER engines and so on. But it is clear now, the name of STIRLING CYCLE Machine is only reserved to a machine possessing particular thermodynamic facilities,e.g. wherethe flow of a working fluid is always controlled by volume changes within their own specific work characteristics.

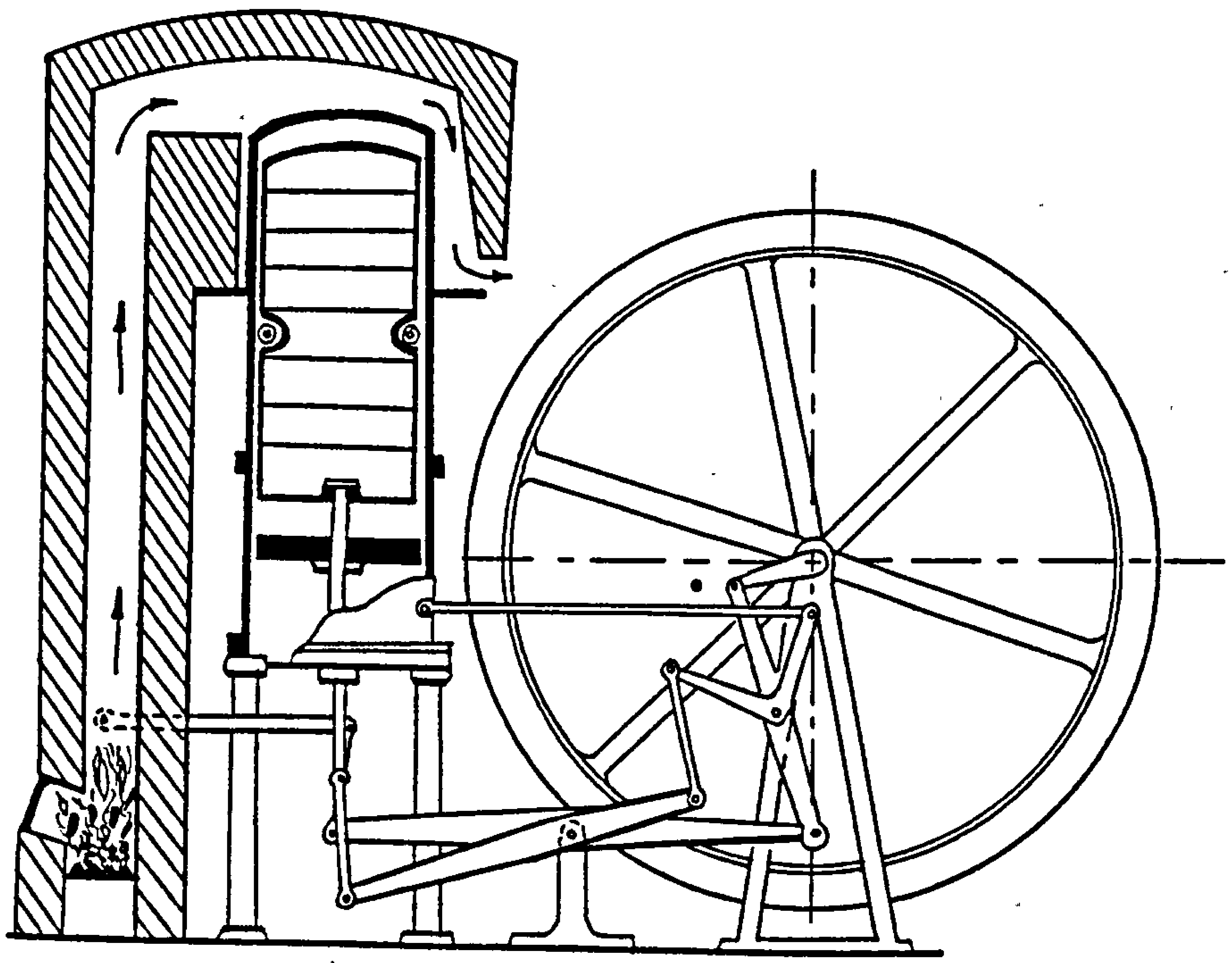

Fig. 2. The Original Stirling Engine. Reproduction of a Drawing Showing the First Stirling Engine from the Original Patent Specifications of 1816. Such an Engine was used in 1818 for pumping water from a quarry. (After Finkelstein 1959 and Walker).

Throughout the whole of the nineteenth century many thousands of Stirling engines, in a variety of shapes and sizes (see Fig. 3a and 3b) were used for different purposes, they were mostly coal fired and used air as their working medium.

The advent of the internal combustion-type engine and the greater availability of mains electricity caused the use of Stirling Cycle Machines to diminish and by 1914 they were no longer commercially available (see Lit.2). Research on Stirling engines commenced at Philips Laboratories in Eindhoven (Holland) in the late 1930's, and has since been in progress continuously, mainly because of Stirling Cycle machine's favourable noise level and low air pollution characteristics, being environmentally increasingly important nowadays.

Stirling engines can use practically any kind of heat input and because of this have a potential application in underwater power systems, vehicles (buses, urban lorries), boats, artifical heart pumps etc. The heaters of modern Stirling Engines are designed to operate on the following fuels: Diesel Oil, Non-leaded petrol, Kerosene, Alcohol, L.P.G. and L.N.G. The possibility exists of building solar-powered Stirling engines also for operating water pumps, for example for use in water pump batteries in underdeveloped tropical countries (see Lit.1). Recently a Nuclear reactor heated Stirling engine (Chalk River Nuclear Laboratories; Atomic Energy Authority of Canada) was announced and this brings forward an exciting possibility especially for multi-megawatt capacity powered generating stations.

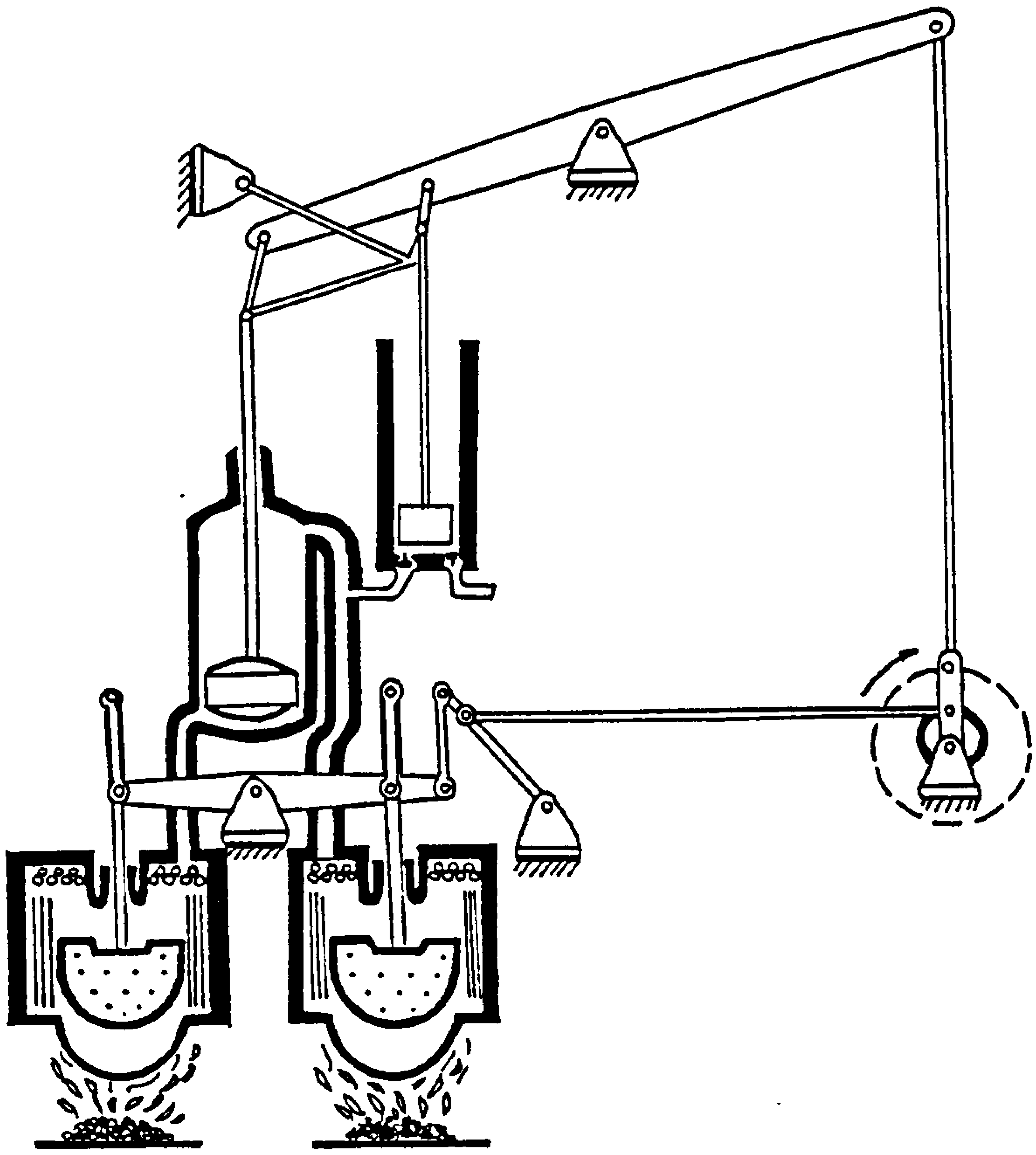

Fig. 3a. Diagrammatic cross-section showing arrangement of the Drive Mechanism (after Finkelstein)

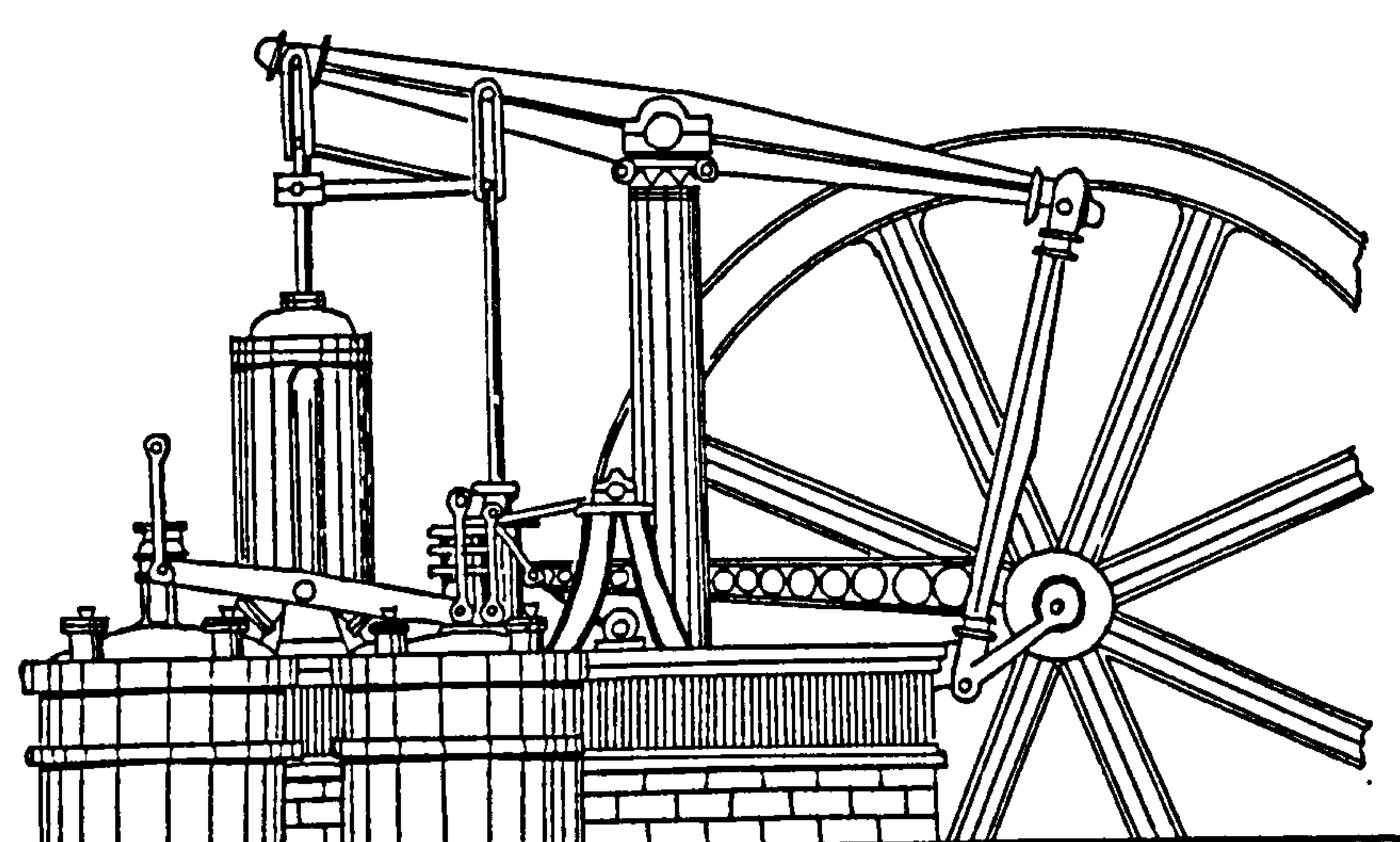

Fig. 3b. An example of an early Beam engine circa 1827 (after Finkelstein)

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by T. Finkelstein
1959

## (1.2) PRINCIPLES OF STIRLING CYCLE MACHINE OPERATIONS

The Stirling Engine Cycle is illustrated in Fig.1.

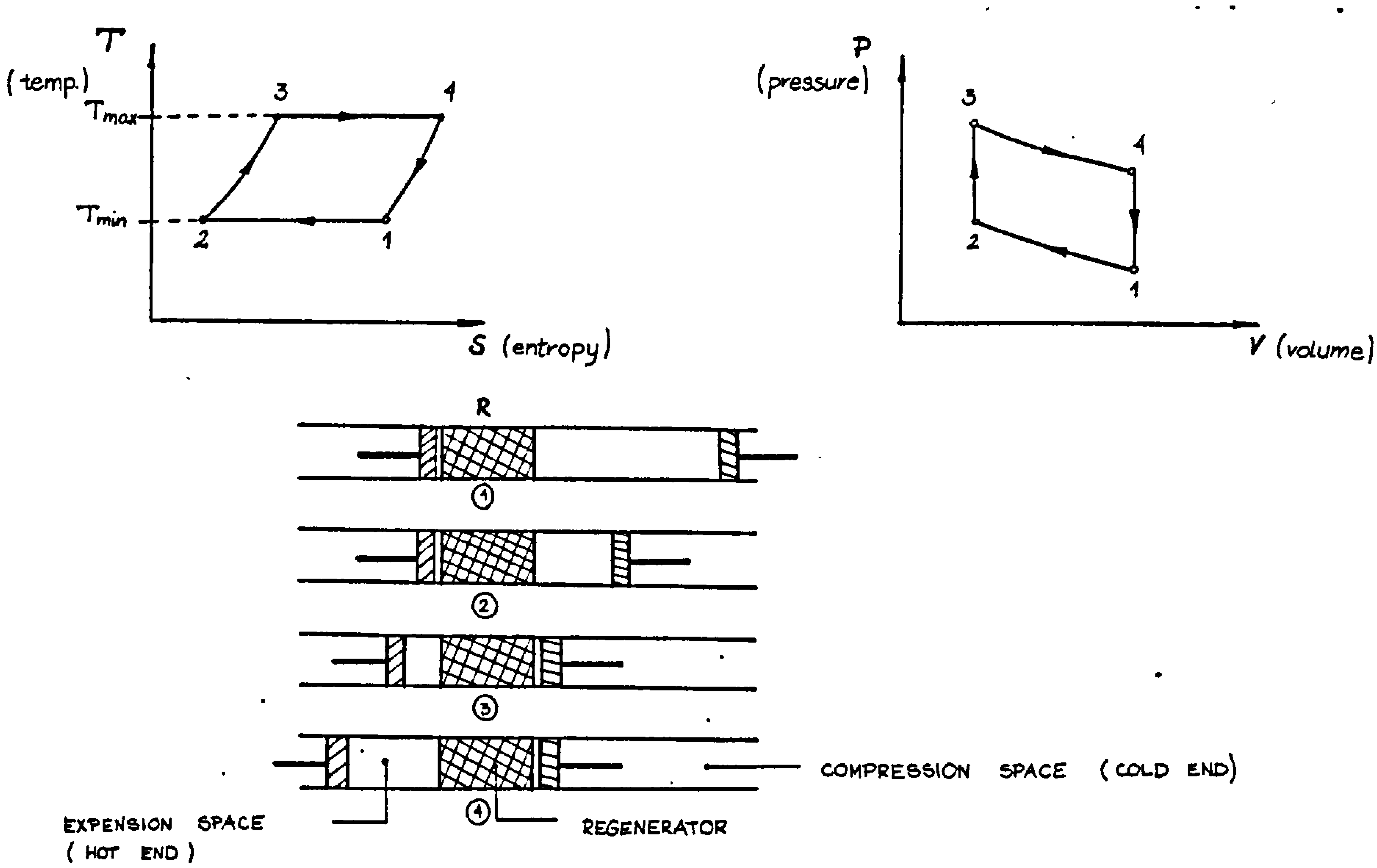


Fig. 1. "T-s" and"P-v" graphs for Stirling Cycle.

Of course, both "T-s" and "P-V" graphs describe the ideal Stirling Engine Cycle although in practice this can naturally be slightly different from these idealised plots. The working fluid is usually air, however, some experiments have been carried out on Helium, Hydrogen and other gases. If we consider the working fluid enclosed in a cylinder between two moving pistons seperated by a regenerator, we find that our working medium could be transferred through the regenerator from the compression space (cold end) to the expansion space (hot end) with the system flow controlled by volume changes, so that we observe a NET conversion of heat to work and vice versa. The engine regenerator is one of the most important parts of this machine and the main function of this element is releasing and absorbing heat during the cycles of Stirling engine operations. The specific work of the regenerator as the 'internal heat source' as well as ' heat sink' requires it to be built from metal wires/strips (formed into the relevant matrix form), supplying heat to the cold fluid and on the other hand storing the heat taken from the hot fluid. Even this simplified engine diagram (Fig. 1.) can be useful to explain the following engine states of operations.

| | |
|---|---|
| Process 1-2 | Cold working medium is compressed isothermally with heat rejection to an external source. |
| Process 2-3 | The working medium absorbs heat at constant volume when passing through the regenerator into the expansion space |
| Process 3-4 | The heat is added (isothermal expansion) from an external source. |
| Process 4-1 | The working medium is cooled at constant volume, giving up heat to the regenerator when passing from expansion to compression space. |

The thermodynamic Stirling Cycle has found so many different applications that it is now possible to build not only Stirling Cycle prime movers but also Stirling Cycle refrigerating machines, heat pumps and finally Stirling Cycle pressure generators. The first two reached a degree of popularity amongst constructors i.e. cryogenic cooling systems developed by Dr. J.W.L. KOHLER and available commercially from Philips Research Laboratories in Holland.

When discussing various aspects of Stirling Cycle Machines, a word must be said about the comparison of Stirling and Carnot cycles which are quite similiar and as the Carnot cycle possesses an idealised highest known thermodynamic efficiency (with no practical application) but which always has been one of the most exciting features of new engine designs. This is especially true nowadays when improved engine efficiency can produce a realistic saving of fuel and therefore simultaneously cutting fuel costs. A comparison of "P-V" as well as "T-S" diagrams was done by Prof. G. Walker (see Fig. 2) for both Stirling and Carnot cycles which were superimposed with common values for the maximum and minimum volumes, temperatures and pressures.

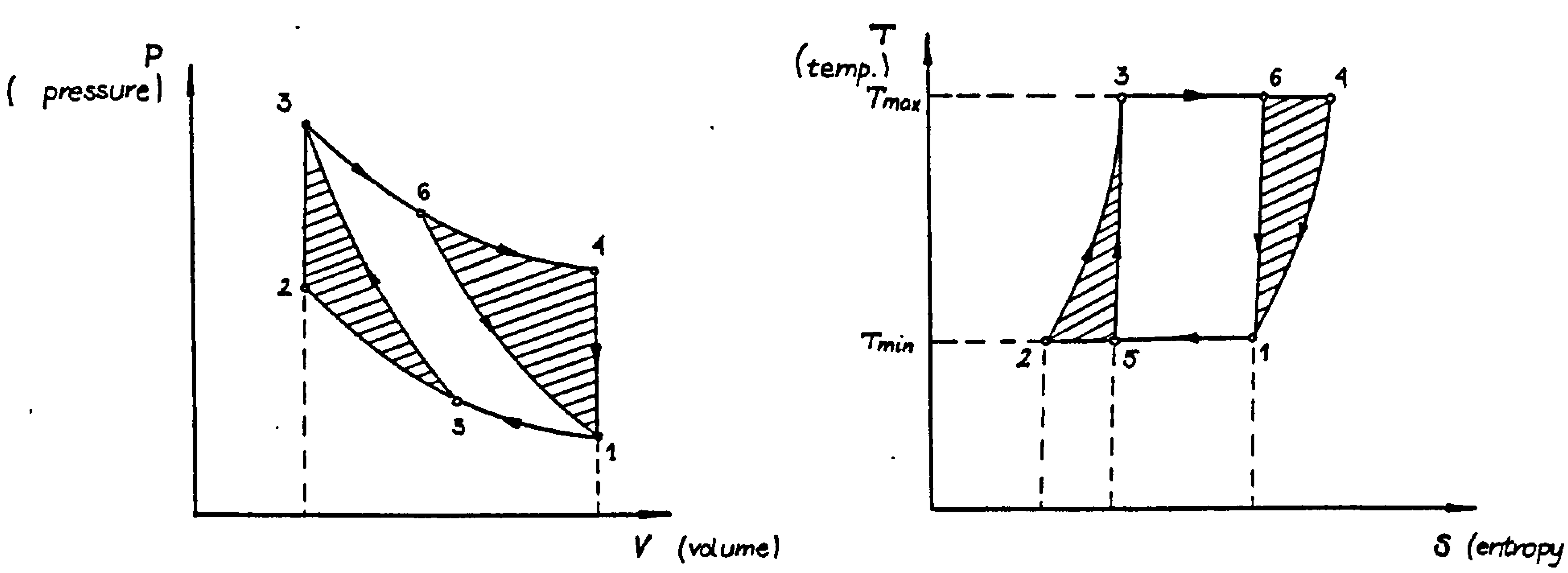


Fig.2

Shaded areas on the P-V plane represent the increased work output of the Stirling Cycle and on the T-S, the increased heat transfer of the Stirling Cycle. The additional work made (the areas 5/2/3 and 1/6/4) was available by substituting constant volume processes for isentropic processes.

The total efficiency (heat which is converted to work) is the same in both Stirling and Carnot Cycles. If the regenerative heating/cooling of the working medium during processes 2-3/4-1 are equal in magnitude the Stirling Cycle thermal efficiency can be expressed with a formula known for thermal CARNOT efficiency.

$$\text{efficiency } \eta = 1 - \frac{T_{min}}{T_{max}}$$

and the Stirling Cycle work output will be greater than for the Carnot engine.

Most types of Stirling Engines can be classified into three basic groups:
a) Piston -DISPLACER in the same cylinder
b) Piston -DISPLACER in seperate cylinders
c) Two piston Machine.

These three basic arrangements are shown in Fig. 3.

Assuming an ideal Stirling Cycle with particular ref. to Fig.1. the principle of this cycle's operations can be explained using a simple set of energy equations which are presented here in a very short form and a more sophisticated thermodynamical analysis can be found in Lit. 1,2,3,4. As mentioned before the whole cycle of Stirling Engine operations can be divided into four cyclic processes (after Walker; Lit.1.)

(a) Isothermal Compression Process (1-2) where work is done on the working fluid, and this work is equal in magnitude to the heat rejected from the cycle. There is a decrease in entropy but no change in internal energy. Heat which is extracted from the working medium is rejected from the cycle at the Tmax (maximum cycle temperature).

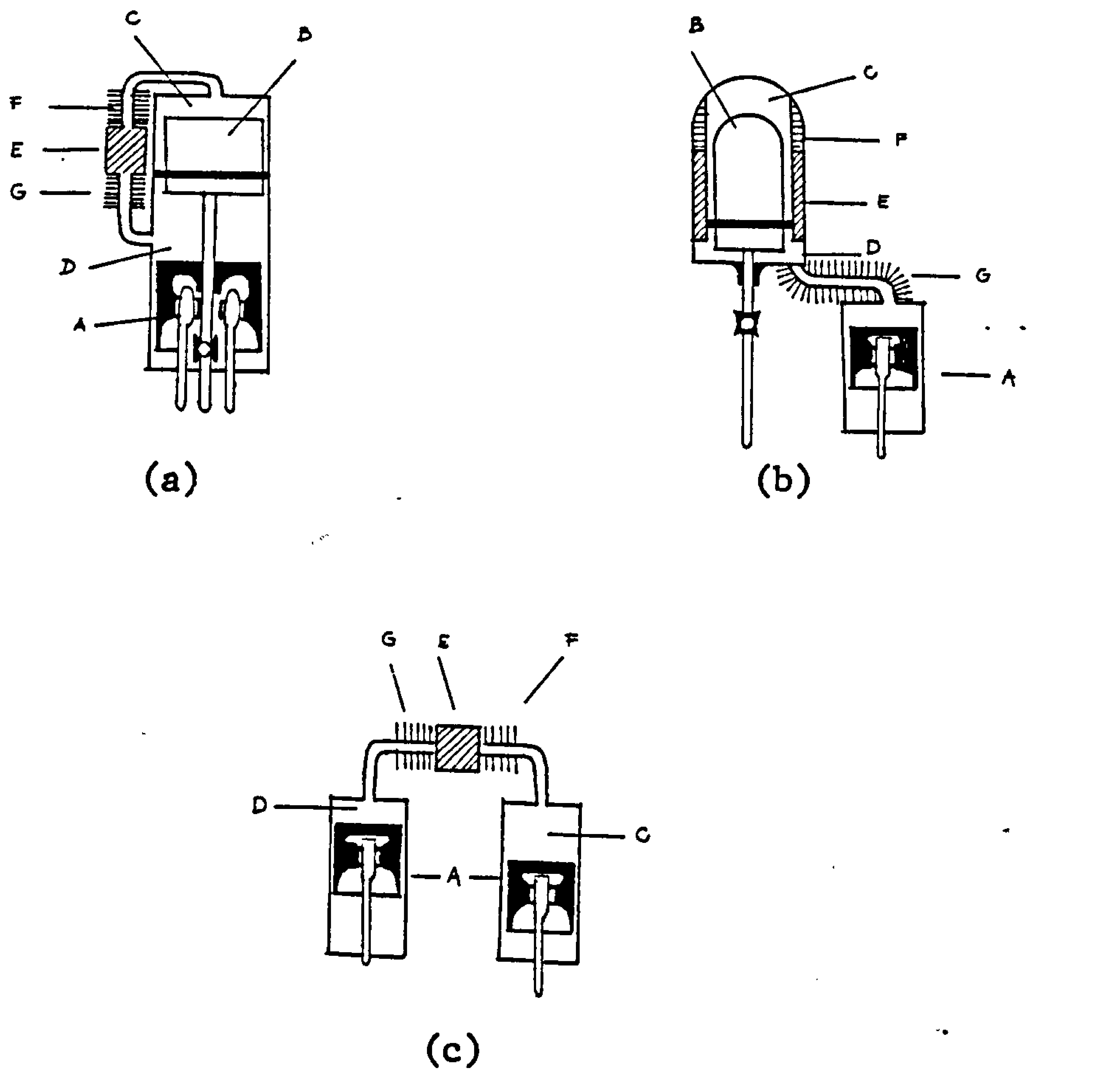


Fig. 3. Three basic arrangements by which most types of Stirling engine may be calculated (after Walker).

(a) Piston-displacer in the same cylinder
(b) Piston-displacer in seperate cylinders
(c) Two-piston machine

A - piston, B - displacer, C - expansion space
D - compression space, E - regenerator,
F - heater, G - cooler.

Assuming that :

1) $\tau$ = Tmin/Tmax = Temperature Ratio
2) r = Vmax/Vmin = Volume Ratio
3) Working medium is the perfect gas.
$V_1 = RT_1/P_1$ (characteristic gas equation)

Heat Transfer (Q) = work done (W) = $p_1V_1 \ln (1/r) = RT$

$P_2 = P_1V_1/V_2 = P_1 . r$

$T_2 = T_1 = Tmin$

change in entropy $(S_2 - S_1) = R \ln (1/r)$

(b) <u>Constant Volume Regnerative Transfer process (2-3)</u> where heat from regenerative matrix is transferred to the working medium. The temperature is increased from Tmin to Tmax with NO work done but with increase in the entropy and internal energy of the working medium.

Heat Transfer $Q = C_V (T_3 - T_2)$

Work done $W = 0$

$P_3 = P_2T_3/T_2 = P_2/\tau$

$V_3 = V_2$

change in entropy $(S_3 - S_2) = C_v \ln (1/\tau)$

(c) <u>Isothermal expansion process (3-4)</u>
At Tmax heat is supplied to the cycle during expansion of the working medium. Heat supplied is equal in magnitude to work done but there is no change in internal energy, however working medium possesses a higher entropy level.

Heat Transfer Q = work done W = $P_3V_3 \ln r = RT_3 \ln r$

$T_4 = T_3 = Tmax$

$P_4 = P_3V_3/V_4 = P_3 (1/r)$

Change in entropy $(S_4 - S_3) = R \ln r$

(d) Constant Volume regenerative transfer process (4-1)
In this process the regenerative matrix possesses a higher temperature level as heat is transferred from the working medium decreasing, because of this, its own temperature from Tmax to Tmin. Internal energy and entropy of the working medium are moved into the higher level and NO work is done.

Heat Transfer $Q = C_v (T_1 - T_4)$

$P_1 = P_4 T_4 / T_1 = P_1 \tau$

$V_1 = V_4$

Change in entropy $(S_1 - S_4) = C_v \ln \tau$

In conlcusion to the above, there is no net gain/loss of heat by the working medium or matrix as heat transferred from the matrix to the working medium in process 2-3 is completely restored from the working medium to the matrix in process (4-1) and thermal efficiency = $\frac{\text{heat supplied-heat rejected}}{\text{heat supplied}}$

$$= (RT_3 \ln r - RT_1 \ln r) / RT_3 \ln r = 1 - \tau$$

which corresponds to the Carnot efficiency between the same temperature limits. This is, of course, a purely idealised theoretical analysis of Stirling Cycle and with all its hypothetical assumptions and consequences. An improved theory in which both pistons move with simple harmonic motion was published by Gustav von Schmidt in 1871 (Lit.5) and has become the classical theoretical analysis of the Stirling Cycle. It was assumed by Schmidt that both compression/expansion temperatures remain constant and the regenerative process is perfect. The Schmidt theory can still be a useful tool for a preliminary engine design, however this type of Stirling Cycle analysis is not too accurate and therefore is of a limited value nowadays.

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## 1.3. REVIEW OF RECENT RESEARCH TOPICS

Modern technical facilities and current thermodynamic and aerodynamic knowledge have made it possible to make the old-fashioned hot-air engine a promising modern source of power, and one that is suitable for any level of power from a few hundred Watts to the Megawatt range. Most modern prototypes are of the displacer type using helium or hydrogen as a working medium, at 110 atm. average pressure and 700 - 900 $^{o}$C heater temperature, and the use of specially developed heat-resistant and creep-resistant materials will make it possible to double the pressure at 800$^{o}$C (see Lit. 1). The efficiency and specific power output are now equal to those of Diesel Engines but will according to expectations, exceed those of all thermal prime movers.

One of the promising avenues is the use of atomic energy. Radio-isotope heat sources could, for example, be used for low powers (up to 10KW) and combination type engines, when using a reactor might be suitable for a higher power range. Prototypes of the radio-isotopes heated engines were built (see Lit.1) and the two alternative methods of coupling the engine to the heat source are shown in Fig. 1 (a and b).

The heat transporting medium (liquid metal or salt) would serve partly as a heat store and this could be extended as desired. It would also be possible to insert the radio-isotope (for example Cobalt 60) inside the Stirling engine's working space, thus providing direct contact between the working gas and the heat source. Such an engine would be extremely compact and could find a lot of applications, for example; for underwater application (observation station), space travel (power supply systems), weather stations and ship propulsion including research and naval submarines.

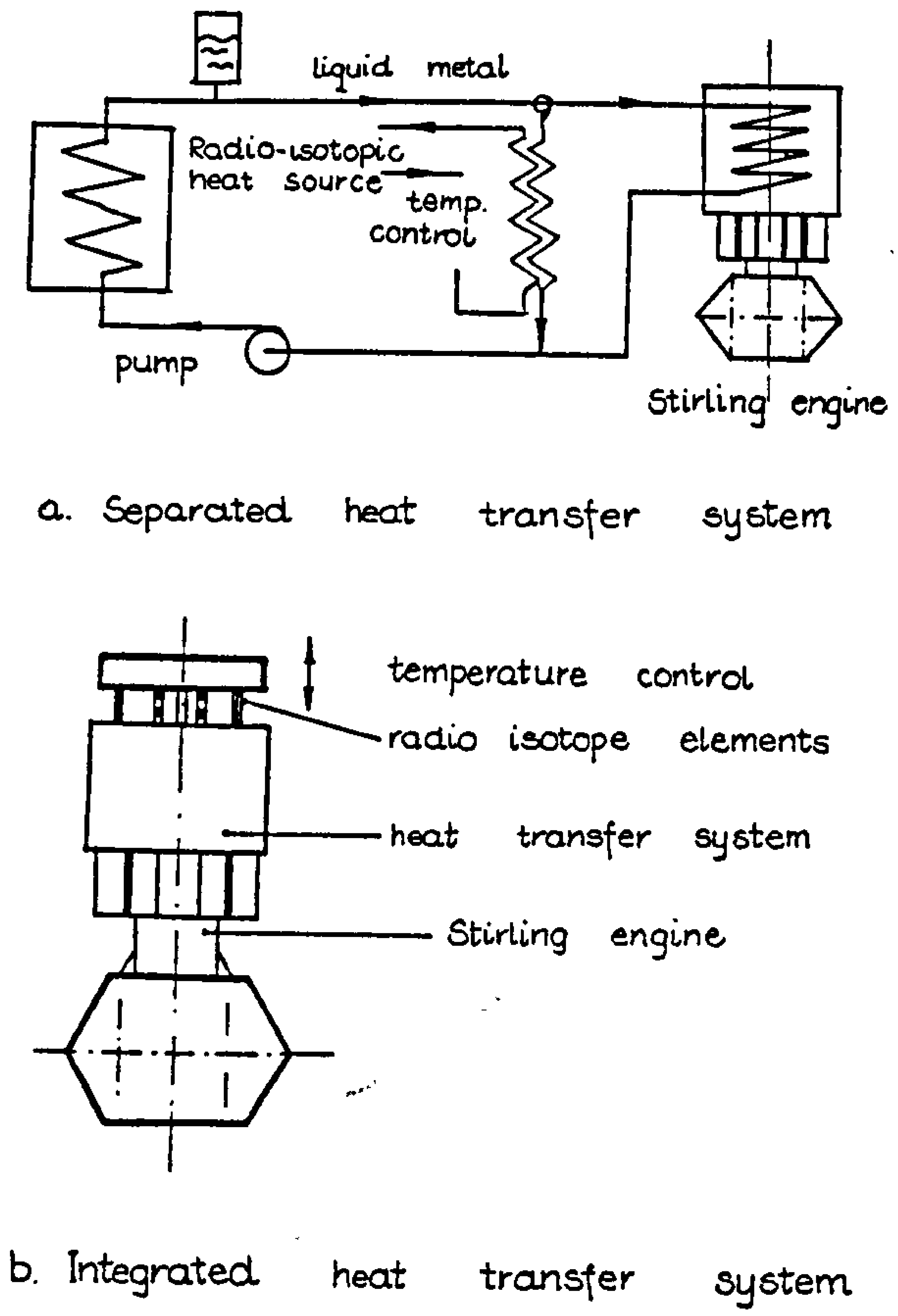


Fig. 1. Possibilities for radio-isotopic heating of Stirling engine.

The sun may also be used as a source of energy. Heat collected can be supplied to the engine via a liquid metal circuit. General Motors (Allison Division) demonstrated this practically with a Stirling engine designed for a satellite. (see Lit.2).

A very interesting solar engine was constructed by Prof. W.T. Beale (see Lit. 3). For effective operation on solar energy, the engine will require an accurate paraboloid reflector and a tracking mount. The reflector must be able to deliver about 700 Watts to the cavity at 500$^{\circ}$C or more, with the reflector-absorber efficiency of approx. 50%.

Fig. 2 shows s solar Stirling engine system which would require a tracker, clockwork being the simplest, but probably ineffective compared to an active sun seeker.

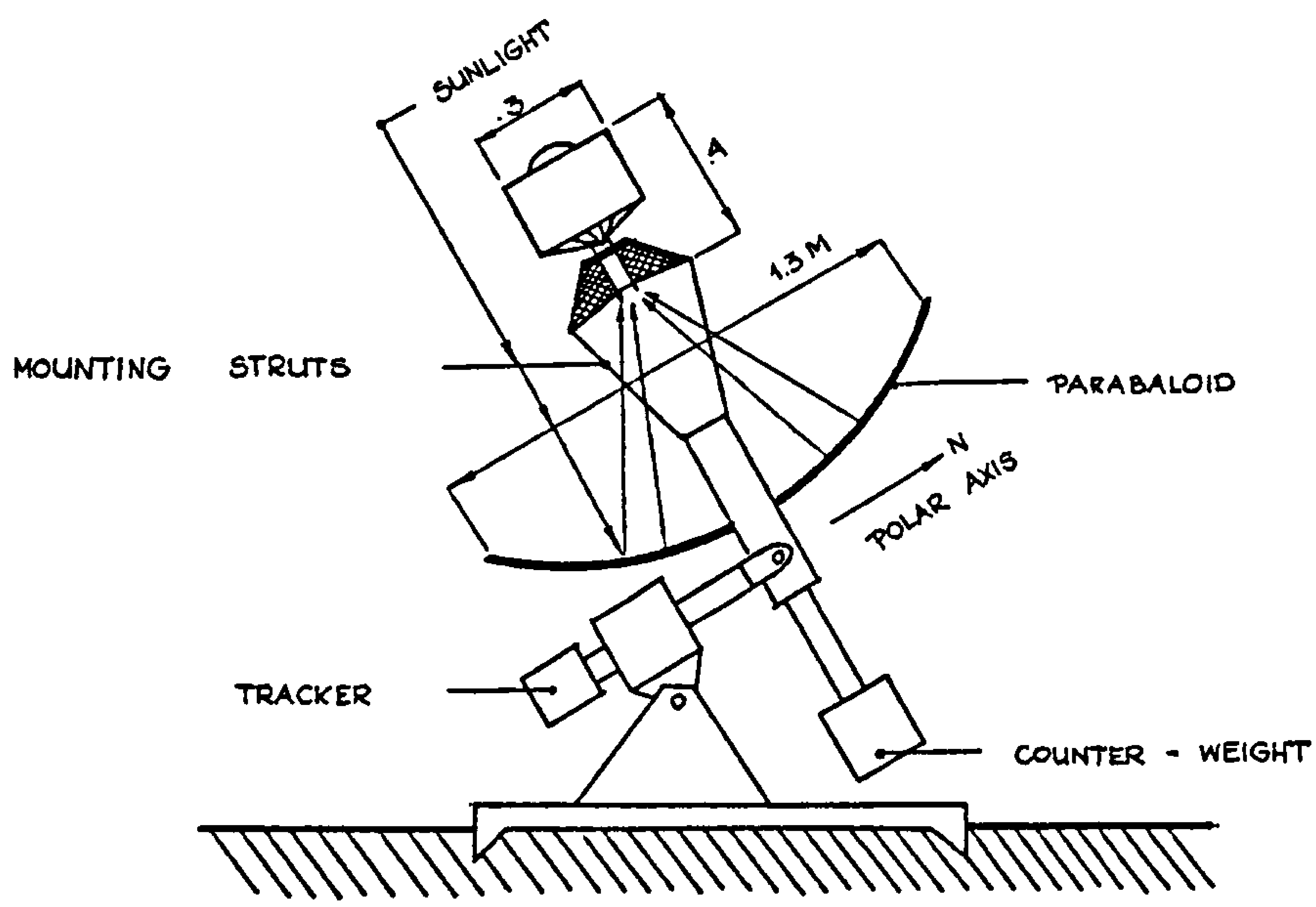


Fig.2. Solar Stirling Engine.

The Stirling Engine used in Prof. Beale's project was a free piston engine which has been under development for over 10 years. In this particular engine, power is taken from the motion of a totally sealed cylinder as it reacts against its'heavy' piston. These are very simple and durable engines and are well adapted for use as solar powered systems. Fig. 3 shows the basic free-piston engine alternator.

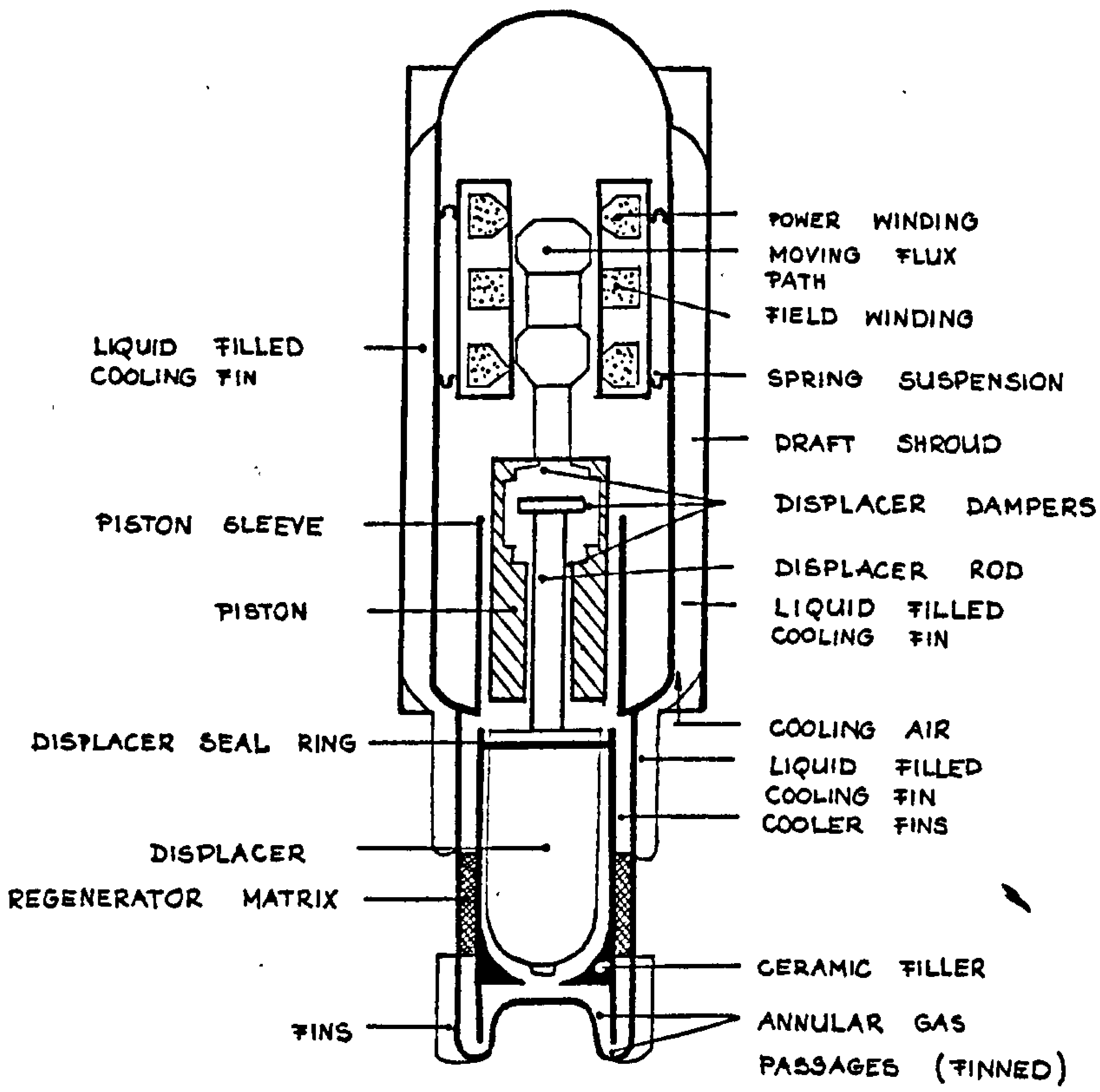


Fig. 3 "Free-piston" engine- alternator

The heat absorbing surface is a cup similar to that used in the original 1816 Stirling engine, which is a convenient form for low power engines and is adaptable to both solar and fuel heat sources.

Recently announced two-component, two-phase working fluid is being studied at the University of CALAGARY, with a view to using it for increasing specific output, as an alternative to using extreme pressurization. While the volume excursion increases and the P-V diagram is larger, thus more work is produced and the specific power output is largely increased. Supplementary advantages of the two-phase working fluid might include improved heat-transfer and a closer approach to isothermal compression and expansion, resulting from the vapourization and condensation processes.

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## 1.4. INVESTIGATION INTO THE POSSIBLE AUTOMOTIVE STIRLING ENGINE APPLICATIONS

By the 1960's the Stirling Engine developed in Philips Laboratories in Eindhoven had led to the construction of prototype engines having power outputs of up to 400 KW with a volume, efficiency and weight comparable to that of a Diesel engine of a similiar power output and nominal speed (see Lit. 1), subsequently Philips granted licencees to GENERAL MOTORS (USA) and the M.A.N. concern (West Germany), UNITED STIRLING (Sweden) AB & Co. As a consequence new alternatives arrived in employing the external combustion Stirling Cycle Engine for automotive applications because of its attractive pollution facilities, quiet and exceptionally efficient level.The almost unmeasurably low emissions of the Stirling engine were highly attractive to FORD (USA) and tests carried out in the Philips Laboratories evaluated a single-cylinder test engine for emissions and extrapolated the results to a 4000 lb car running under specified test conditions. The results include the effect of exhaust gas recirculation to reduce the $NO_x$ of the Stirling Philips engine.

See Tab. 1.

Tab.1.

Emissions in (Grams/Mile) (After Lit. 2)

| | (HC) | (CO) | ($NO_x$) |
|---|---|---|---|
| 1. 1976 U.S. Federal Standards | 0.41 | 3.4 | 0.40 |
| 2. Philips Stirling Engine Test (cold start not included) | 0.018 | 0.60 | 0.20 |
| 3. Lear Vapour turbine (based on bus combustor) | 0.10 | 2.0 | 0.35 |
| 4. TOYO KOGYO "WANKEL" (1975 experimental average) | 0.17 | 2.2 | 0.93 |

These figures were slighly changed when tests were carried out, including a cold engine start, but even so it performed very well. Another interesting statistic showing the comparative rates of emission products per horsepower (see Tab.2) were published by Prof. Meijer of Philips in 1968 (see Lit.5)

Tab.2

| Compound $mgs^{-1}(h,p)^{-1}$ | Stirling Cycle Engine | Gas Turbine | Diesel Engine |
|---|---|---|---|
| CO | 0.1 - 0.3 | 2.0 - 3.6 | 0.2 - 5.0 |
| $C_xH_y$ | 0.003-0.006 | 0.036 | 0.6 -12.0 |
| $NO_x$ | 0.7 -0.02 | 0.7 -2.0 | 0.4 - 2.0 |

The absence of carbon monoxide as well as unburned hydrocarbons in the Stirling Engine exhaust is due to the fact that the combustion processes take place continuously in a hot walled chamber and air may be supplied in any access of quantity required thus eliminating unburned residual gas. On the other hand, it is extremely interesting to compare the typical Stirling engine and other types of popular car-engines from the heat balance point of view (after Lit.3) - Tab. 3.

Tab. 3

| | Diesel | Stirling | Wankel |
|---|---|---|---|
| Effective Energy | 37% | 36% | 27% |
| Exhaust gases | 35% | 9% | 52% |
| Cooling Water | 19% | 47% | 17% |
| Friction | 7% | 4% | 4% |
| Auxiliaries | 2% | 4% | - |

Heat balances for both Stirling and Diesel engines can be presented in a graphical form (see Lit.6) in Fig. 1.

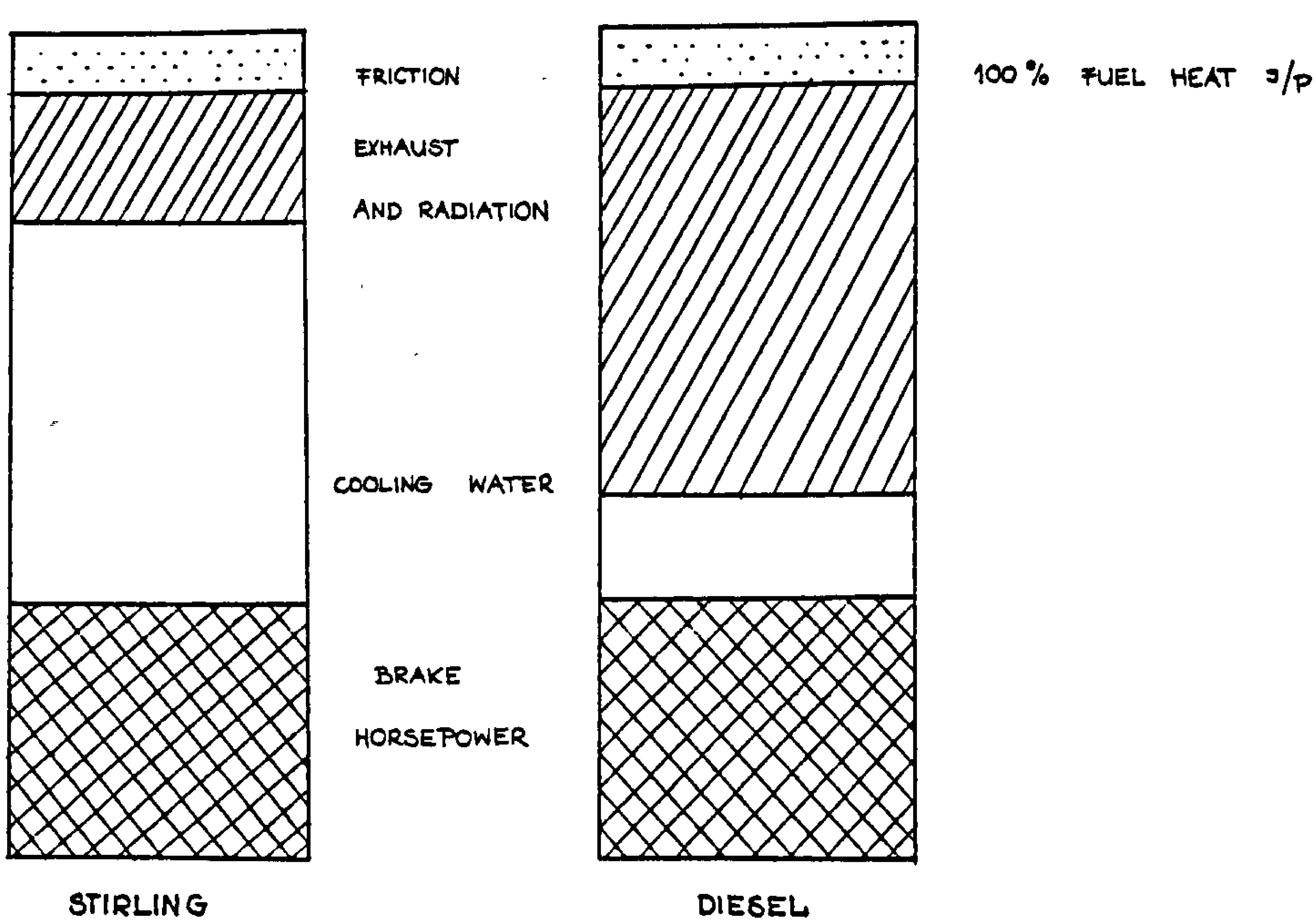



Fig.1. Comparison of heat balance for Stirling and Diesel engines.

Other interesting points of Stirling Engines for automobile applications were stated by other scientists, (Neelen, Ortegren, Kuhlman & Zacharias) in 1971 (See Lit.5)

1. Very low level of vibrations by any car engine standards (compared with Diesel approx. 20 - 40 dB) mainly because of the absence of any valves and periodic explosions. This allows for a nearly perfect balancing.
2. The specific power output and efficiency of the Stirling Engine is comparable to a Diesel cycle with a very good efficiency characteristic for part-load conditions.
3. Engine Breaking can be done, simply by negative torque up to 80% of full load torque and Torque characteristic as well as speed characteristics are satisfactory on actual experiments.
4. There is a very low oil consumption (practically nil) simple maintenance, high level of reliability with very long service life.
5. Finally - a very useful feature - the multifuel capability An important fact in these days of fuel crisis.

During 1970 -1971 the new 4-64, 60 bhp Philips engine was constructed and tested. From the constructional point of view, it was the first car Stirling engine - double acting cylinder type, used more effectively with a special swash plate drive system in which a rotating angled disc driven by reciprocating rods through sliding bearings replaces the usual crankshaft and connecting rods. (see Fig.2)

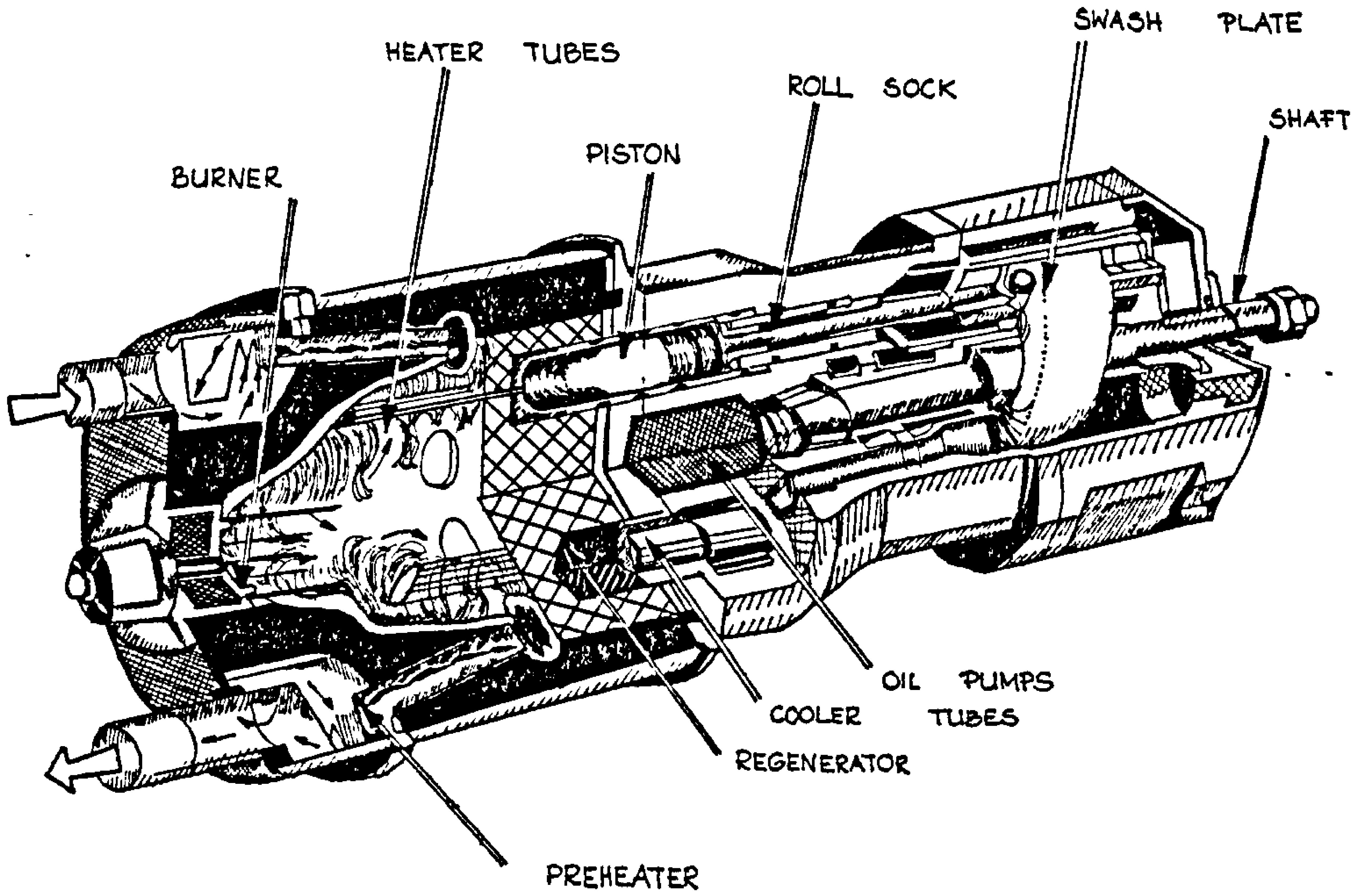



Fig.2 Cutaway drawing of the four cylinder,double-acting swashplate drive Philips Stirling engine.

The engine's outside dimensions were three feet in length and about a foot and a quarter in its basic diameter. Its operational speed was up to 5000 RPM which proved during several thousand hours of bench tests, both the double acting principle and the swashplate drive for the Stirling car engine. Philips developed a whole family of successful Stirling Engines and some of them reached quite an advanced stage of development. Most of them are highly pressurized and compact in design, where the Philips 4/235 Stirling Engine performance can be taken as an example of modern Philips design. A summary of technical features of the Philips 4/235 Stirling engine (after Walker Lit.5) is presented in Tab. 4 below.

Tab. 4

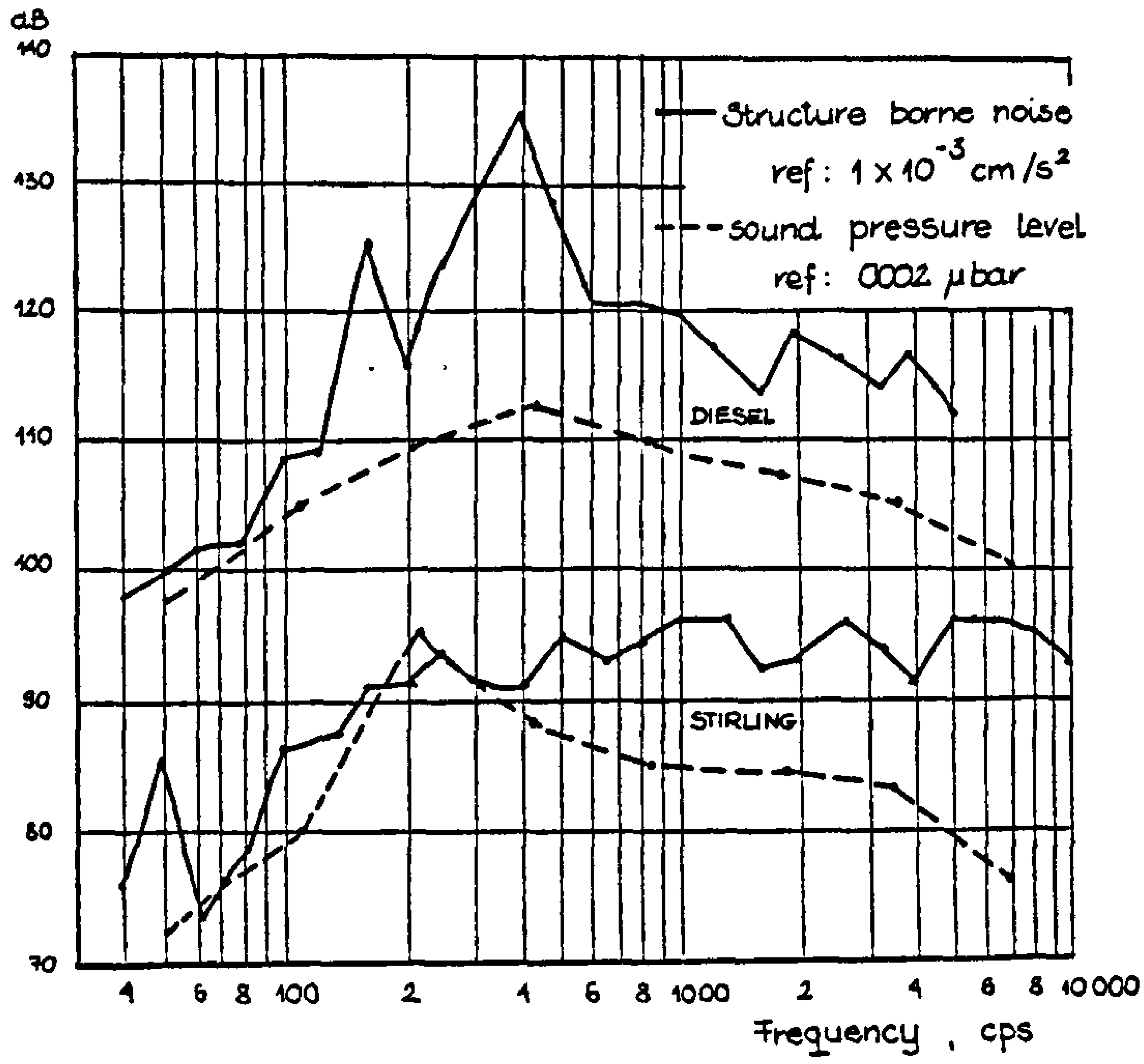
| | |
|---|---|
| Number of cylinders | 4 |
| Combustion system | External |
| Fuel | Diesel fuel |
| Bore & Stroke | Ø 77.5mm x 49.8mm |
| Total piston displacement | 940 $cm^3$ |
| Working medium | Helium |
| Maximum Output | 200 b.h.p. @ 3000RPM ) final |
| Mean pressure of medium | 220 ATM ) ratings. |
| Maximum Output | 100 b.h.p. @ 3000 RPM ) prelimin- |
| Mean pressure of medium | 110 ATM ) ary ratings |
| Normal Temperature | 700°C |
| Normal Radiator Temperature | 60°C |
| Max. Torque | 35 KGf.m @ 1000RPM |
| Efficiency | approx. 30% @ 100 h.p. |
| Lubrication system | Dry sump with scavenge pump |
| Oil filter | Bypass (replacement element) |
| Cooling System | Water cooled with centrifugal pump |
| Dry weight | 760 Kg |
| Dimensions | 1250mm long x 1100mm high |

The 4/235 Philips Stirling Engine built in 1971 - 72 was originally designed for either horizontal or vertical mounting and in April 1972 was installed in the rear of a D.A.F. bus chasis - becoming the first Stirling power unit to drive a vehicle through a conventional transmission, an automatic VOITH DIVA type 502-3.

Some other Philips licencees, as mentioned before, were sold to United Stirling (Sweden), M.A.N. of Augsburg, (West Germany) and M.W.M. of Mannheim (West Germany). The last two formed the Stirling Engine Development Group M.A.N. - M.W.M. in the course of research on 4-440 Stirling engine which is a four cylinder, 120 b.h.p. machine, and the group's special point of interest was an engine built with cost reduction as well as improvement in control of the engine in a vehicle. As far as United Stirling (Sweden) AB & Co is concerned - the company was formed in 1968 and started with licences bought from FORD (USA) and Philips, Holland and their program concerns mainly the four cylinder heavy-duty 4 -615 type (200 b.h.p) engine for marine as well as town communication systems use. Apart from those "orthodox" Stirling constructions, new concepts are still being studied, for example, Prof. Meijer's of Philips concept of a small vehicle engine with use of a thermal storage battery that could be charged overnight by electrical heat energy and discharged by daytime use of the engine, thus converting the stored heat to work for driving a vehicle. Prof. Meijer and his team at Philips Laboratories have done a lot of research work on this possibility using Lithium fluoride as the thermal storage medium. They have found this concept suitable for automotive applications as far as commuter cars. city buses, taxis and delivery vans are concerned. As mentioned earlier because of absence of internal combustion a Stirling type engine performs more quietly than most of the other available engines today.

An interesting comparison of structure-borne and air-borne noise from a comparable four cylinder 360 b.h.p. engines for both Diesel and Stirling type of machines (after Lit.6) is shown in Fig. 3.

Fig. 3.




Structure-borne and air-borne noise from a four-cylinder 36o hp engine and that from a comparable diesel.

The chance of prospective Stirling car engine applications is nowadays under tremendous social pressure from environmentalists and new standards for the controlling of air pollution (Clean Air Act 1970 - USA), which should make the engine competitive for both Diesel and petrol engines. It seems highly reasonable to expect a general take-over of Stirling engine on a large scale in the automotive industry about 1985.

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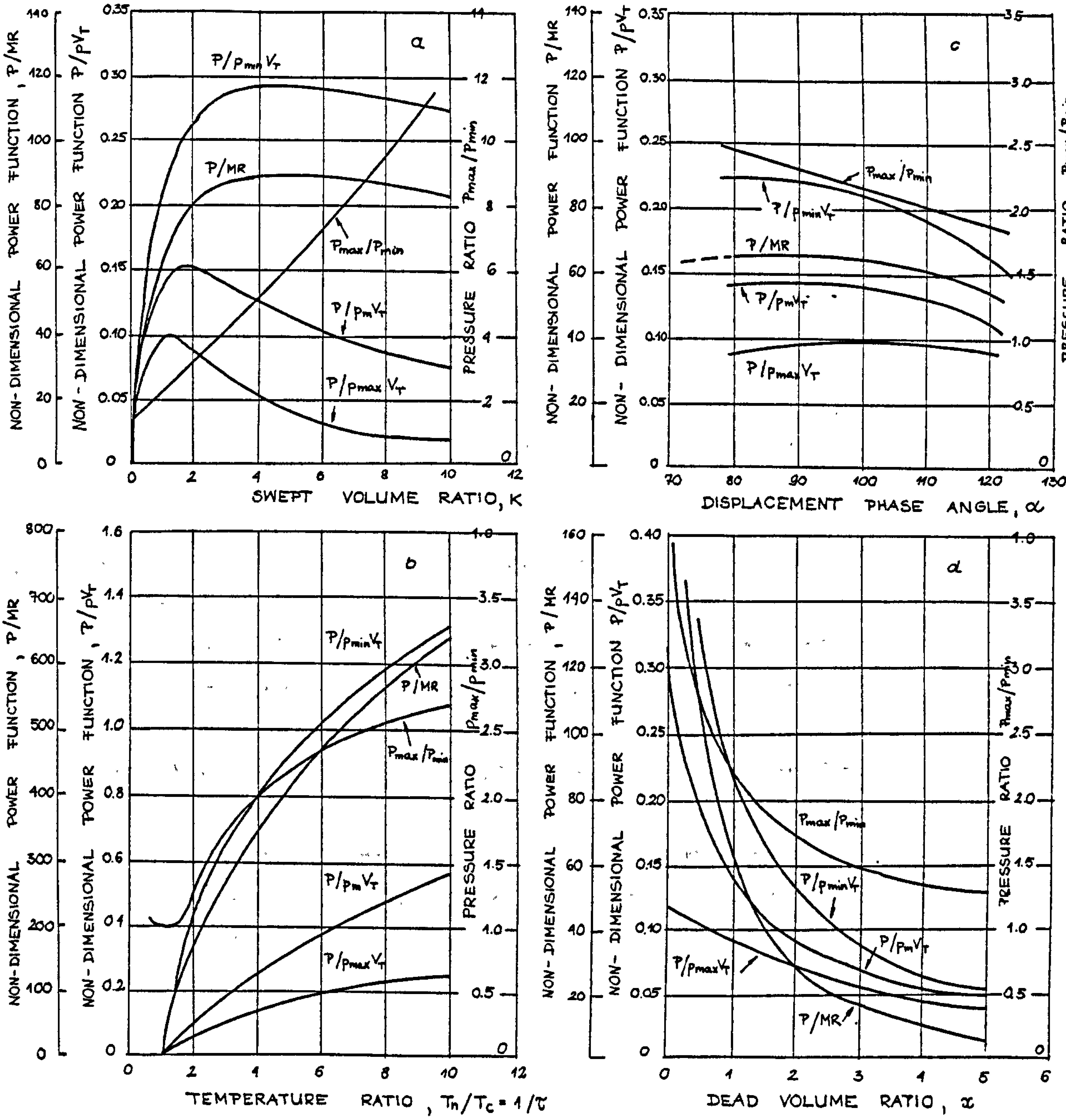
## 1.5. INTRODUCTION TO CONTROL OF STIRLING CYCLE ENGINES

The efficiency of a Stirling Engine calculated using the theory due to Schmidt, corresponds to the theoretical CARNOT efficiency and is unrealistically high. Even so Schmidt's theory demonstrates the significance of certain Stirling Cycle parameters which must be selected in the engine design and carefully considered in any preliminary discussion of the control system proposals. These specific parameters are:

| | | |
|---|---|---|
| 1. | The temperature ratio | $\tau$ |
| 2. | The swept volume ratio | K |
| 3. | The dead volume ratio | $x$ |
| 4. | The phase angle | $\alpha$ |
| 5. | The speed | $\Upsilon$ |
| 6. | The operational pressure | P |

It is possible to derive an equation describing the power output from the engine and hence determine optimum values for certain non-dimensional parameters such as : $\tau$, $\kappa$ , $x$ . Professor Walker proved (See Lit.1.) the above, discussing theoretical aspects of engine performance and some of his findings are presented in Fig.1. Each of the four seperate plots (a,b,c,d) show the effect on the engine's power output with variation in one of the above mentioned four parameters (1,2,3,4), with the values of the other three being fixed, as indicated beneath each of the graphs. For reference, the pressure ratio $\frac{P\ max}{P\ min}$ is also present on all four graphs.

Summing up the performance presented in Fig.1., it is clear that the engine's specific power output is not greatly sensitive to the phase angle $\alpha$ , but does tend to reach a maximum value. According to theoretical studies




<u>Fig.1</u>. Variation in the non-dimensional power and pressure ratio with change in the four main design parameters according to the Schmidt isothermal analysis.

The power has been made non-dimensional on four different bases. It should be noted that the optimum value of or depends on the base selected.

a $\tau = 0.5$ $\alpha = 90^{\circ}$ $x = 1.0$
b $\tau = 0.5$ $\varkappa = 1.0$ $x = 1.0$
c $\varkappa = 1.0$ $\alpha = 90^{\circ}$ $x = 1.0$
d $\tau = 0.5$ $\varkappa = 1.0$ $\alpha = 90^{\circ}$

by Walker, the swept volume ratio $K$, for the best performance should be adjusted in a 1-4 region and the displacement phase angle $\alpha$ should be about $100^{o}$. The other cuves confirm obvious Stirling Cycle facts that the dead space should be small and the temperature ratio should be as high as is consistent with metallurgical limits.

For many years now, constructors were completely convinced that the Stirling Cycle Engine should be ideally used at constant speed runs, i.e. for power generation applications and it was not popular to alter its speed, torque and consequently the power output. At present it is widely accepted that the power output and hence also the engine's torque can be easily controlled, for example, by altering the operational gas pressure of the working medium, while the efficiency remains more or less constant. As an example, Let us consider the practical results as described in Lit.4 and shown in Fig. 2.

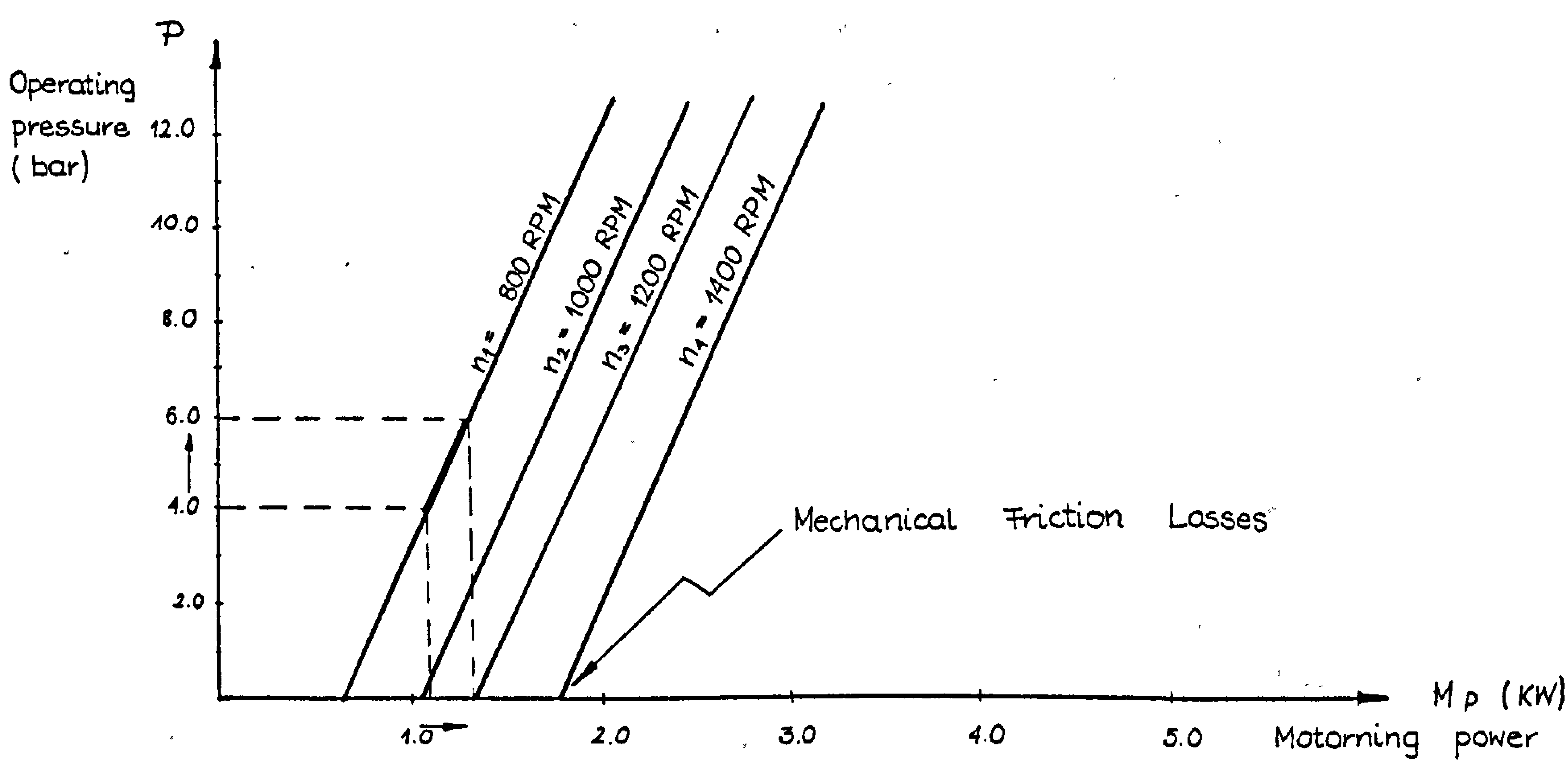



Fig. 2. Experimental Inter-relation between operation pressure motoring power and the engine's RPM speed levels.

This shows the engine's operation pressure (P) in bars expressed as a function of the motoring power (Mp) in KW. Selecting any of the constant speed run characteristics, for example 800 RPMs, and increasing the pressure level, it is evident that a substantial increase of motoring power will occur. The motoring power is believed to be the combined total of mechanical friction work and gaseous pumping work. One possible interpretation of these results , assuming that for each constant speed condition the motoring power line has been extrapolated to the zero operating pressure axis, is that at zero operating pressure, the motoring power of the engine is equal to the mechanical friction work of the engine.

The power regulation of the Stirling engine, which is based on this, easily fulfils all normal control requirements, including those affecting the A.C. generator set, which is a classical example of a constant speed run. As the power increases, the gas is allowed into the working space from a pressurised storage reservoir. As it decreases, gas is pumped back into the reservoir. In order to limit the physical size of the compressors, power should be temporarily dissipated by connecting the working space to another space. The engine torque responds immediately to a change in gas pressure, and acceleration is therefore governed only by the moments of inertia to be overcome (see Lit.2). From practical experiments, we know also, that after the engine has been started, full power can safely be used straight away, which is highly important in all automotive and other applications. As mentioned earlier the engine power can also be controlled by shifting the phase angle between the power piston and the displacer, which allows the direction of rotation to be reversed. The basic difficulty in process control of the Stirling Engine depends on the fact that because of its specific character, the control system will incorporate a number of various sub-control loops acting simultaneously and amongst the most important and widely used are :

1. Air/Fuel control systems
2. Temperature Control systems
3. Power Control Systems

With varying demand for heat in the working cycle of the engine, the Air/Fuel flow is controlled in such a way that heater temperature is kept constant, and for this reason the amounts of combustion air as well as fuel supplied to the combustion system must be governed by a temperature control system. At the same time the ratio of the mass flow rates of combustion air and fuel must also remain constant.
In fact in many commercially available Stirling engines, the air flow was adjusted to obtain the maximum possible cylinder head temperature for a given fuel flow, which ensured optimum utilization of the fuel material. In practice, there is one principle difficulty in realisation of this type of control - the modern Stirling engine's cylinder head temperature is kept at a very high level, for example 1000$^{o}$C. As far as the power control system is concerned, there are many ways of controlling the engine's power output as explained earlier in this section, however, the method depending on the variation of the cycle pressure to which the shaft power is almost proportional, seems superior to others because of :

Simplicity.
Wide control span.
Reasonably compact in comparison with other systems.
Fast and accurate action.
Extensive experience in this type of control.

In a properly designed pressure power control system (MEAN PRESSURE SYSTEM), no torque drop could be measured, and the lag time between accelerator depression and valve response is very short. Also the dead volume power control system seems to be quite an adequate solution in many Stirling engine applications. Its main principle is variation of the cycle amplitudes at constant mean pressure. This is made by connecting different combination of external volumes to the cold space of the cycle when increasing dead volume, decrease of output shaft power should be observed and vice versa. This method was used in a few commercial automotive constructions (see Lit. 3), and drivability was generally good. It was not possible to feel the power steps as the manufacturer claims (see Lit.3) when acceleration position was changed.

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## 1.6. REVIEW OF SELECTED STIRLING CYCLE MACHINE LITERATURE

An example of practical automotive applications of Stirling engine and the successful development of a 150 KW (200 HP) prototype can be taken from the earlier mentioned Swedish company KB United Stirling (Sweden) AB & Co. Their V4X engine is designed for medium duty automobile applications. Since early 1971 (see Lit.1.) this company developed a whole range of Stirling Cycle Machines, i.e.

| | |
|---|---|
| Model: 4-615 | United Stirling (USS) displacer type rhombic drive, 4 cylinder in line engine. Engine displacement is 615cm$^3$ (37.52 cu.in) per cylinder. Net shaft power 140 KW @ 2400 RPM. |
| Model: 4-66 | Philips double-acting swash-plate 4 cylinder engine. Engine displacement 66cm$^3$ (4.03 cu.in) per cylinder. Net shaft power 30 KW @ 3500 RPM. |
| Model: V4XI | USS double-acting V4 experimental engine with next to conventional crankshaft. Engine displacement 66 cm$^3$ (4.03 cu.in) per cylinder. Net shaft power 30 KW @ 3500 RPM. |
| Model: V4X3 | USS double-acting V4 engine, third generation. Engine displacement 90 cm$^3$ (5.51 cu.in) per cylinder. Net shaft power 40 KW @ 3500 RPM. |

| | |
|---|---|
| Model: V4X31 | The FORD PINTO engine. |
| Model: V4X32 | Dead volume power control test engine, later endurance test engine. |
| Model: V4X35 | Mean Power (pressure) control test engine . |
| Model: P150 | Uss double-acting V8 engine, based on two V4 - 75 KW modules in line. Engine displacement is 188 $cm^3$ (11.53 cu.in) per cylinder. Net shaft power 150 KW @ 2400 RPM. Mean pressure power control system. |

The basic concept and part of the subassemblies were developed on a small 40KW (55HP) experimental V4 configuration . The latest version was installed in a FORD TAUNUS car and tested under real road conditions. A simplified cross section of one of the "Swedish" V8 P150 (150KW @ 2400 RPM) automotive engines is shown in Fig.1.

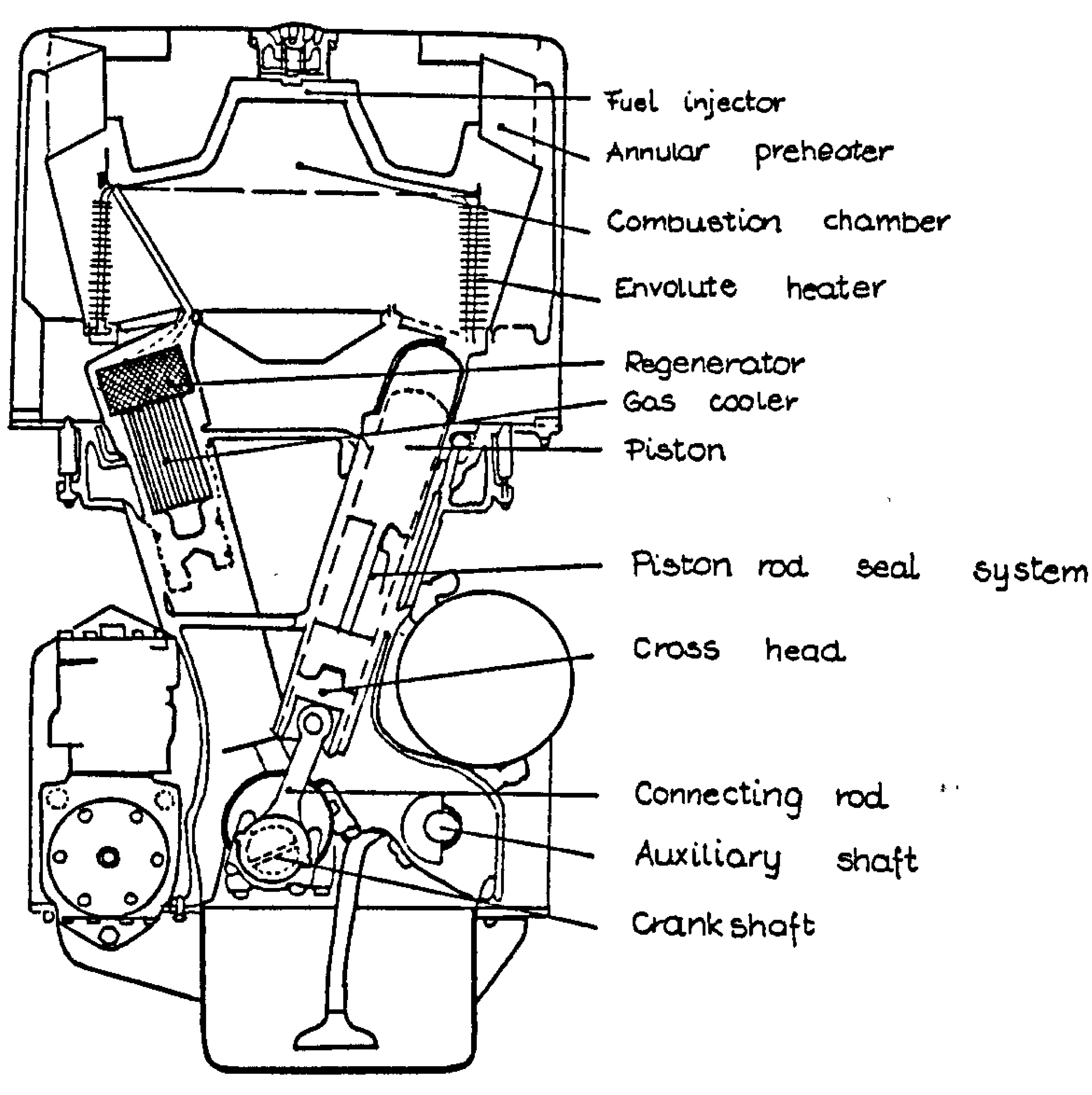



Fig.1. Cross section of double-acting V8 P150 engine developing 150 KW at 2400 rpm.

Most interesting (apart from its highly original mechanical design) for prospective automobile applications seems the two types of control systems developed, namely:

1. Temperature and Air/Fuel control system
2. Power Control system

The first system is schematically presented in Fig.2.

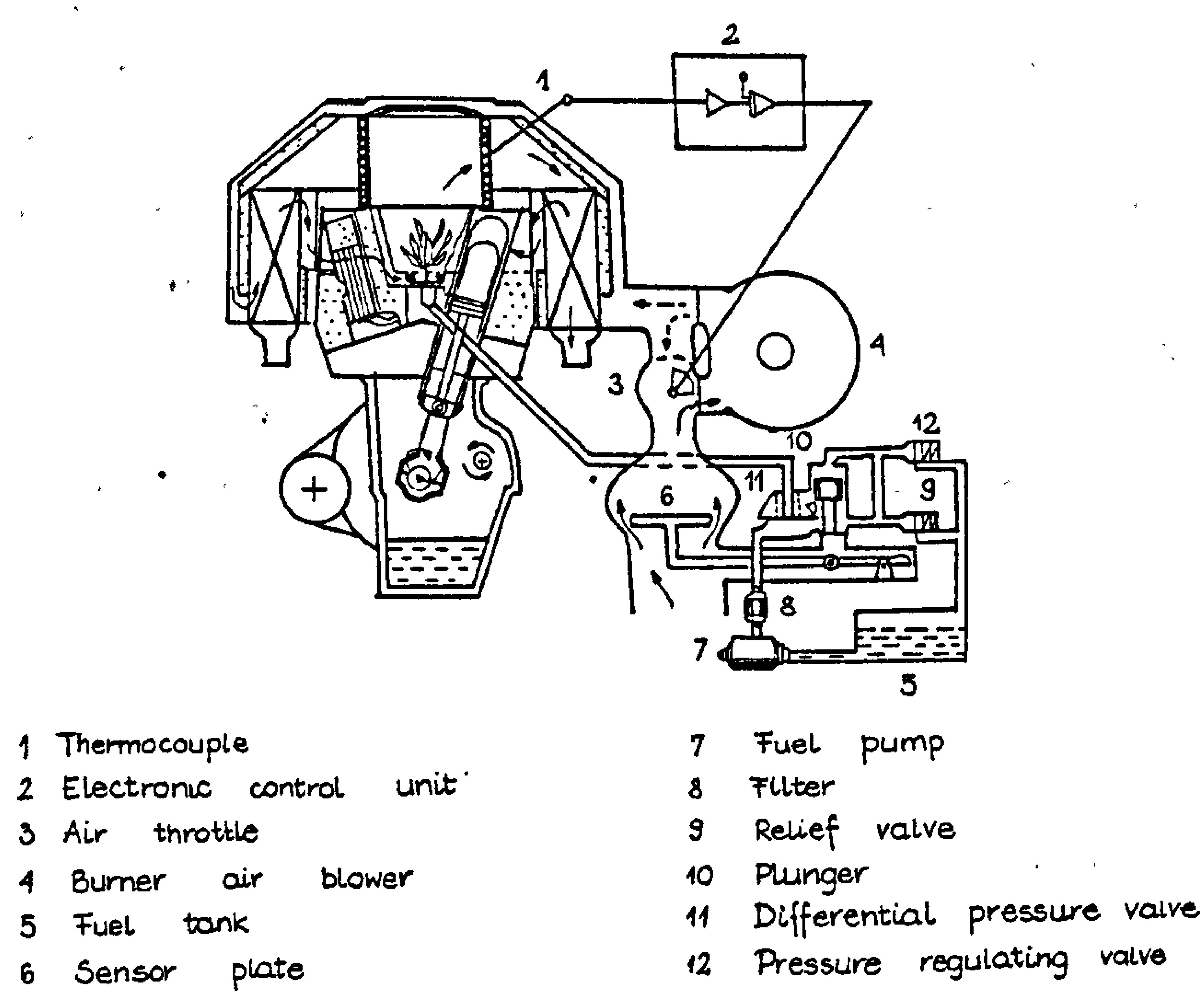



Fig.2 Temperature and air-fuel control

As mentioned earlier, with varying demand of heat in the working cycle, the air/fuel flow is controlled in such a way that the heater temperature is kept virtually constant, therefore the air/fuel control is individually governed by the power control system. In addition the air/fuel ratio is controlled with regard to emissions. Referring to Fig.2. the temperature of the heater tube is measured by a thermocouple (1). The signal of the thermocouple is amplified and converted in the electronic controller (2) into a signal controlling the position of the air throttle (3). The appropriate amount of air is delivered to the combustion chamber via the burner and air blower (4). The fuel from the tank (5) passes an electric pump (7) fitted with a fuel filter (8). The fuel pressure is held approx constant by a relief valve (9), and the relative position of the sensor plate controls, via a plunger (10), the amount by which a fuel metering port is opened. The fuel flow to the atomizer depends upon the amount the fuel metering port is opened only as the differential pressure across the metering port is maintained at a constant value by a valve (11). Finally, the air/fuel ratio depends upon the hydraulic counter pressure controlled by a pressure regulating valve (12). If any adjustment of ratio over the entire load range is required this can be achieved by a modification of the shape of the conical air passage.

The second - the power control system, is based on the fact that the power output of the Stirling Cycle machine can be controlled by varying the mean cycle pressure to which the shaft power is approximately proportional. The schematic diagram of a simplified mean pressure power control system is shown in Fig. 3.

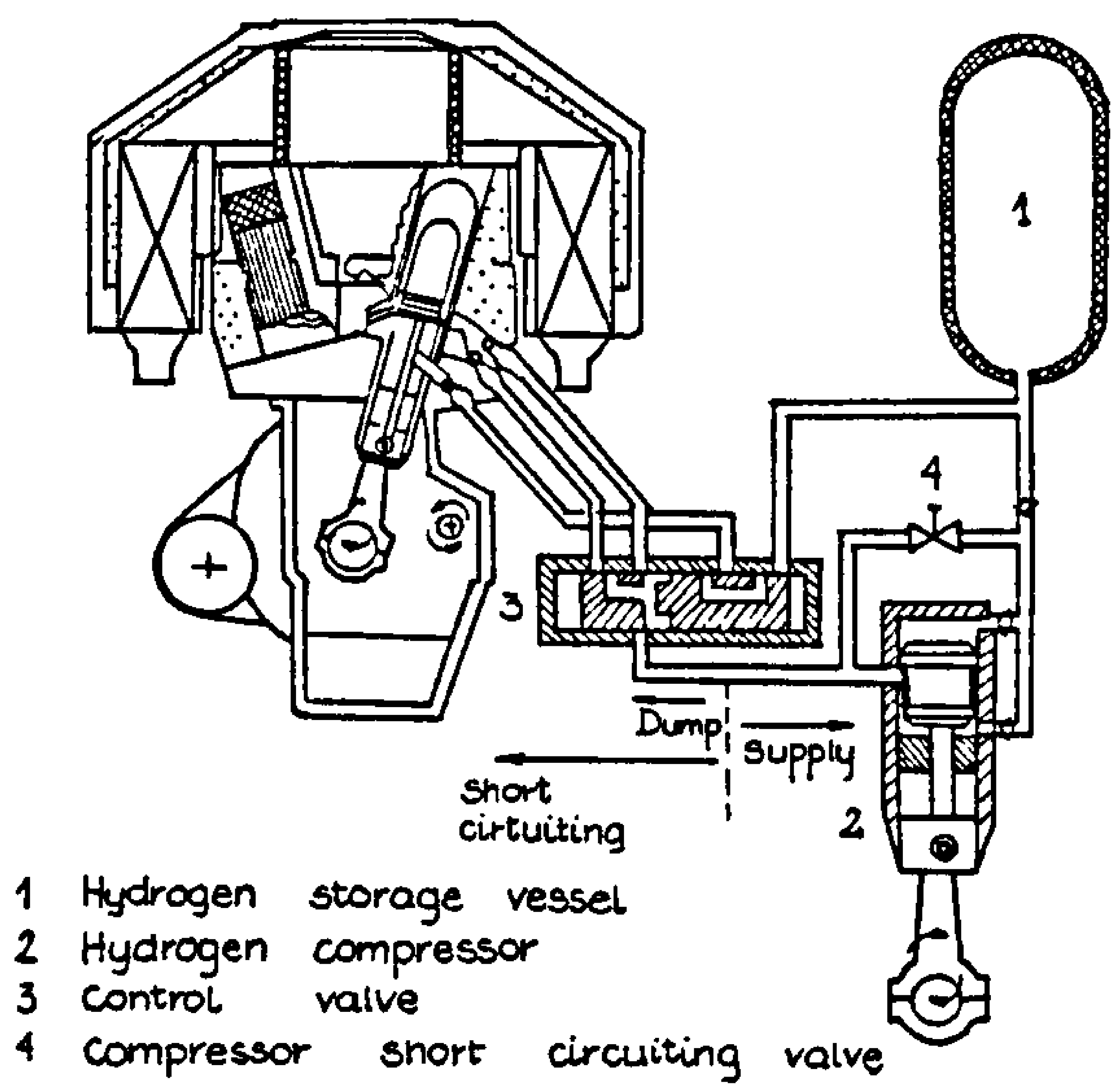
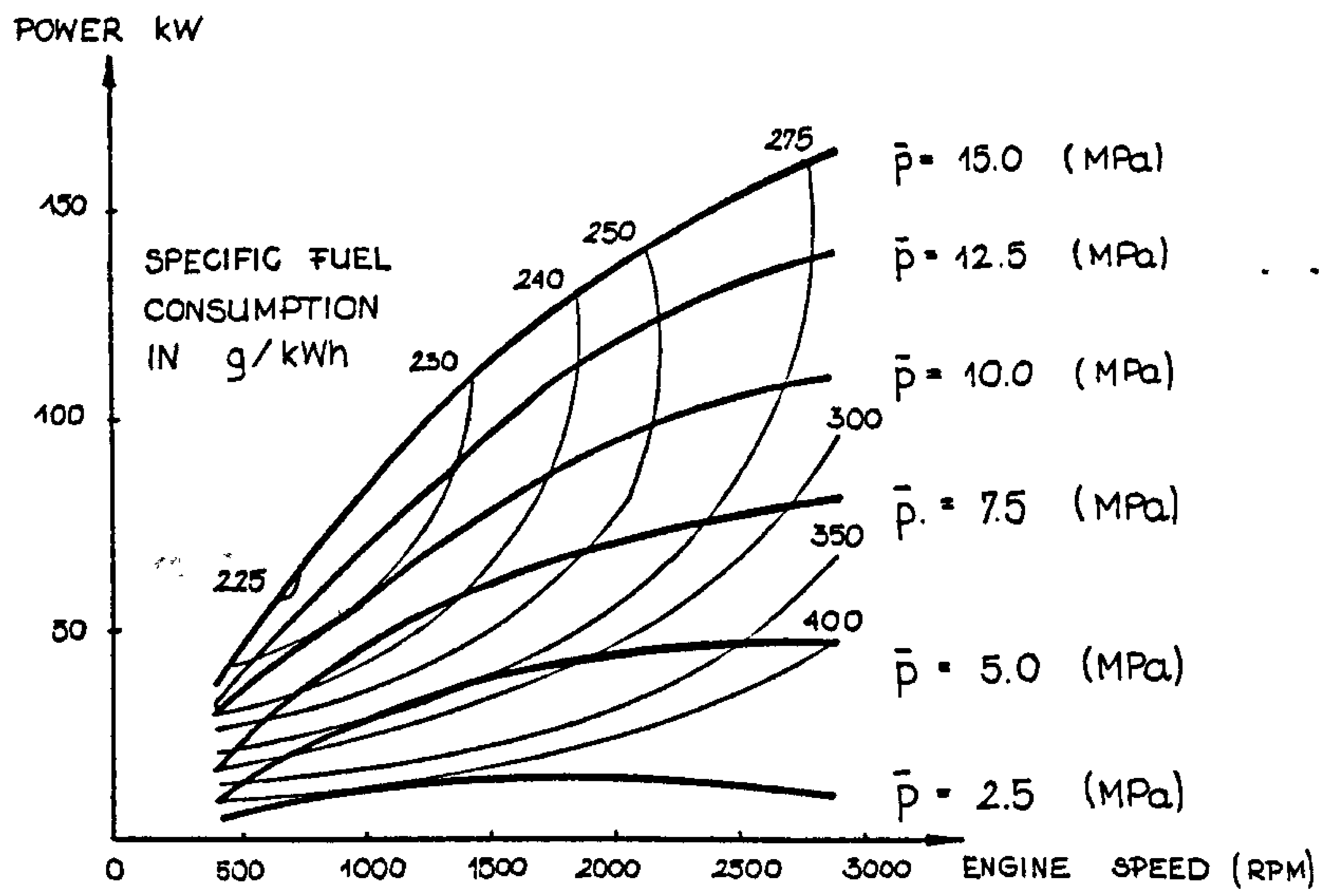



Fig. 3. Simplified diagram of the power control system.

The most important components of this system are :

- hydrogen storage vessel
- hydrogen compressor
- control valve block
- servosystem which controls the position

of the control valve. In order to increase the output power, the control valve slide is moved to the right(Fig.3), and hydrogen flows from the high pressure storage vessel to a timed supply system built into the engine. The timed supply system supplies hydrogen to the working cylinders mainly when the mean pressure is close to its maximum level. In order to decrease power the slide should be moved to the left (Fig.3). The link between the accelerator pedal and control valve is a servosystem one, which for different accelerator settings moves the control valve slide, resulting in a corresponding pressure change, which will correspond to the required power output level. The mean pressure power control system performance of the P150 Stirling Engine is shown in Fig.4.




Mean Pressure Power Control
Radiator fan driven, performance related to ambient air Temp = 30°C and altitude = 150m

Fig.4.

Simultaneously N.V. Philips of Holland had been working with the FORD motor company developing their 170HP, four cylinder-double acting 4-215D.A. Stirling Engine with a swash-plate drive mechanism suitable for passenger car installations. This co-operation produced a lot of valuable results (see Lit.2), and as far as the engine's control side is concerned, a similiar temperature and air/fuel control system as well as power control systems were developed. Both control systems being specifically designed for automobile applications. A cross-section of the 4-215 D.A. Stirling engine is shown in Fig.5.

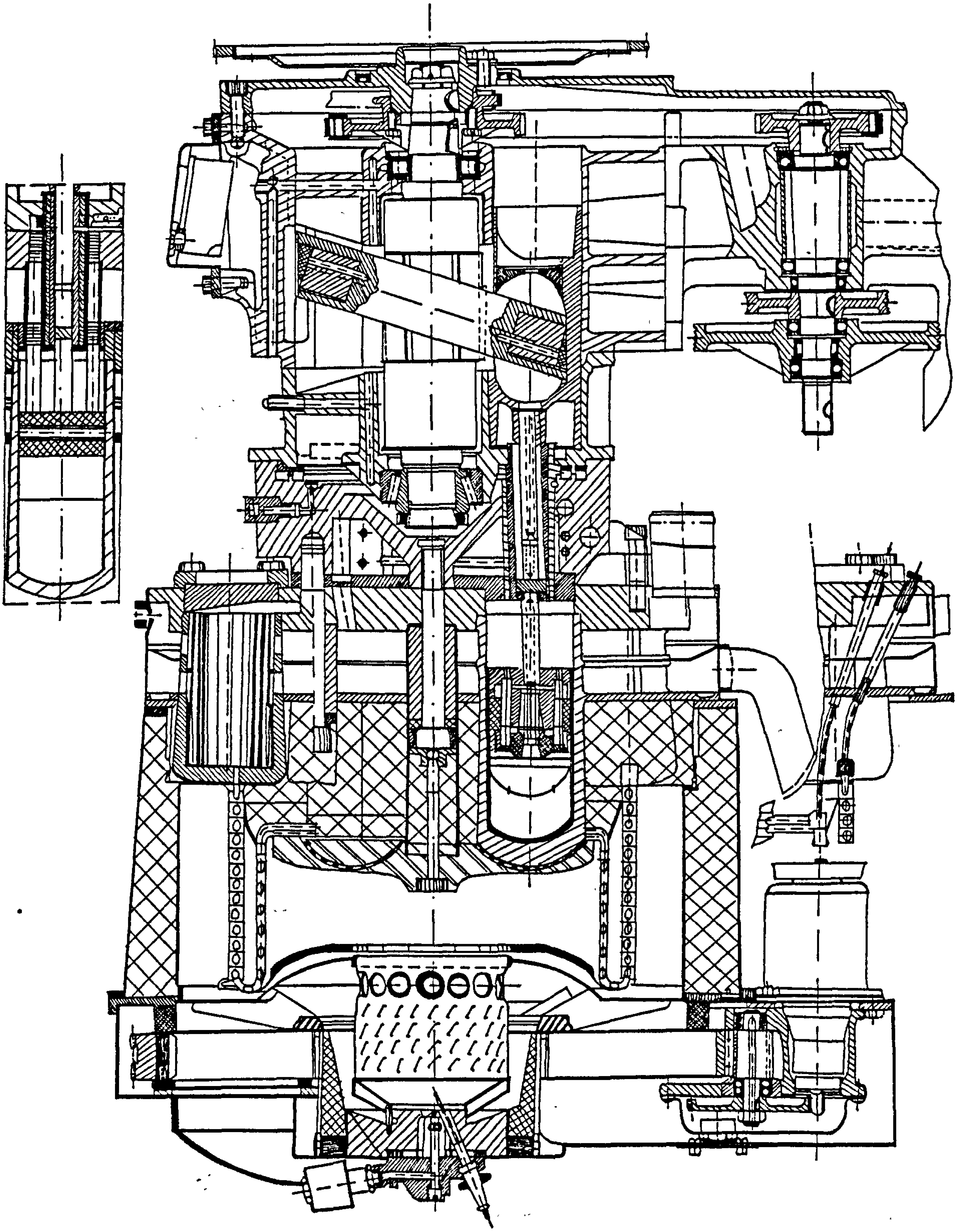


Fig.5. Cross section of the 4 - 215 D.A. Stirling engine.

The temperature and air/fuel control system basically controls the specific heat input of the engine so that the temperature of the working gas in the heater tubes is kept at a desired constant level and therefore the amounts of combustion air and fuel are governed by a special temperature control system with the ratio of air to fuel remaining constant at approximately 20:1. The block diagram for this type of control system is shown in Fig. 6.

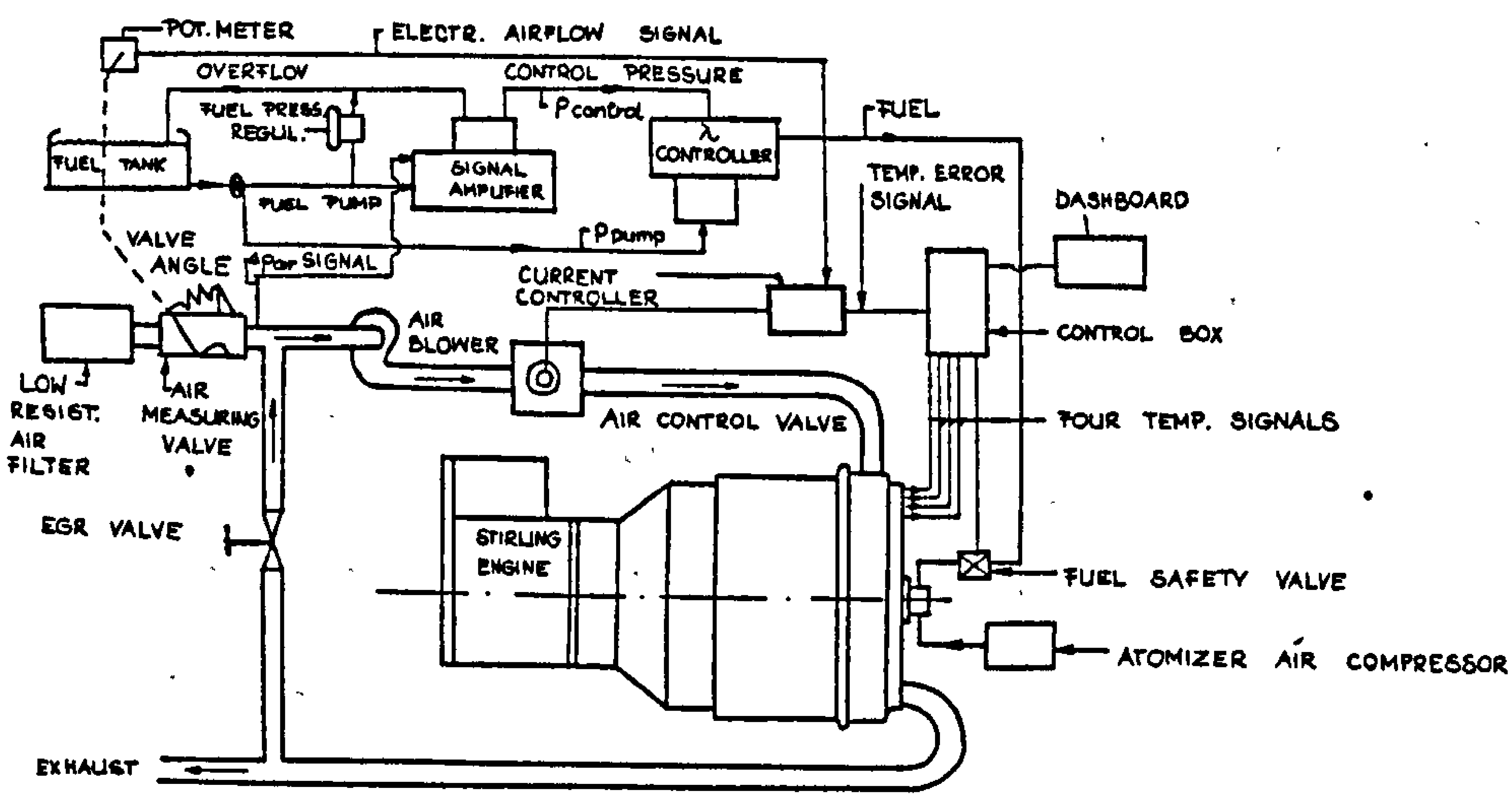



Fig. 6. Scheme of system for control of the temperature and of the air-fuel ratio.

The temperature of the working gas in the heater tubes is measured by a set of four chromel-alumel thermocouples, and afterwards amplified in a quadruple form amplifier fitted inside the control box. The highest temperature is selected and after comparing with a desired value, the temperature error signal is fed to the current controller, which at the same time receives an electric air flow signal from the air measuring valve i.e.

$$\triangle P_{air} = C_{air} * (\dot{m}_{air})^n$$

where $C_{air}$ and n, are chosen ref. to combustion processes.

The resultant control current is passed to a rotating magnet acting as throttle air control valve. The magnitude $\triangle P_{air}$ is transferred into a fuel pressure signal. (see Fig. 7)

$$P_{control} = P_{pump} - \alpha * \triangle P_{air}$$

where $\alpha$ - gain factor
$P_{pump}$ - fuel pressure behind the pump.

Then the magnitude of $P_{control}$ is fed into a $\lambda$ type controller (see Fig. 7), in which a pressure drop across a variable resistance can be expressed as:

$$\triangle P_{fuel} = C_{fuel} * (\dot{m}_{fuel})^n \quad \underline{\text{OR}} \quad \triangle P_{fuel} = P_{pump} - P_{control}$$

and finally:

$$\frac{(\dot{m}_{fuel})^n}{(\dot{m}_{air})^n} = \frac{\alpha * C_{air}}{C_{fuel}}$$

The last equation makes the following facts obvious :

1) the air/fuel ratio remains constant and does not depend on $P_{pump}$.
2) the air/fuel ratio does not depend on the back pressure of the system of burner and atomizer.

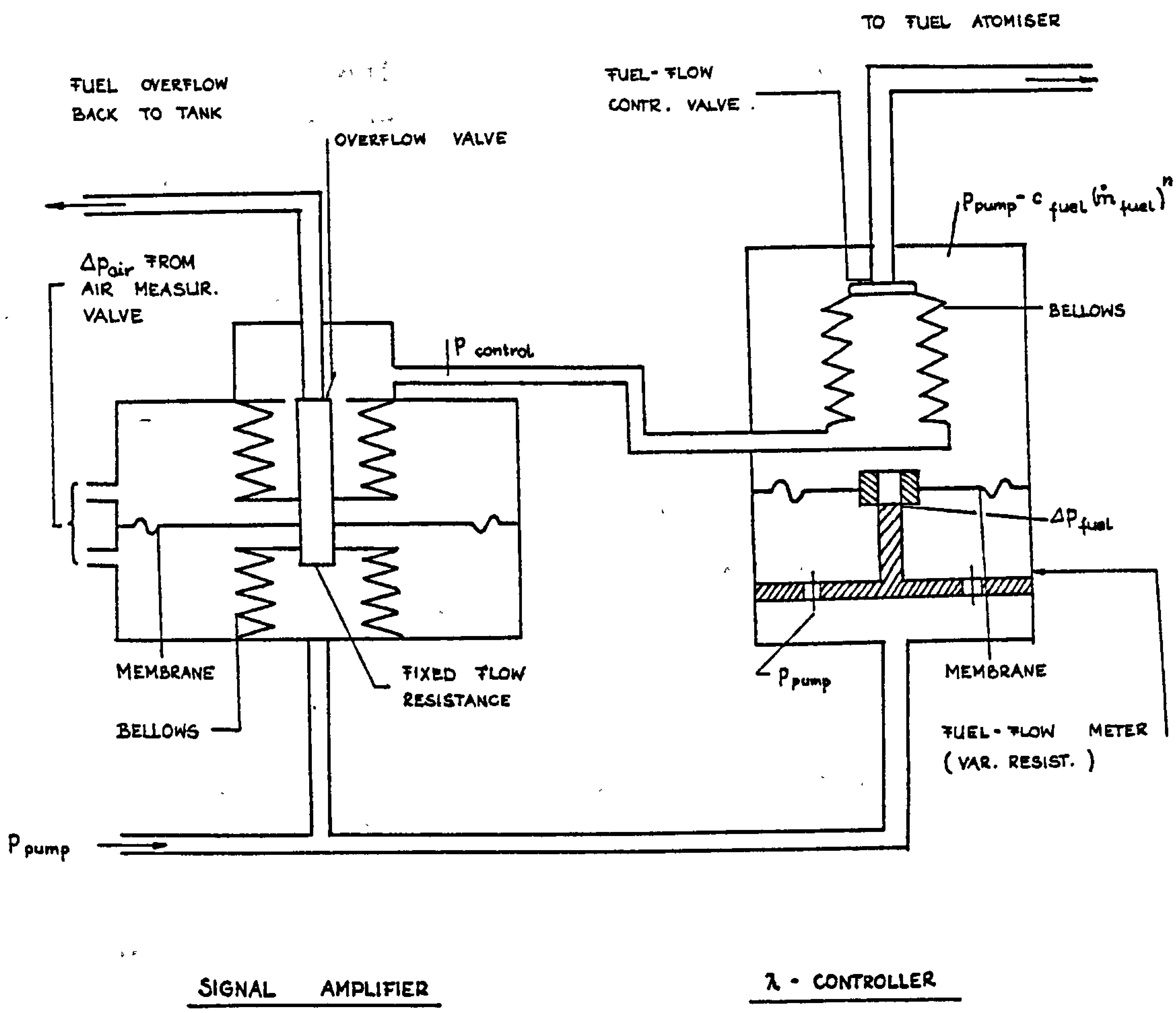



Fig.7. Diagram of fuel control system.

The second type of control system employed for the 4-215 D.A. engine is the power control system in which the specific output power is controlled by varying the mean cycle pressure. To satisfy the torque response as well as torque/speed (RPM) requirements, the system has been developed as illustrated in Fig. 8 in simplified block diagram form.

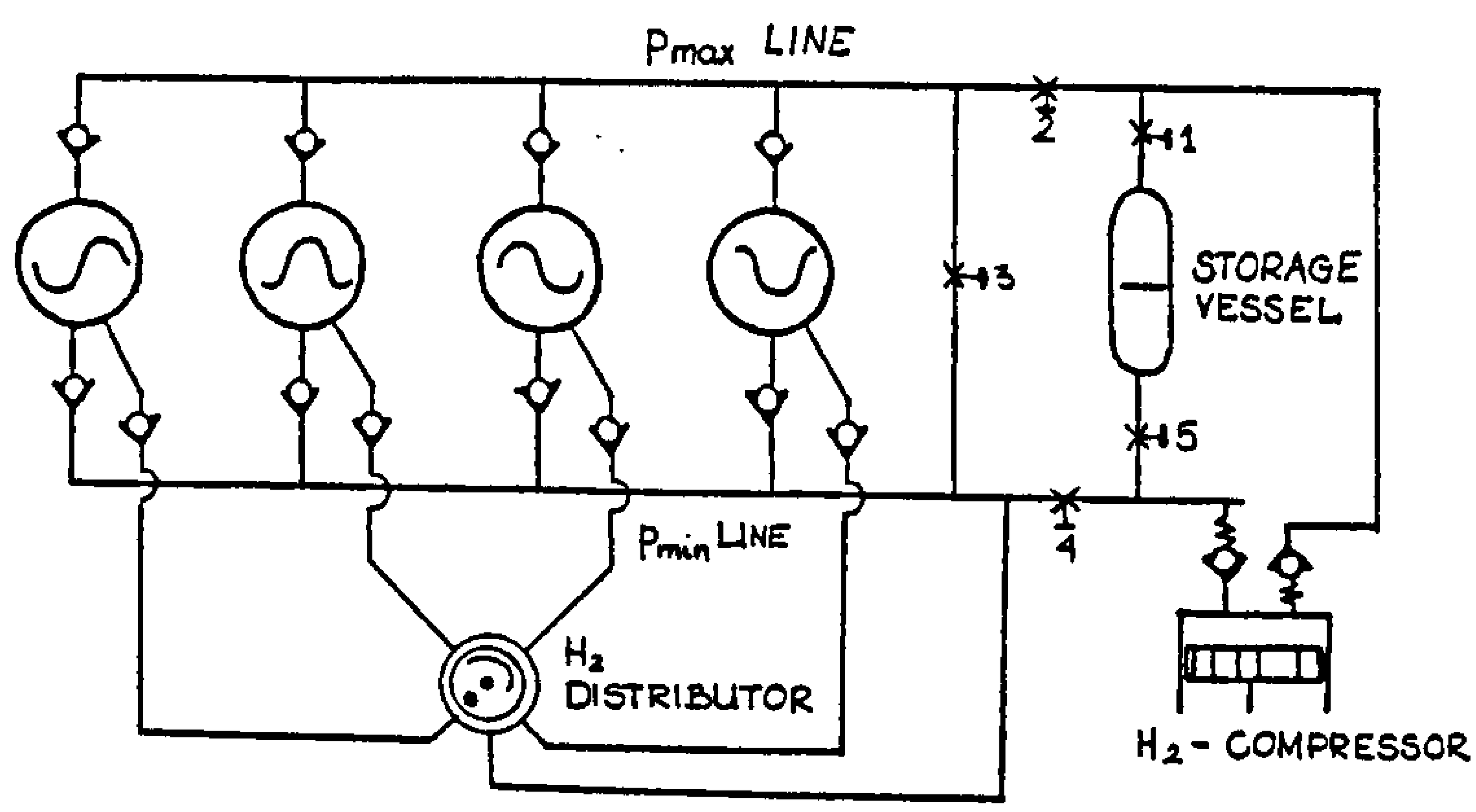



Fig.8. Diagram of modified pressure control system with hydrogen distributor.

The mean pressure power control system has been described previously, however, this particular one (see Fig.8) possesses a major difference i.e. a hydrogen distributor has been incorporated in the supply line and also the hydrogen compressor is used both for replenishinf and for draining. The specific timing of the hydrogen distributor i.e. the cycles are replenished at the moment that cycle pressure reaches its maximum value is shown in Fig. 9.

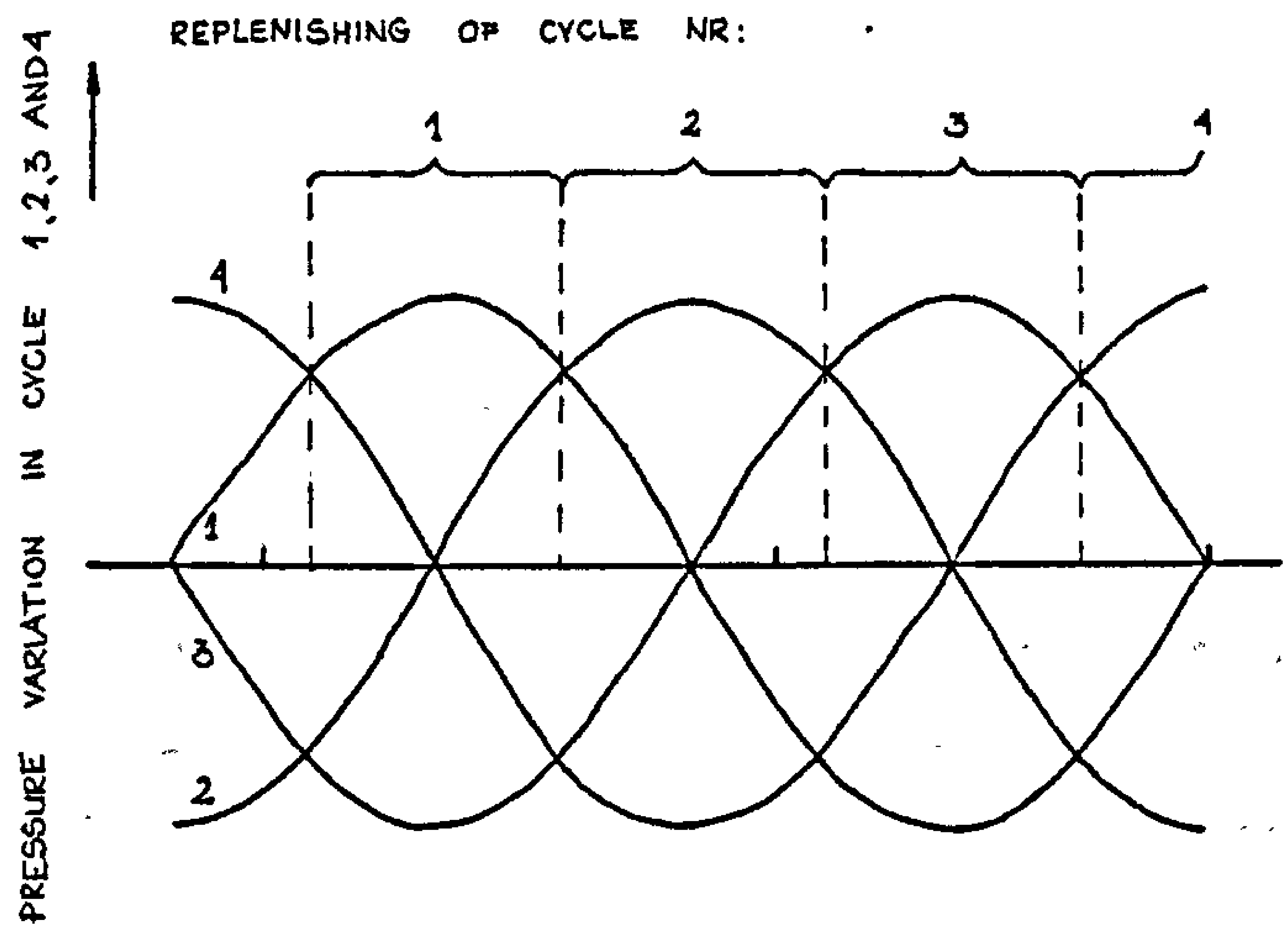



Fig. 9. Timing of hydrogen distributor

This system's modes of operation consist of the following:

1. replenishing
2. short-circuiting
3. draining
4. braking

And again the switching pattern of control valves (as per Fig.8) which emphasises the control valve function as well as modes of the system's operation is shown in Fig. 10.

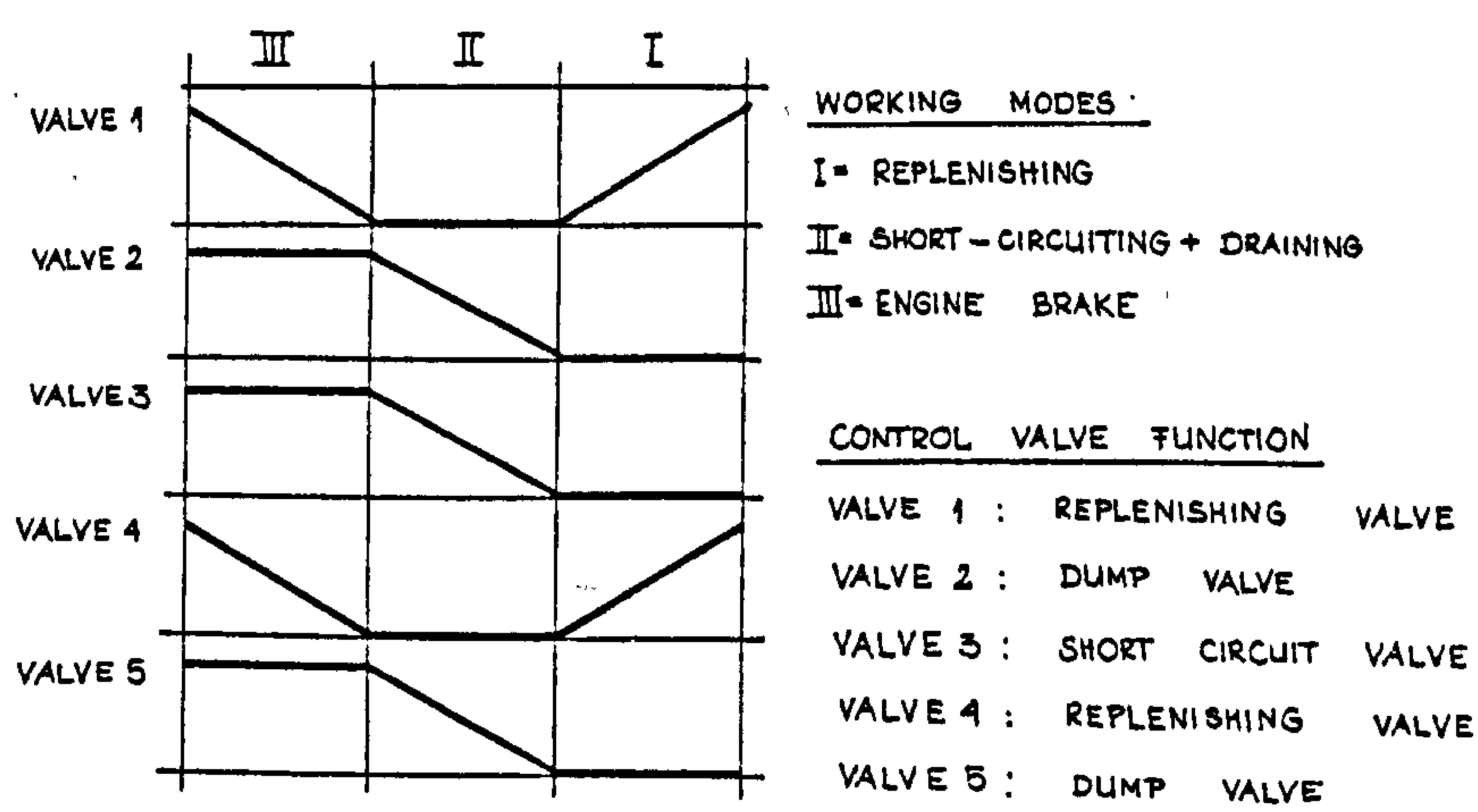



Fig. 10. Switching pattern of control valves.

As Van Giessel and Reinink proved, it is possible to raise and to lower the mean cycle pressure thus adjusting the power/torque levels according to the actual situation and applied load requirements.

Referring back to Fig. 8, it now remains to be decided how the driver should operate the control valves No. 1 -5. A direct link between the accelerator pedal and the operating mechanism of the valves seems unlikely to offer a solution. We should then have a system working as an integrator and thus with the accelerator in a certain position (set point of the control), resulting in the operating pressure of the engine to rise or fall in order to achieve torque/speed characteristics similiar to those of internal combustion engines. The final block diagram of speed and pressure control described above is shown in Fig.11 and contains the following : Acceleration Block, Control Box, The Engine Block, Pressure Transducer and Amplifier Block and finally the Speed Transducer and Amplifier Block. Use is made of the two negative feedback signals, i.e. The speed of the engine and the maximum cycle pressure in the engine - by using the appropriate pressure/speed transducer equipped with relevant signal amplifiers.

Fig.11

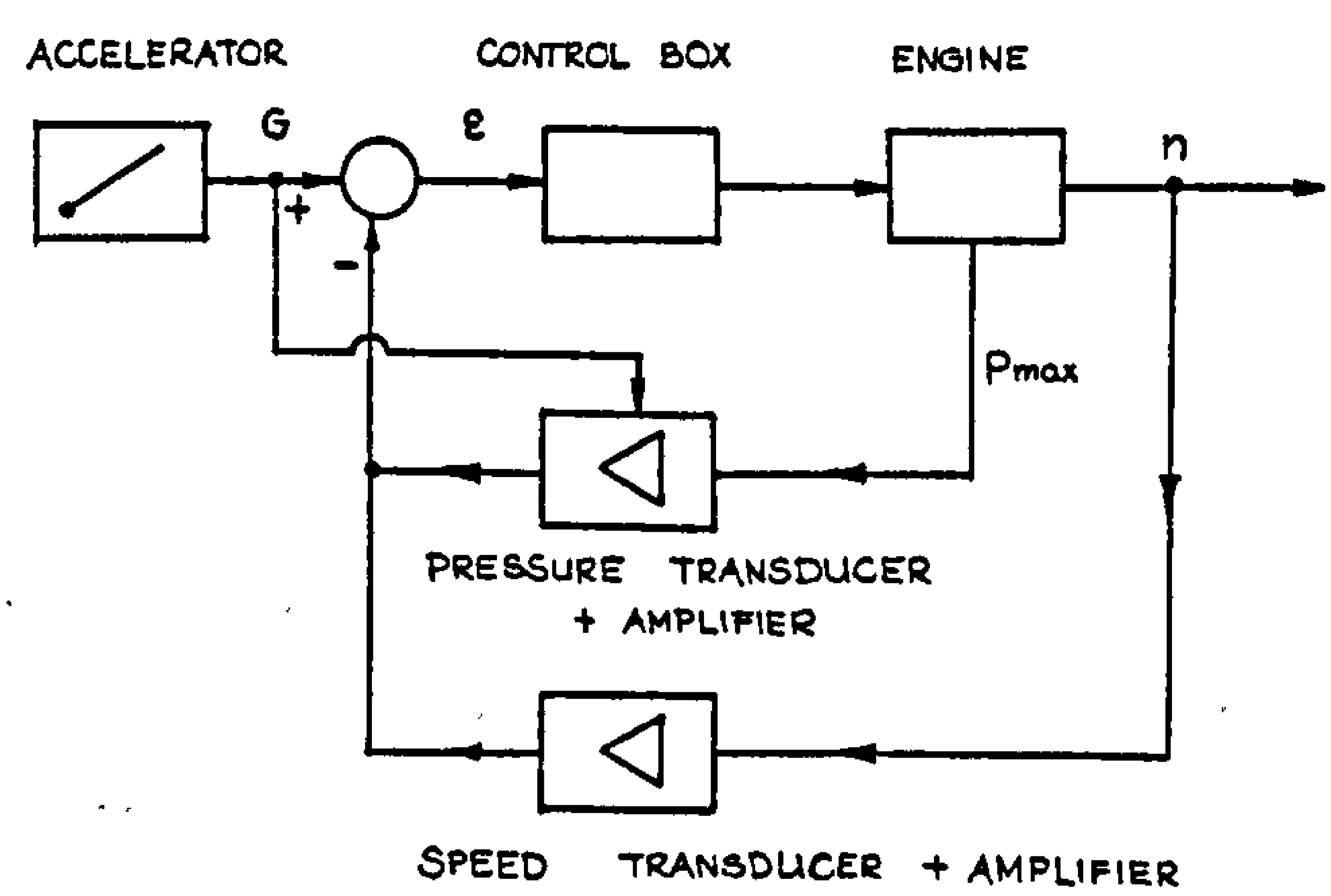



Diagram of speed and pressure control

The working range of this particular power control system applied in the 4-215 D.A. Stirling engine is presented in Fig. 12 and shows it at its maximum torque level - a well known "automotive" character, with a generally recognised higher torque value at lower speed regions and correspondingly lower torque values at higher speed regions.

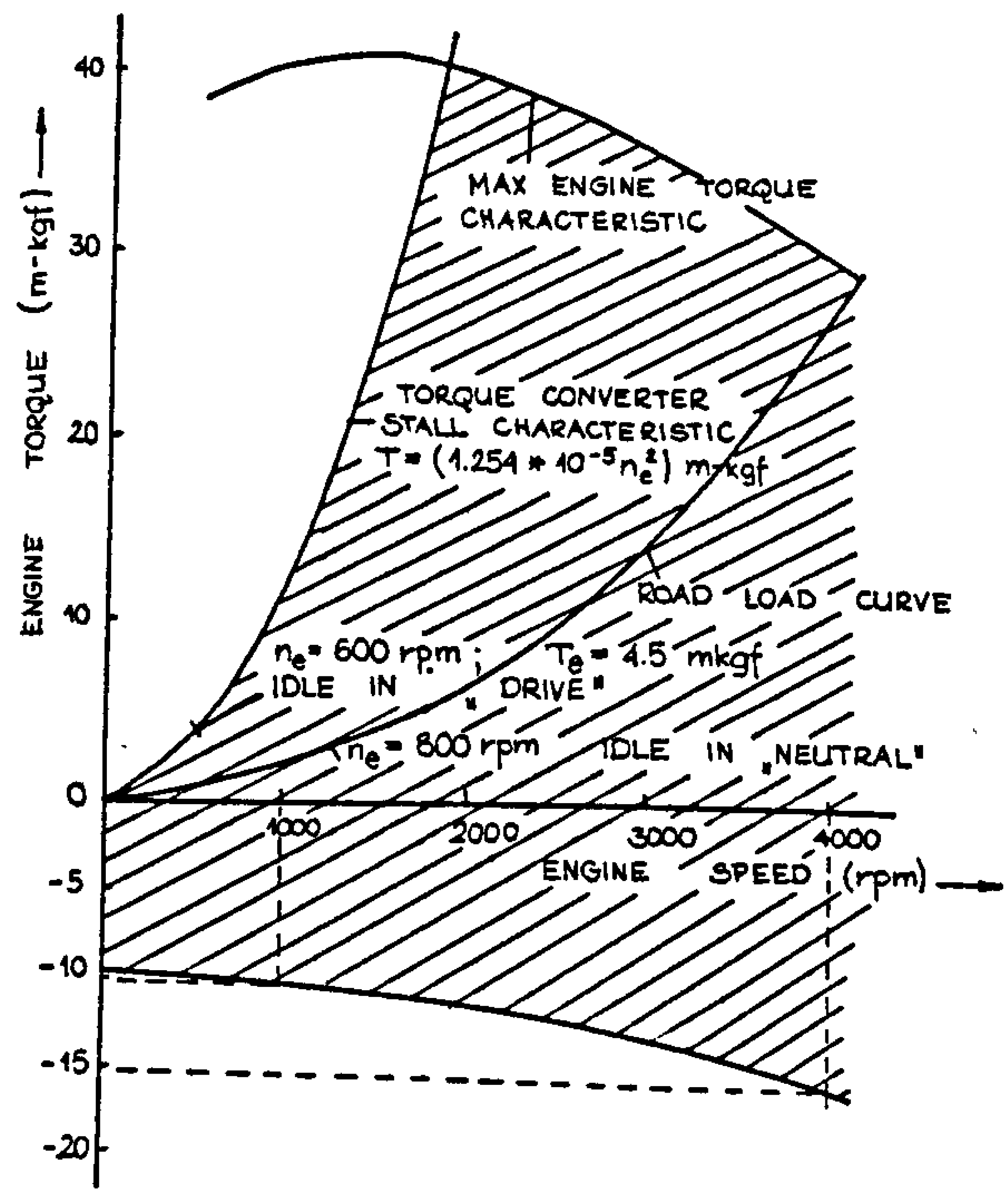



Fig.12. Working range of power control system.

As far as engine torque time response is concerned as well as the torque lag time, this power control system has to fulfil the following requirements:

Engine Torque response : Time to reach 90% W.O.T.
(wide open throttle) Drive shaft torque $\leqslant$ 0.6 sec.
Time from pressing of accelerator pedal until engine reaches idle torque again $\leqslant$ 0.2 sec.
Deceleration response time $\leqslant$ 0.2 sec.

The transient characteristics derived from actual experiments on 4 - 215 D.A. Stirling engine are shown in Fig. 13a and 13b.

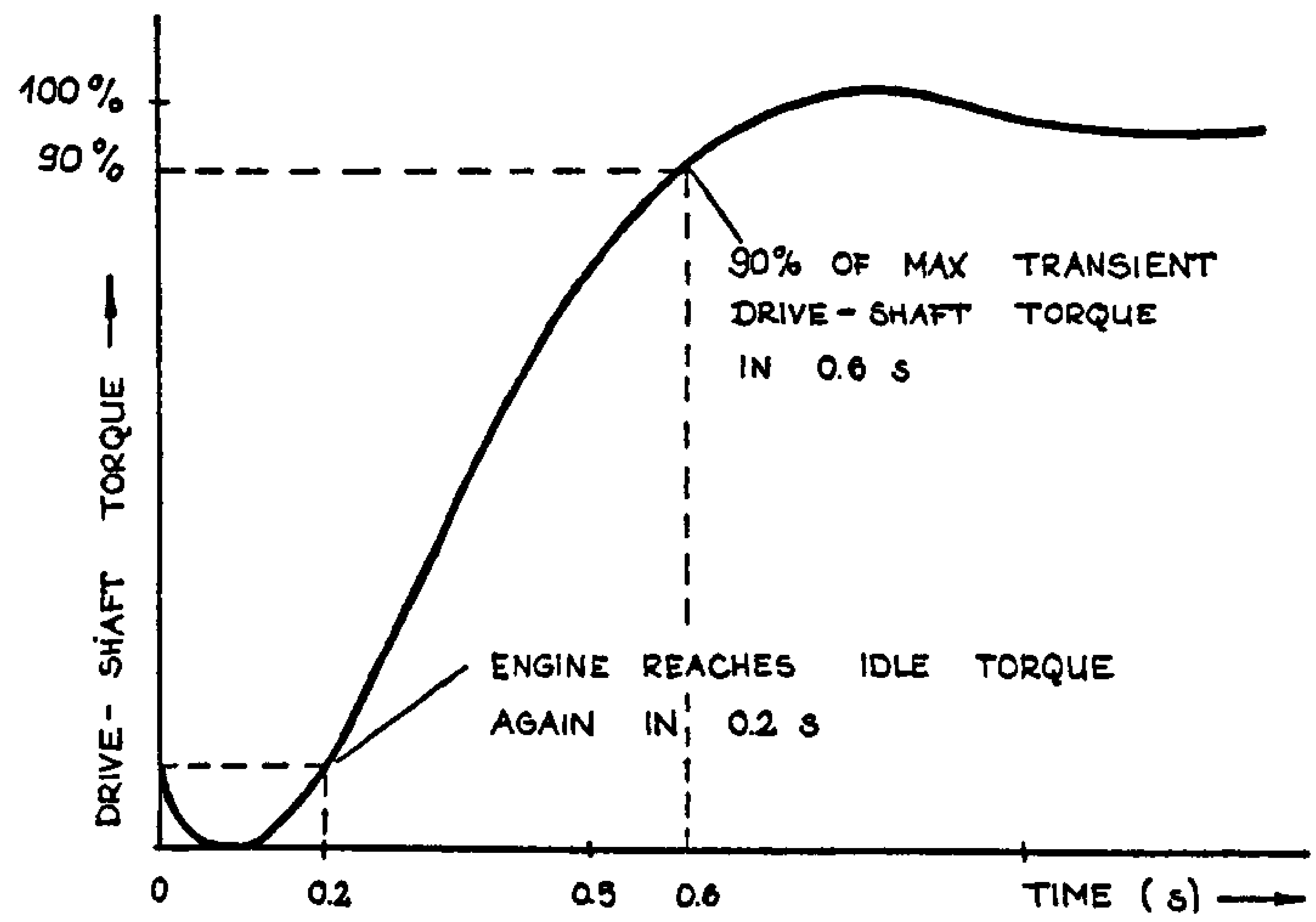



Fig. 13a.
Torque response time and torque lag time.

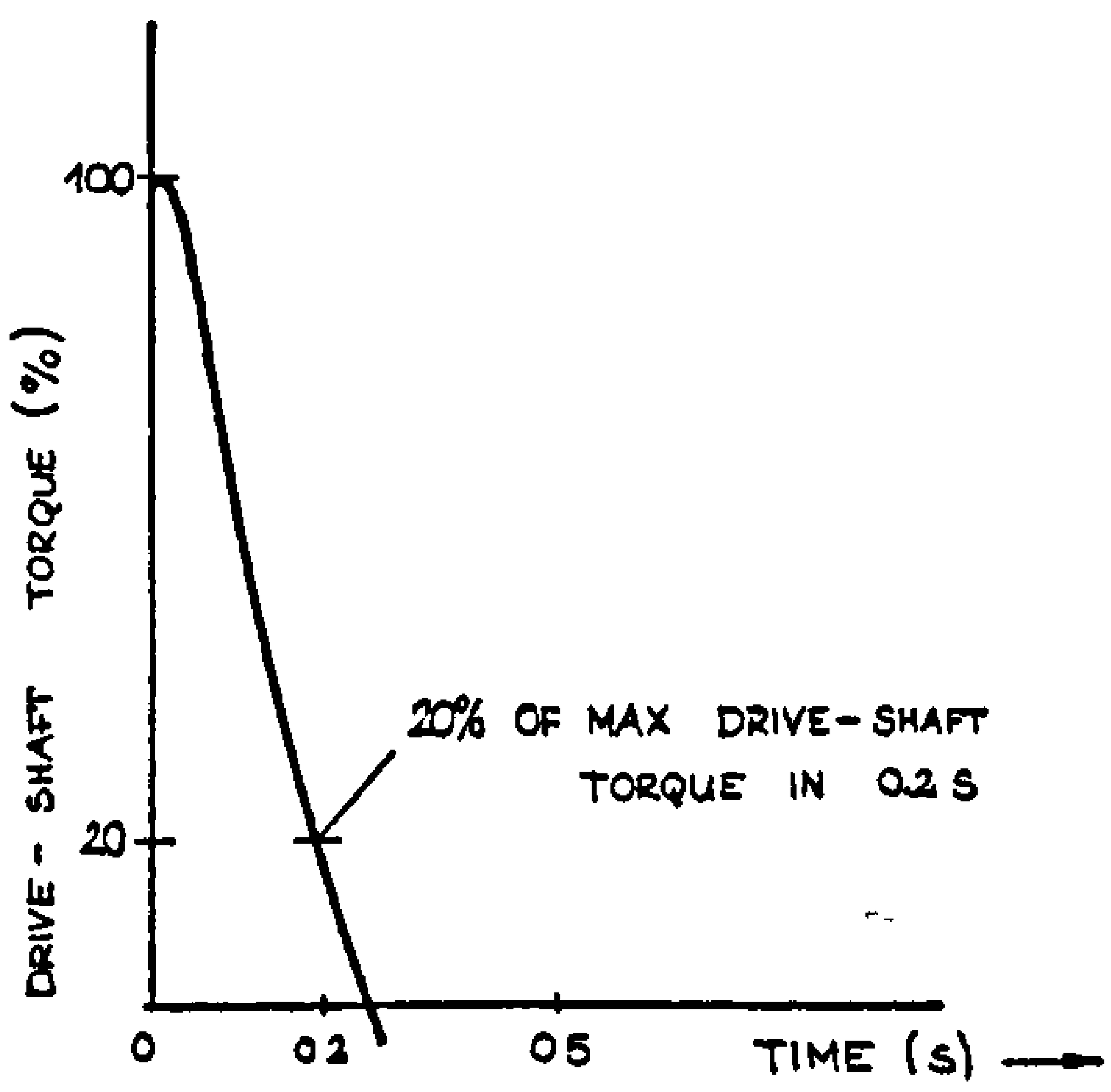



Fig. 13b.
Deceleration response time.

In conclusion to the above, it is quite obvious that the mean pressure power control system applied to this engine fulfils its role satisfactorily, being reasonably compact and uncomplicated and providing a wide control span over the regulated quantities. It is worthwhile to underline the following : all controls and including the high pressure vessels being here accomodated in the engine compartment make this particular design an extremely compact construction.

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SAE No. 770082
by R. Van Giessel and F. Reinink
N.V. Philips "Gloeilampen fabrieken" (Germany)

## 1.7. SUMMARY

The purpose of this chapter is to provide an introductory treatment of the Stirling Cycle machine philosophy, including specific Stirling Cycle nomenclature, its historical background and also the principles of Stirling machine operations.

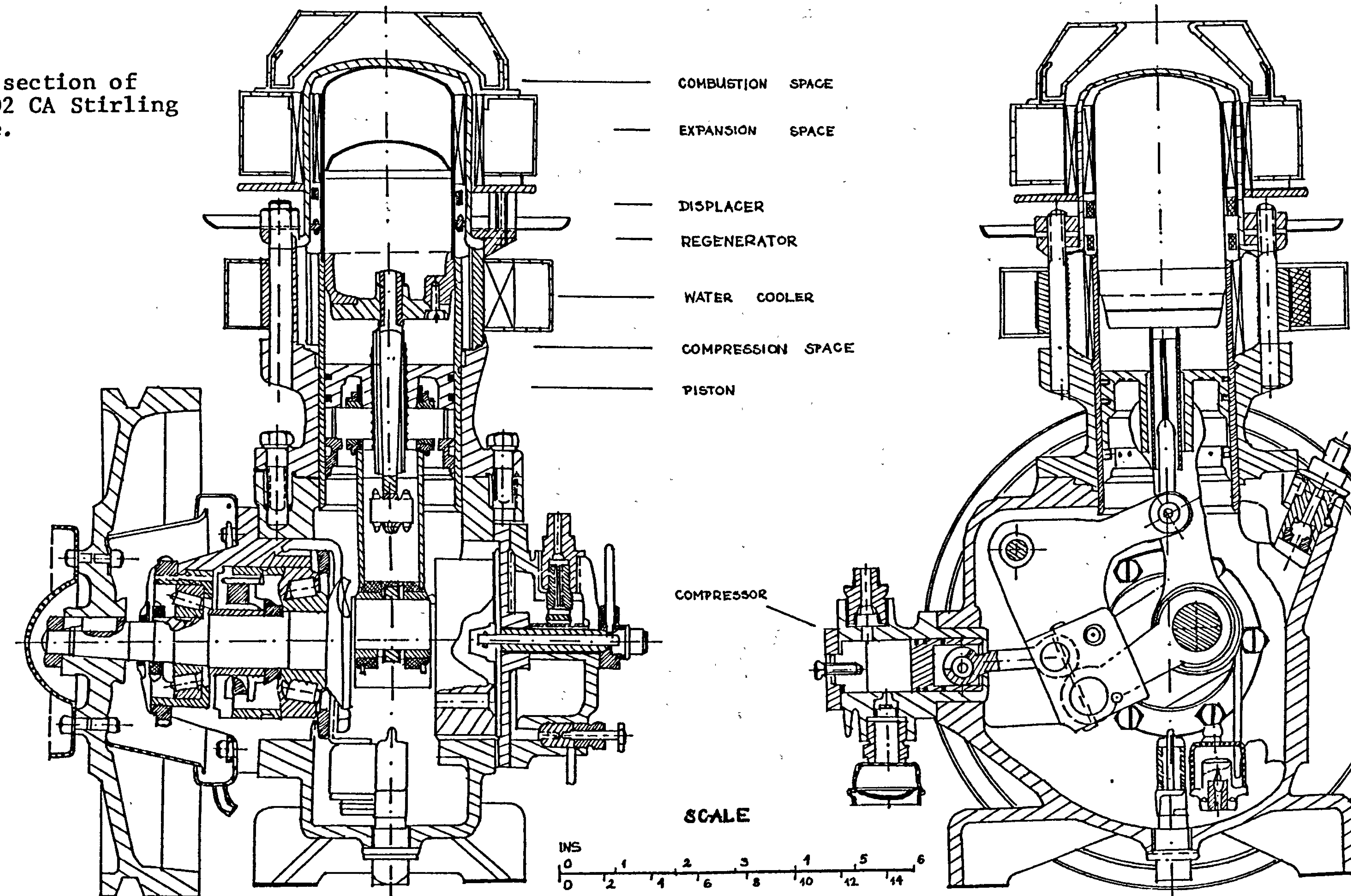
The field of "Stirling Cycle" engineering has made many notable advances in the last few years, and these are included in the form of a review of recent research topics, followed by an investigation into the possible Stirling Engine automotive application. The field of automatic control applied to Stirling engine is investigated on carefully selected examples coming from proven and highly reputable organisations and manufacturers and also an outline of a theoretical introduction to the general control of the Stirling Cycle Engines. Chapter one is completed by a review of selected Stirling Cycle machine literature which includes principles of operations of the two most popular Stirling Control Systems i.e. Temperature and Air-fuel control as well as Pressure/Speed (Power/Torque) control systems.

In addition, Transient behaviour of considered systems are illustrated with work dynamic characteristics, illustrating the torque response time, torque lag time, deceleration time all measured on one of the most up to date "automotive" design engines.

# 2. THE PHILIPS TYPE: MP 1002 CA STIRLING MACHINE AND VARIOUS EXPERIMENTS ON MODIFIED ENGINES OUTSIDE THE NETHERLANDS

Fig.1.

Cross section of MP 1002 CA Stirling engine.



## 2.1. THE ORIGINAL CONSTRUCTION

Work on Stirling Engines at the Philips Laboratories, Eindhoven, Holland has continued since 1938 and contributes substantially to the present state of "Stirling Research". The initially developed small one horsepower engine was a single-cylinder piston-displacer machine incorporated in an approx. 200 Watts generator set provided as a multi-purpose electric power source for radio, and other small electrical equipment, for use in under-developed countries. There were several hundred similiar engines made about 1950 and many of them were disposed of to universities, technical colleges, mainly for teaching purposes and experimental work. Basically, the engine was designed for Kerosene/gasoline running, however, many were converted into other fuel materials and used satisfactorily. The summary of technical data is given below and more details of the MP 1002 CA engine can be found in Lit. 1.

THE MP 1002 CA TECHNICAL DATA

| | |
|---|---|
| Fuel consumption (kerosene); | approx.0.4 litre/hour |
| Fuel tank capacity | approx. 3.3 litres |
| Running hours per filling | approx. 8 |
| Lubricant | Shell Turbo 27 (S.A.E.20) |
| Lubricant capacity in crankcase | approx. 0.16 litre |
| Nominal engine speed | 1500 RPM |
| Generator output | 180 Watt/220V/50Hz @ 3000RPM |
| Weight (without fuel) | approx. 30 Kg. |
| Dimensions | length: 45cm, width:27cm<br>Height: 40cm |

A cross-section drawing of the engine is shown in Fig.1. and a brief outlay of the whole generator set is presented in Fig. 2.

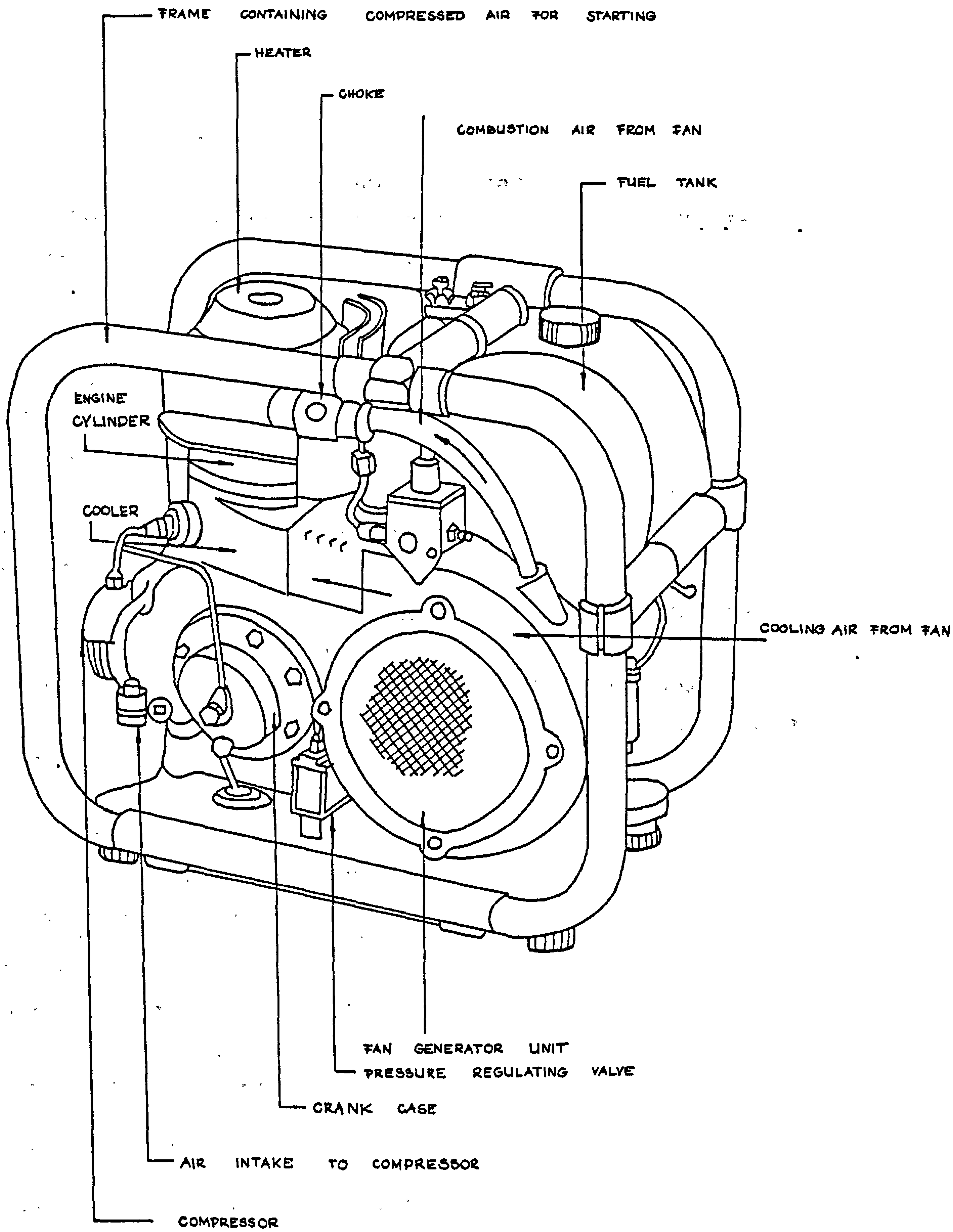



Fig.2. Philips engine-generator set

The Philips MP 1002 CA Stirling engine is characterised by a number of favourable properties making it particularly suitable for various purposes. Amongst others, it possesses the following characteristics and advantages.

1. Non-critical on fuel specification
2. Silent Operation, no explosions, no valve noise, the combustion is continuous. Non-poisonous flue gases.
3. Uniform torque owing to uniform expansion and an extremely uniform load on the driving gear.
4. Little wear, because the piston walls do not come into contact with the combustion products.
5. Low friction losses.
6. Extremely low lubricant consumption and no contamination of the oil, partly because the temperature of the sliding surfaces of the piston walls is kept low.
7. Long life. The engine can be left running for very long periods without supervision.
8. Great reliability.
9. Simple operation and little maintenance.
10. Constant speed, independant load, no readjusting needed

Tab.1. THE PHILIPS MP 1002CA Stirling Engine Particulars

| | |
|---|---|
| Cylinder bore: | 0.056 (m) |
| Stroke of piston: | 0.027 (m) |
| Stroke of displacer: | 0.025 (m) |
| Swept Volume in expansion space: | $6.38 \times 10^{-5}$ ($m^3$) |
| Swept volume in compression space: | $6.71 \times 10^{-5}$ ($m^3$) |
| Total dead volume(estimated): | $7.97 \times 10^{-5}$ ($m^3$) |

The displacement vs. Crankangle Diagram for the Philips MP 1002 CA Stirling Engine is shown in Fig. 3 (after Lit. 3).

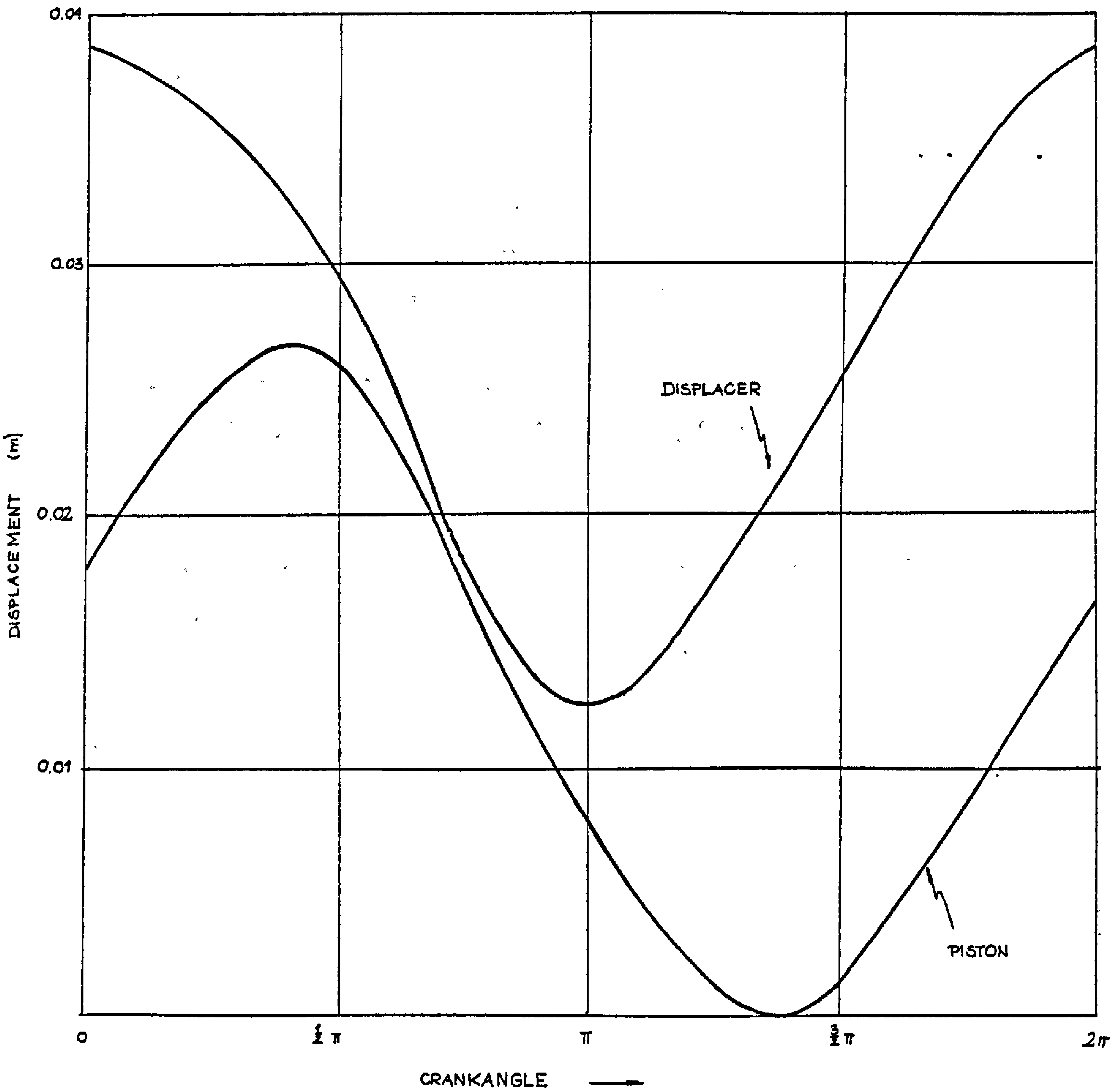



Fig.3. Displacement/crankangle diagram for the Philips MP 1002 CA Stirling engine.

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2. Stirling Cycle Machines
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3. An Investigation of the Effect of an Exhaust
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   Engine.
   Bath University of Technology
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## 2.2. RESEARCH WORK AT BATH (J.B. PHILLIPS/T.J. REID AND G.L. WARD).

As mentioned earlier, the Philips Type MP 1002 CA Stirling Engine, being a single-cylinder piston-displacer machine is the ideal type of Stirling Cycle Engine for experimental and research work. The basic conception of having a single, uncomplicated and inexpensive engine must naturally imply its own limitations, in that one cannot incorporate the usual "luxury" features of many more advanced Stirling engines, whilst simultaneously being economically viable. However, after studying various alternatives, the decision to work on the MP 1002 CA was undertaken in Bath, (the faculty of Engineering, Bath University of Technology) in 1972 by G.L. Ward, in 1973 D.M. Conlin and L.H. Reed and continued in the following years by J.B. Phillips and T.J. Reid. The first project carried out by G.L. Ward investigated the performance of MP 1002 CA Stirling engine when fuelled with liquid petroleum gas. The control parameters were cylinder head temperature, minimum cycle pressure and engine speed. Engine performance was analysed in terms of the Schmidt cycle analysis which predicts, for the conditions studied (see Lit.1), that performance can best be improved by increasing the cycle pressure. The least improvement would be gained by increasing the cylinder head temperature. The results indicated a strong and detrimental effect of mechanical power losses. On the power output of the MP 1002 CA Stirling engine and the mechanical friction losses which increase with engine speed and were particularly serious. In a limited set of tests, the mechanical efficiency ranged 25.0 to 62.2 per cent, with variation of cylinder head temperature provided as the simplest method of power control. Temperature control was easily and accurately attained with the gaseous fuel and an increase in temperature was not accompanied by a corresponding increase in mechanical power loss.

Finally, three recuperative heat exchangers were designed, built and tested, unfortunately their effectiveness was very much less than predicted due in part to the lack of design data. The main difference from the original Philips version was the water-cooling as well as combustion air which was supplied under pressure from the mains.

The next reserach study concerning the performance of a Philips Stirling Engine model MP 1002 CA using an exhaust gas recuperator was carried out by D.M. Conlin and L.H. Reed (see Lit.2) in 1973. However, after limited testing only, somewhat unexpected results were obtained. The purpose of J.B. Phillips and T.J. Reid's project was to ascertain whether Conlin and Reed's results were sound and then to explore the problems with further preheating. On both above mentioned projects, the engine was modified by incorporating water instead of air cooling and by liquid petroleum gas as a fuel instead of parafin. A series of tests was conducted using an exhaust gas recuperator to preheat the incoming combustion air. The basic modifications improved the efficiency of the standard engine and increased the rated power output from the nominally stated 200 Watts up to 600 Watts. Preheating the combustion air was found (see Lit.3) to increase the thermodynamic efficiency of the engine for a given power output and increasing the working pressure resulted in an increase in the brake power output as well as the brake thermal efficiency of the MP 1002CA Stirling Engine.

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byJ.B. Phillips and T.J. Reid.
1974

## 2.3. "PROFESSOR WALKER'S" EXPERIMENT

Under this heading - the performance of the Philips Type MP 1002 CA Stirling engine was presented by Ward, Slowley and Walker (see Lit.1.).

For test purposes the engine was modified to operate on liquid petroleum gas fuel and with water cooling rather than air cooling. The maximum brake power output of the engine was found to be approximately 0.5KW @ 1400 RPM with a mean operating pressure of 12.4 bar, ref. cylinder head temperature of 900$^{o}$C. On this particular experiment, there was no attempt made to incorporate an exhaust gas heat exchanger to preheat the incoming air or to minimise thermal losses from the combustion space. For the purpose of this work the engine was mounted on a test rig shown in Fig.1. The combustion equipment was modified to allow the use of liquid petroleum gas (L.P.G.) and air rather than the normal liquid kerosene or gasoline fuels. The output shaft of the engine was coupled to a special swinging field dynamometer and provision was made for accurate measurements of the gas and air flow, the engine output shaft speed (RPM) and finally the brake power input (or output of the engine). Additionally, instrumentation was provided to monitor the temperatures and control flow rates of the cooling water. The hot space temperature was measured by a set of chromel-alumel thermocouples, and for motoring tests, the engine's working space was connected to a large receiver, thereby increasing the internal volume or "dead space" of the engine.

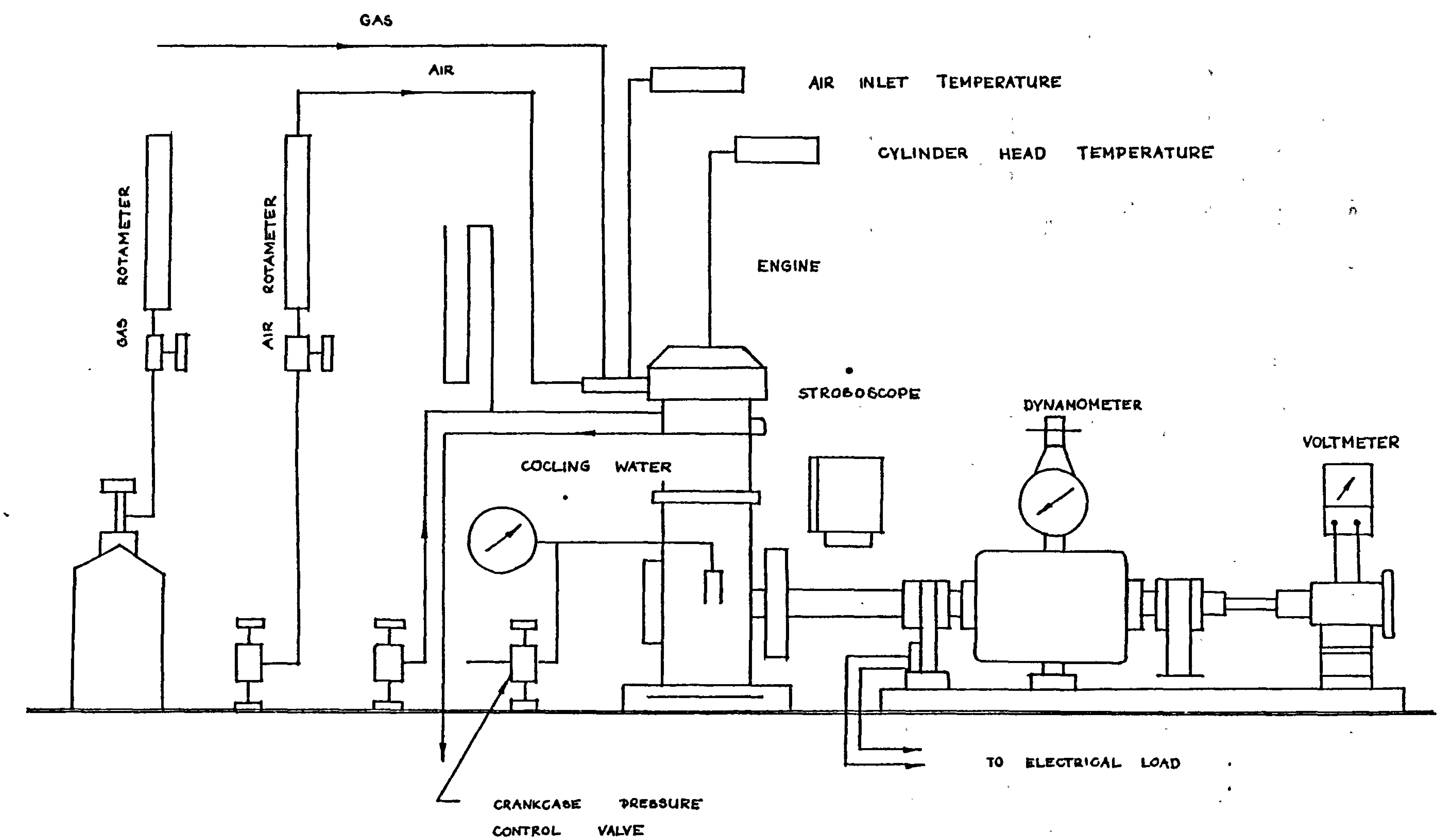



Fig.1. Test rig for the MP 1002 CA Stirling Engine.

The range of test results made at constant speed of 1200, 1400, 1600 and 1800 RPM cover four various cylinder head temperatures ( $^{o}$ C) at 600, 700, 800 and 900$^{o}$C with a mean working space operating pressure of 4.14, 5.52, 6.90, 8.28, 9.66 and 12.41 bar. A summary of these tests is produced in Tab. 1. in a graphical form in Fig. 2a,2b,3a,3b,4,5a,5b. A commercial mixture of butane ($C_4H_{1o}$) and propane ($C_3H_8$) known as 'calor gas' were used in these tests with a typical mixture (by weight) i.e. 90% propane and 10% butane, possessing a calorific value of 46,500 $\left(\frac{KJ}{KG}\right)$. As far as combustion air is concerned, this quantity was adjusted to obtain the maximum possible cylinder temperature for applied fuel flow.

Tab.1. (a) ENGINE SPEED 1800 R.P.M.

| Cylinder Head Temp $^{o}C$ | Operating Pressure Bar | Engine Torque m-n | Power Watts | Fuel Rate g/min | Brake specific fuel consumption kg/kw-hr |
|---|---|---|---|---|---|
| 900 | 12.41 | 2.47 | 465.5 | 7.7 | 0.99 |
| | 11.03 | 2.35 | 442.9 | 7.7 | 1.04 |
| | 9.66 | 2.35 | 442.9 | 7.8 | 1.06 |
| | 8.28 | 2.06 | 388.2 | 7.3 | 1.13 |
| | 6.90 | 1.72 | 324.2 | 6.5 | 1.20 |
| | 5.52 | 1.41 | 265.7 | 6.3 | 1.42 |
| | 4.14 | 0.91 | 171.5 | 6.0 | 2.10 |
| 800 | 12.41 | 1.94 | 365.6 | 7.0 | 1.15 |
| | 11.03 | 2.00 | 376.9 | 7.4 | 1.18 |
| | 9.66 | 1.86 | 350.5 | 6.9 | 1.18 |
| | 8.28 | 1.69 | 318.5 | 6.7 | 1.26 |
| | 6.90 | 1.43 | 269.5 | 6.0 | 1.34 |
| | 5.52 | 1.14 | 214.8 | 5.6 | 1.56 |
| | 4.14 | 0.73 | 137.6 | 5.4 | 2.35 |
| 700 | 12.41 | 1.22 | 229.9 | 5.9 | 1.54 |
| | 11.03 | 1.32 | 248.8 | 6.4 | 1.54 |
| | 9.66 | 1.25 | 235.6 | 6.0 | 1.53 |
| | 8.28 | 1.18 | 222.4 | 5.8 | 1.56 |
| | 6.90 | 1.02 | 292.2 | 5.3 | 1.65 |
| | 5.52 | 0.79 | 148.9 | 4.3 | 1.73 |
| | 4.14 | 0.49 | 92.3 | 4.3 | 2.80 |
| 600 | 12.41 | 0.20 | 37.7 | 5.4 | 8.59 |
| | 11.03 | 0.76 | 143.2 | 5.5 | 2.30 |
| | 9.66 | 0.82 | 154.5 | 5.3 | 2.06 |
| | 8.28 | 0.57 | 107.4 | 4.3 | 2.40 |
| | 6.90 | 0.54 | 101.8 | 3.7 | 2.18 |
| | 5.52 | 0.39 | 73.5 | 3.2 | 2.61 |
| | 4.14 | 0.19 | 35.8 | 2.8 | 4.69 |

Tabl(b) ENGINE SPEED 1600 RPM

| Cylinder Head Temp °C | Operating Pressure Bar | Engine Torque m-n | Power Watts | Fuel Rate g/min | Brake specific fuel consumption kg/kw-hr |
|---|---|---|---|---|---|
| 900 | 12.41 | 2.86 | 479.1 | 7.7 | 0.96 |
| | 11.03 | 2.65 | 443.9 | 7.4 | 1.00 |
| | 9.66 | 2.72 | 455.7 | 7.0 | 0.92 |
| | 8.28 | 2.32 | 388.6 | 6.8 | 1.05 |
| | 6.90 | 2.09 | 350.1 | 6.2 | 1.06 |
| | 5.52 | 1.62 | 271.4 | 5.6 | 1.24 |
| | 4.14 | 1.14 | 191.0 | 5.2 | 1.63 |
| 800 | 12.41 | 2.34 | 392.0 | 6.7 | 1.03 |
| | 11.03 | 2.35 | 393.7 | 7.1 | 1.08 |
| | 9.66 | 2.13 | 356.8 | 6.4 | 1.08 |
| | 8.28 | 1.97 | 330.0 | 6.0 | 1.09 |
| | 6.90 | 1.65 | 276.4 | 5.5 | 1.19 |
| | 5.52 | 1.29 | 216.1 | 5.1 | 1.42 |
| | 4.14 | 0.91 | 152.4 | 4.8 | 1.89 |
| 700 | 12.41 | 1.50 | 251.3 | 6.1 | 1.46 |
| | 11.03 | 1.67 | 279.8 | 5.5 | 1.18 |
| | 9.66 | 1.52 | 234.6 | 5.3 | 1.25 |
| | 8.28 | 1.50 | 251.3 | 5.0 | 1.19 |
| | 6.90 | 1.09 | 182.6 | 4.6 | 1.51 |
| | 5.52 | 0.96 | 160.8 | 4.1 | 1.53 |
| | 4.14 | 0.57 | 95.5 | 3.7 | 2.32 |
| 600 | 12.41 | 0.85 | 142.4 | 4.7 | 1.98 |
| | 11.03 | 0.78 | 130.7 | 4.4 | 2.02 |
| | 9.66 | 0.85 | 142.4 | 4.3 | 1.81 |
| | 8.28 | 0.89 | 149.1 | 3.8 | 1.53 |
| | 6.90 | 0.74 | 124.0 | 3.7 | 1.79 |
| | 5.52 | 0.62 | 103.9 | 3.2 | 1.85 |
| | 4.14 | 0.30 | 50.3 | 2.9 | 3.46 |

Tab.1.(c) ENGINE SPEED 1400 RPM

| Cylinder Head Temp $^{o}C$ | Operating Pressure Bar | Engine Torque m-n | Power Watts | Fuel Rate g/min | Brake Specific Fuel consumption kg/kw-hr |
|---|---|---|---|---|---|
| 900 | 12.41 | 3.19 | 467.6 | 7.2 | 0.92 |
| | 11.03 | 3.16 | 463.2 | 6.9 | 0.89 |
| | 9.66 | 2.77 | 406.0 | 6.2 | 0.92 |
| | 8.28 | 2.51 | 367.9 | 5.7 | 0.93 |
| | 6.90 | 2.24 | 328.3 | 5.5 | 1.00 |
| | 5.52 | 1.69 | 247.7 | 5.3 | 1.28 |
| | 4.14 | 1.30 | 190.6 | 5.2 | 1.64 |
| 800 | 12.41 | 2.87 | 420.7 | 6.9 | 0.98 |
| | 11.03 | 2.49 | 365.0 | 6.2 | 1.02 |
| | 9.66 | 2.38 | 348.9 | 5.5 | 0.94 |
| | 8.28 | 2.26 | 331.3 | 5.1 | 0.92 |
| | 6.90 | 1.94 | 285.0 | 4.8 | 1.02 |
| | 5.52 | 1.56 | 228.7 | 4.6 | 1.21 |
| | 4.14 | 1.02 | 149.5 | 4.1 | 1.64 |
| 700 | 12.41 | 1.94 | 284.4 | 5.5 | 1.16 |
| | 11.C3 | 1.97 | 288.8 | 4.9 | 1.02 |
| | 9.66 | 1.90 | 278.5 | 4.8 | 1.o3 |
| | 8.28 | 1.69 | 247.7 | 4.4 | 1.07 |
| | 6.90 | 1.47 | 215.5 | 4.2 | 1.17 |
| | 5.52 | 1.14 | 167.1 | 3.8 | 1.36 |
| | 4.14 | 0.79 | 115.8 | 3.2 | 1.66 |
| 600 | 12.41 | 1.13 | 165.6 | 4.2 | 1.52 |
| | 11.03 | 1.36 | 199.3 | 4.2 | 1.26 |
| | 9.66 | 1.29 | 189.1 | 3.8 | 1.21 |
| | 8.28 | 1.13 | 165.6 | 3.8 | 1.38 |
| | 6.90 | 1.02 | 149.5 | 3.3 | 1.32 |
| | 5.52 | 0.80 | 117.3 | 3.0 | 1.53 |
| | 4.14 | 0.53 | 77.7 | 2.6 | 2.01 |

Tab.1(d) ENGINE SPEED 1200 RPM

| Cylinder Head Temp $^oC$ | Operating Pressure Bar | Engine Torque m-n | Power Watts | Fuel Rate g/min | Brake Specific Fuel consumption kg/kw -hr |
|---|---|---|---|---|---|
| 900 | 12.41 | 3.64 | 457.3 | 6.9 | 0.90 |
| | 11.03 | 3.56 | 447.3 | 6.7 | 0.90 |
| | 9.66 | 3.21 | 403.3 | 6.0 | 0.89 |
| | 8.28 | 2.74 | 344.3 | 5.8 | 1.01 |
| | 6.90 | 2.38 | 299.0 | 5.4 | 1.08 |
| | 5.52 | 1.91 | 240.0 | 5.2 | 1.30 |
| | 4.14 | 1.36 | 170.9 | 4.8 | 1.68 |
| 800 | 12.41 | 3.11 | 390.7 | 5.9 | 0.91 |
| | 11.03 | 2.89 | 363.1 | 5.4 | 0.89 |
| | 9.66 | 2.68 | 336.7 | 5.2 | 0.93 |
| | 8.28 | 2.42 | 304.0 | 5.1 | 1.01 |
| | 6.90 | 2.03 | 255.0 | 4.5 | 1.06 |
| | 5.52 | 1.59 | 199.8 | 4.6 | 1.38 |
| | 4.14 | 1.18 | 148.3 | 4.0 | 1.62 |
| 700 | 12.41 | 2.32 | 291.5 | 5.1 | 1.05 |
| | 11.03 | 2.24 | 281.4 | 4.7 | 1.00 |
| | 9.66 | 2.24 | 281.4 | 4.1 | 0.87 |
| | 8.28 | 2.03 | 255.0 | 4.0 | 0.94 |
| | 6.90 | 1.72 | 216.0 | 3.9 | 1.08 |
| | 5.52 | 1.39 | 174.6 | 3.4 | 1.17 |
| | 4.14 | 1.00 | 125.6 | 3.1 | 1.48 |
| 600 | 12.41 | 1.52 | 191.0 | 4.1 | 1.29 |
| | 11.03 | 1.51 | 190.0 | 3.7 | 1.20 |
| | 9.66 | 1.43 | 179.7 | 3.4 | 1.14 |
| | 8.28 | 1.30 | 163.3 | 3.1 | 1.14 |
| | 6.90 | 1.10 | 138.2 | 3.0 | 1.30 |
| | 5.52 | 0.82 | 103.0 | 2.7 | 1.57 |
| | 4.14 | 0.57 | 71.6 | 2.5 | 2.10 |

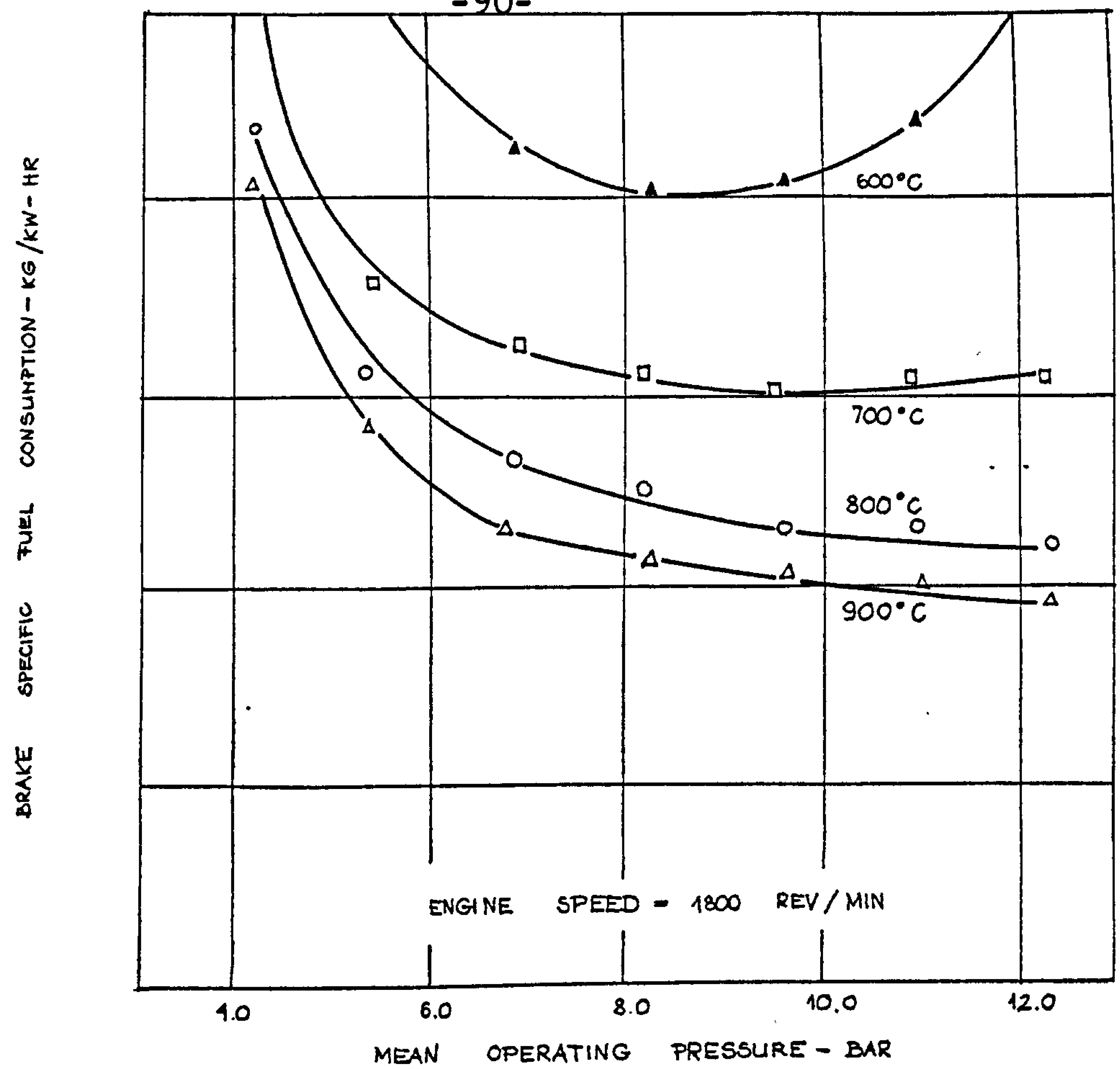



Fig.2b. Brake specific fuel consumption vs pressure

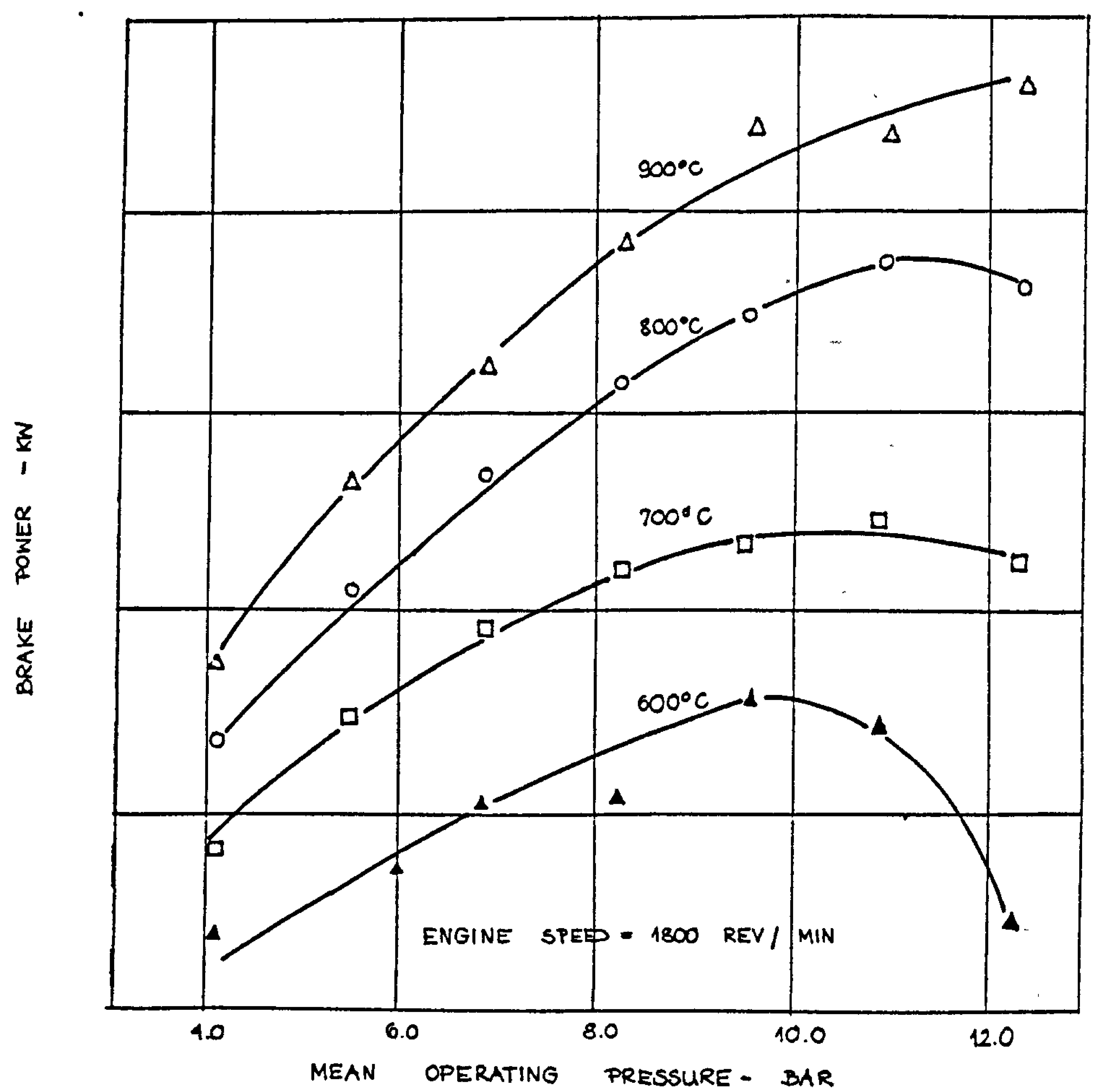



Fig. 2a. Brake power vs pressure

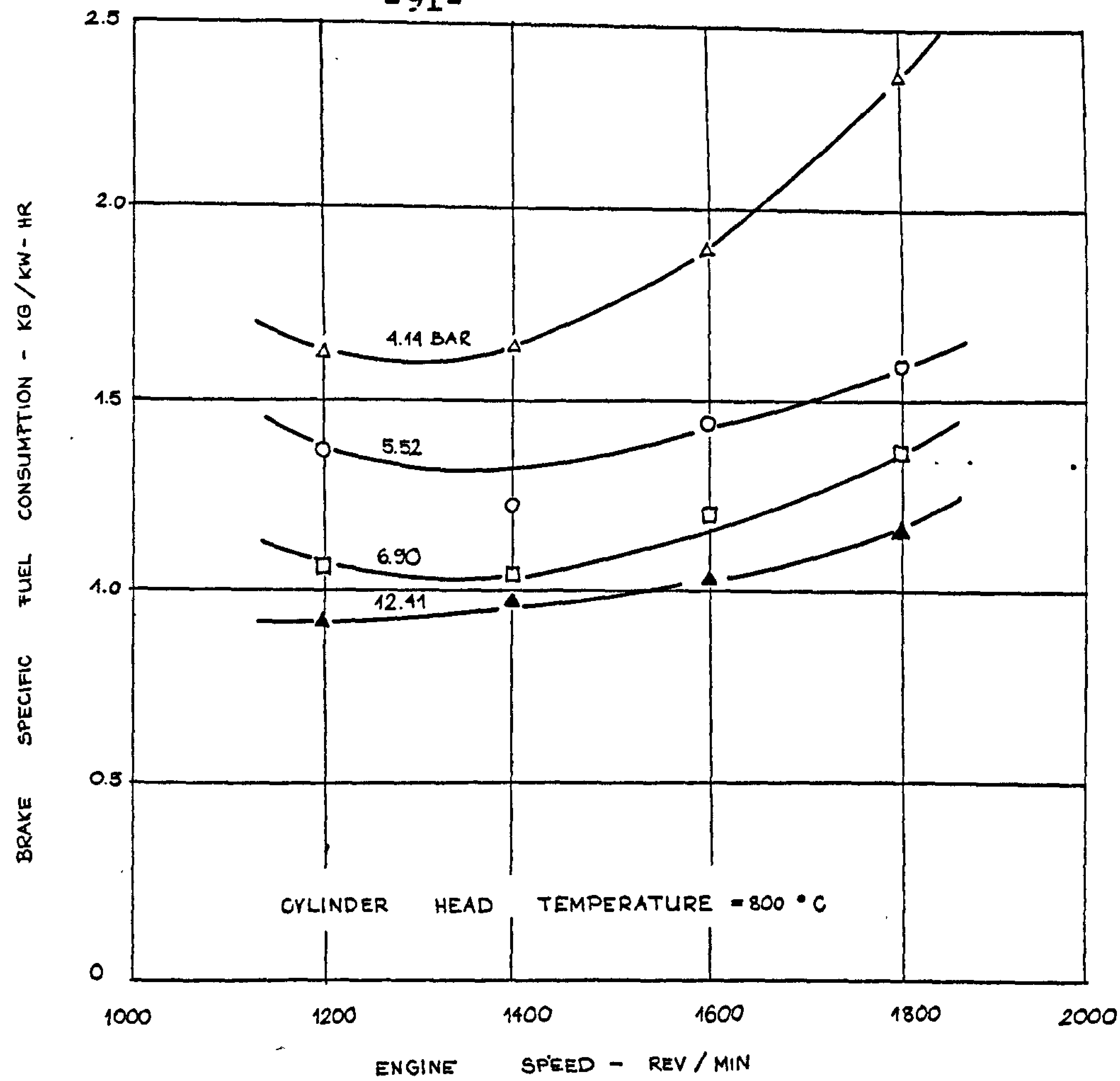



Fig. 3b. Brake specific fuel consumption vs speed

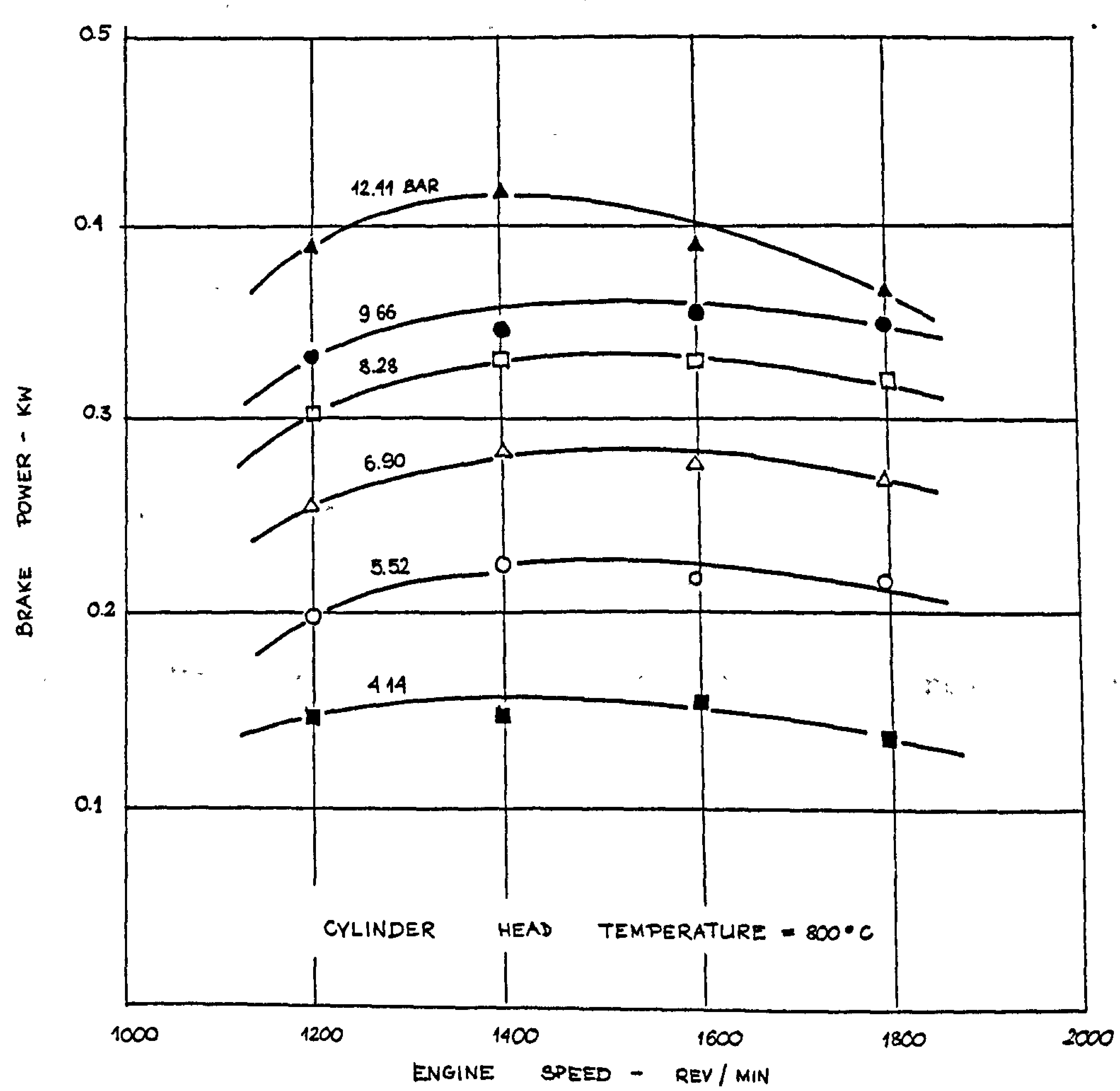



Fig. 3a. Brake power vs speed

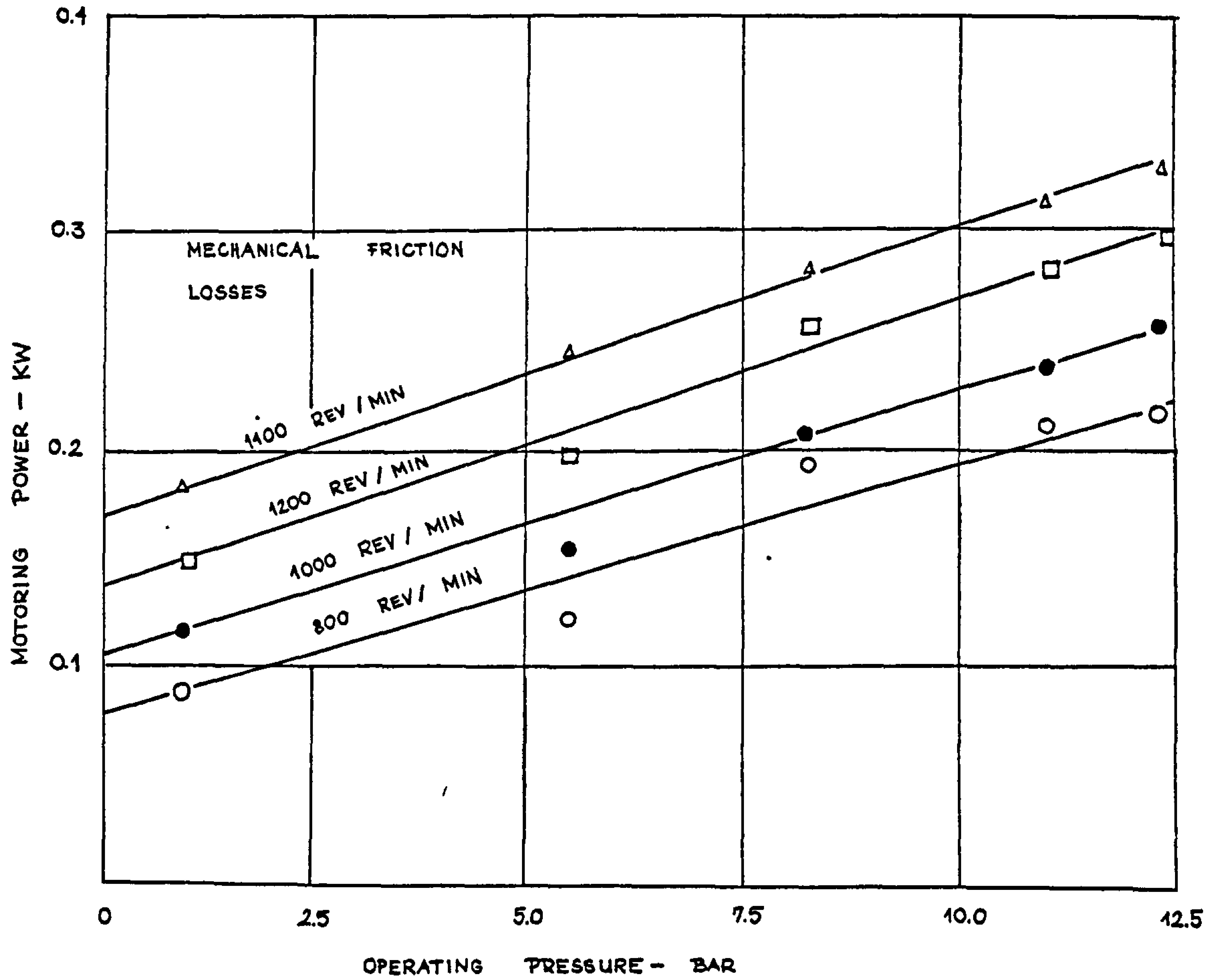
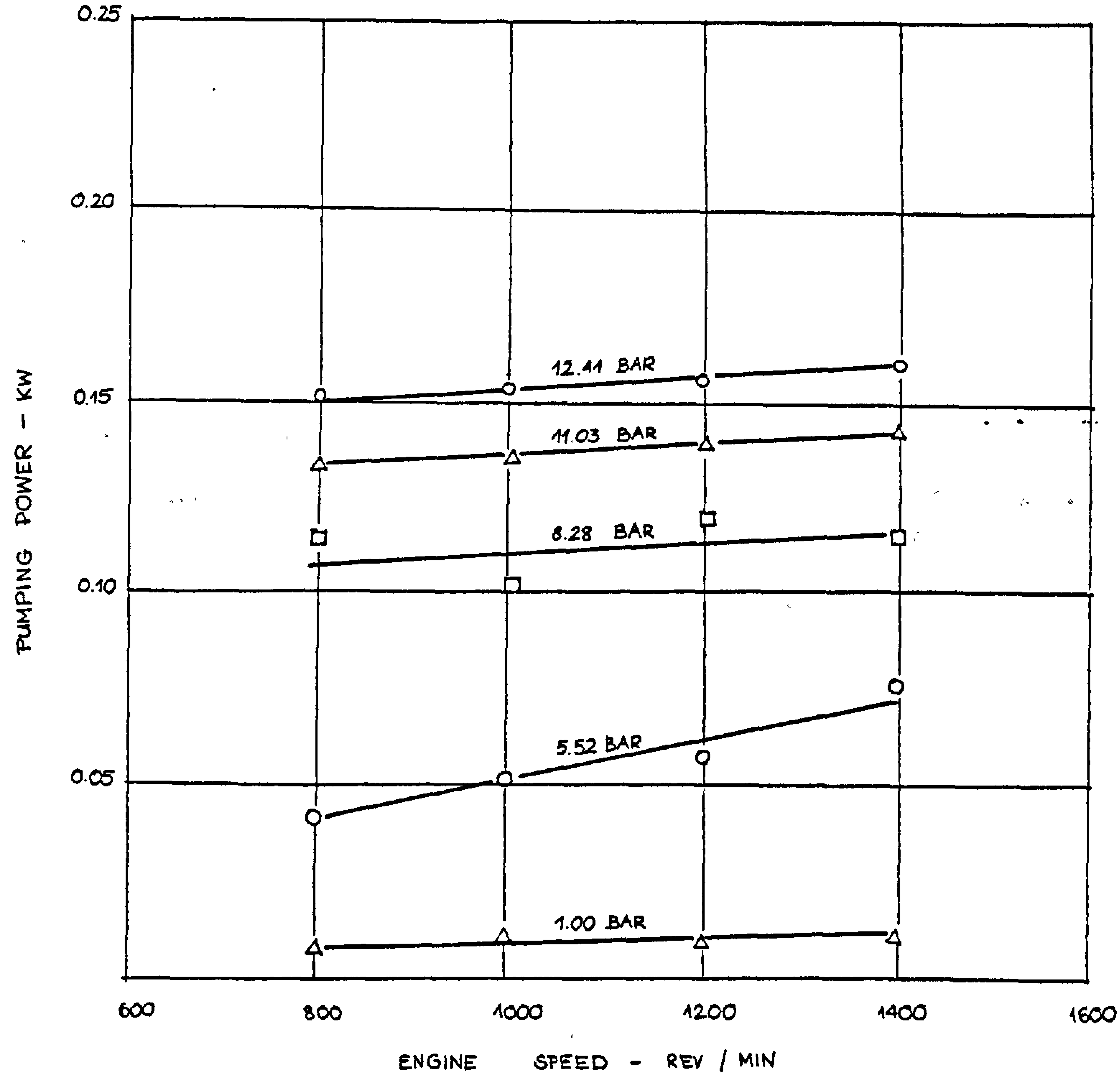
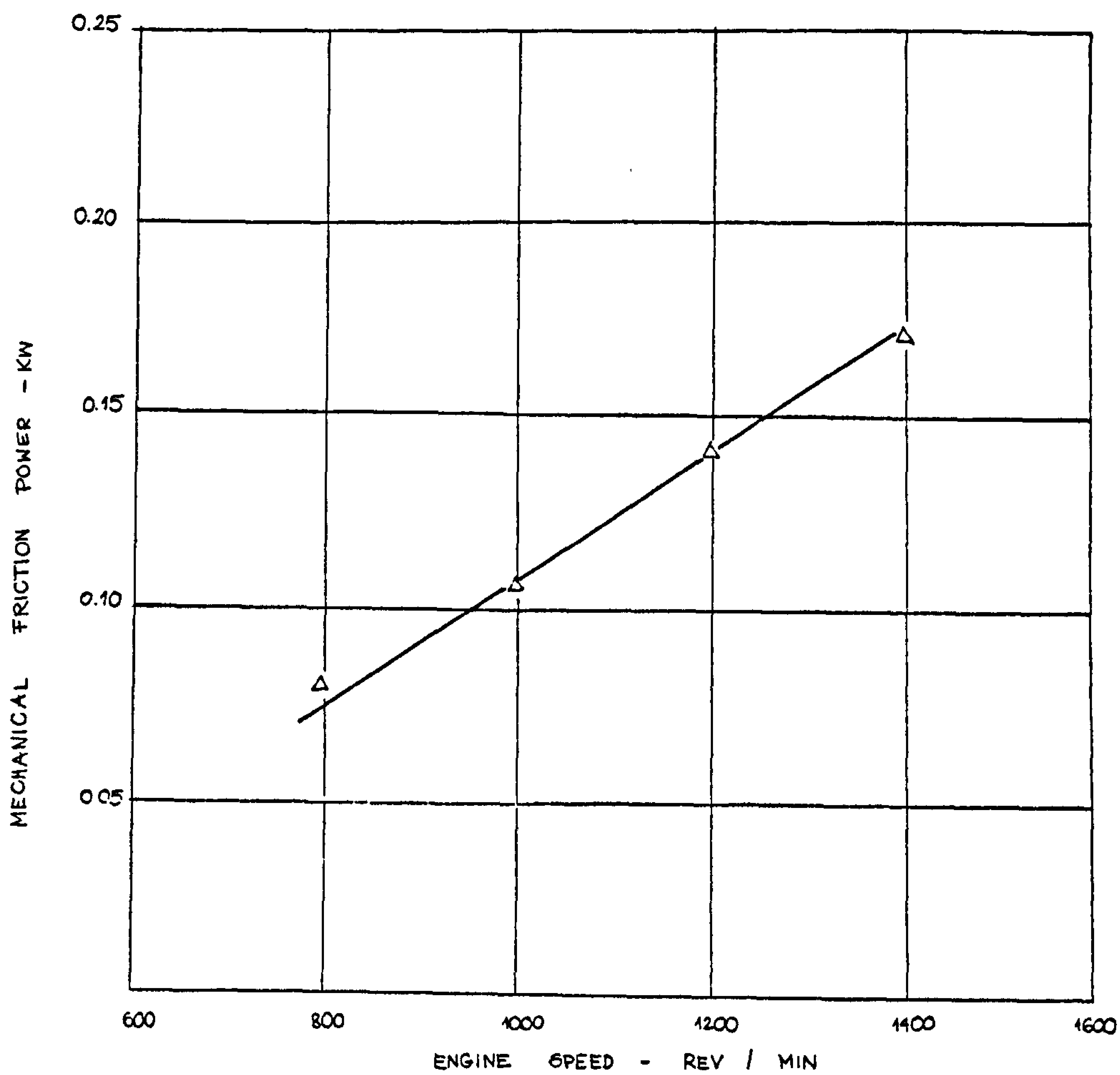



Fig.4. Motoring power in KW vs Operating pressure in bars.




<u>Fig. 5b.</u> Gaseous pumping power vs speed




<u>Fig. 5a.</u> Mechanical friction loss vs speed.

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## 2.4. SUMMARY

One of the earliest revolutionary research programs undertaken on the Stirling Cycle machine was started about 1938 at the Philips Research Laboratories, Eindhoven, Holland, and this has contined since that date. Amongst many of the highly successful Stirling engine models developed by the Philips research team, is one very special one is the Philips MP 1002 engine, incorporated originally with a 200 Watts air-cooled generator set. A pre-production run of about 400 of these machines was produced during the early fifties and disposed of several years later to universities and technical colleges for use as teaching and experimental aids. Considerations of this particular engine and various experiments on modified engines outside the Netherlands are the main subject of the second chapter which also includes the original MP 1002 construction and a feature report of research work at Bath , and Professor Walker's experiment on one of the modified MP1002 engines.
The experimental data from Walker's experiment was used in the computer simulation model which is described in greater detail in part two of this thesis.

# 3. SELECTION OF AVAILABLE DATA MATERIAL FOR ELEMENTARY STEADY-STATE CONSIDERATION

## 3.1. EXPERIMENTS WITH FINKELSTEIN'S DATA

For the purpose of this analysis (see Lit. 2) the most essential parts of a practical Stirling Cycle engine (as explained in Section 1.2) are shown in Fig. 1. The engine is split into five main parts i.e.

1. The expansion space where Volume $V_{we}$ is periodically varied at temp. $T_{we}$.
2. The compression space where volume $V_{wc}$ is periodically varied at temp $T_{wc}$.
3. Volume of the gas enclosed in the regenerator $V_r$.
4. Volume of the gas in left communication duct - $V_{he}$@$T_{he}$
5. Volume of the gas in right communication duct -$V_{hc}$@$T_{hc}$

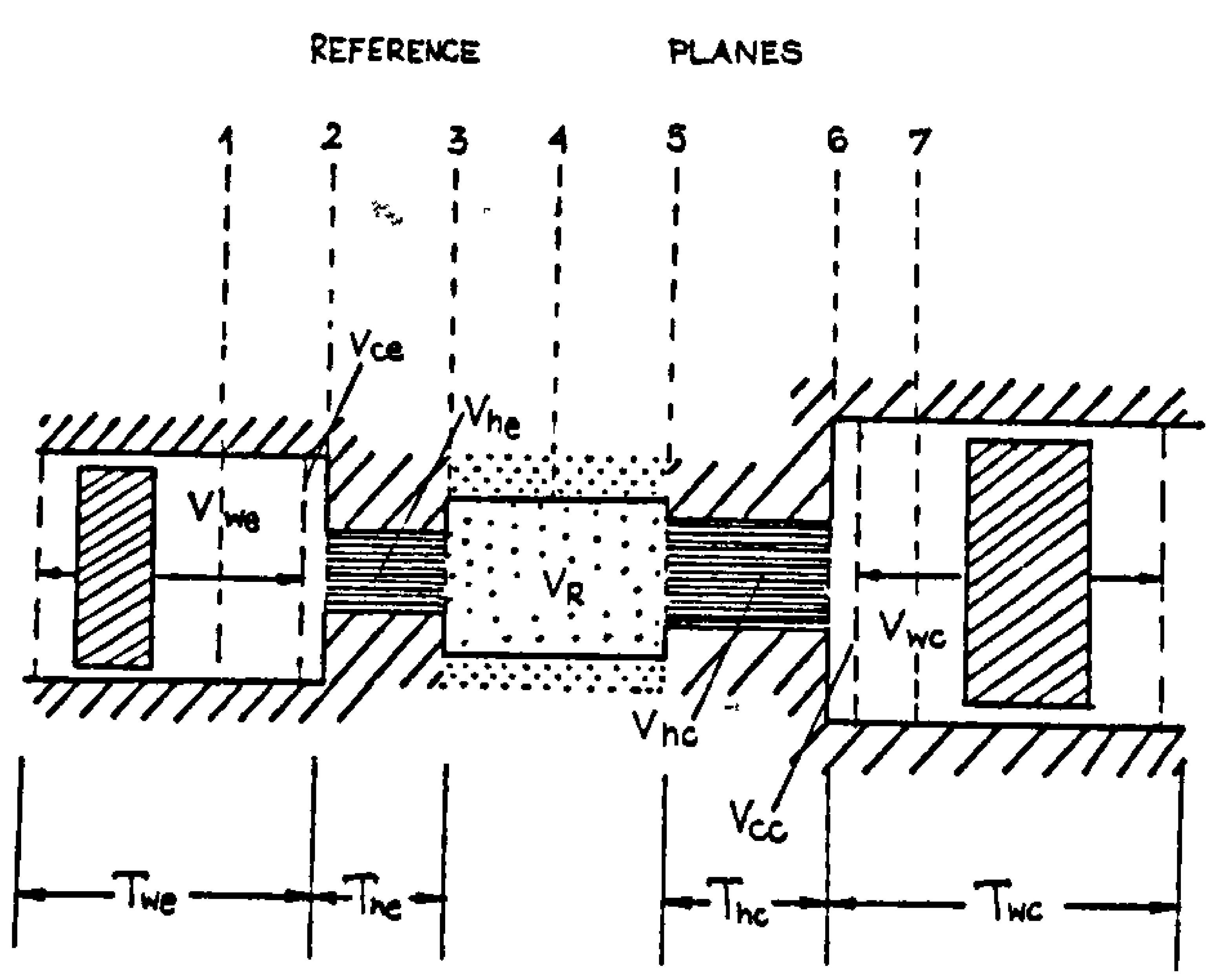


Fig.1. Diagrammatic representation of a closed regenerative gas cycle machine showing the symbols used in the analysis

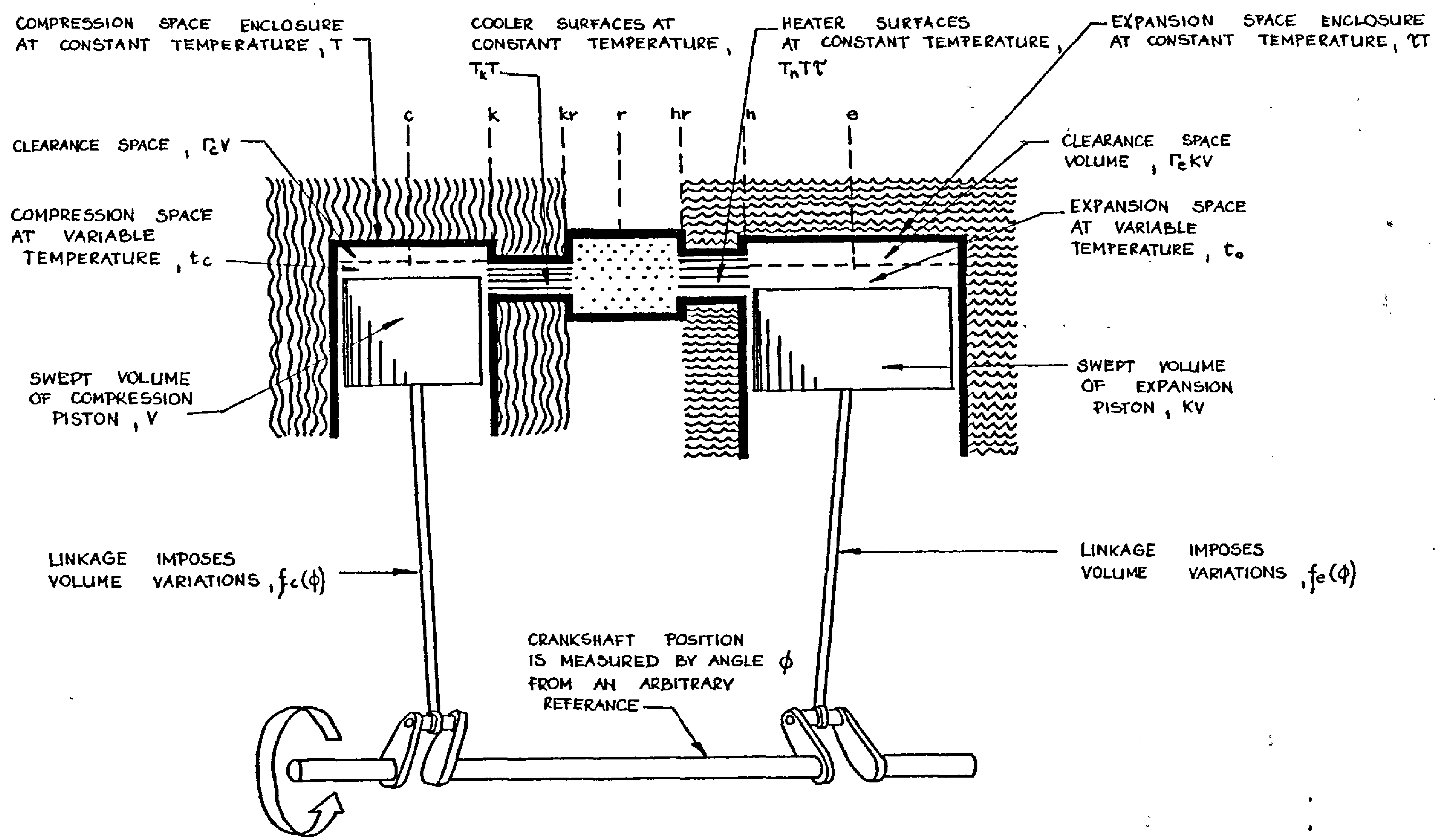


**Fig.2** Configuration diagram of a general closed regenerative-cycle machine defining volumes, temperatures, and reference planes.

The basic energy balance can be written as :

(Heat added to the system) + (Enthalopy added to the) =
by heat transfer system by mass flow

= (Mechanical work done ) + (Increase in mechanical )
by the system energy in the system

And a set of eleven equations which were derived, (see Lit2) govern the cyclic variations in pressure, temperature and mass distribution within the system under steady-state operating conditions. Using the nomenclature at Lit. 1,2, these equations are presented in Tab.1.

Tab.1. Eleven Basic System Equations for the Cycle Processes

| No. | Significance | Equation |
|---|---|---|
| 1. | Expansion space energy balance | $\dot{\tilde\tau}_e = \frac{1}{\tilde\sigma_e}\left\{\left[\gamma\underline{T}_h(\tilde\tau_h+1)-(\tilde\tau_e+1)\right]\tilde\sigma_e - \gamma\underline{H}_e\left[\underline{A}_e+\underline{B}_e f_e(\varphi)\right]\tilde\tau_e - \frac{(\gamma-1)\underline{K}}{\underline{T}}\dot{f}_e(\varphi)\,\psi_e\right\}$ |
| 2. | Compression space energy balance | $\dot{\tilde\tau}_c = \frac{1}{\tilde\sigma_c}\left\{\left[\gamma\underline{T}_k(\tilde\tau_k+1)-(\tilde\tau_c+1)\right]\dot{\tilde\sigma}_c - \gamma\underline{H}_c\left[\underline{A}_c+\underline{B}_c f_c(\varphi)\right]\tilde\tau_c - (\gamma-1)\dot{f}_c(\varphi)\,\psi_c\right\}$ |
| 3. | Heater assembly flow loss | $\dot{\tilde\sigma}_e = \Phi_e\psi_r(\psi_r-\psi_e)$ |
| 4. | Cooler assembly flow loss | $\dot{\tilde\sigma}_c = \Phi_c\psi_r(\psi_r-\psi_c)$ |
| 5. | Heater heat transfer | $\tilde\tau_{hr} = \tilde\tau_h e^{\underline{H}_h/\dot{\tilde\sigma}_e}$ |
| 6. | Cooler heat transfer | $\tilde\tau_{kr} = \tilde\tau_k e^{\underline{H}_k/\dot{\tilde\sigma}_c}$ |
| 7. | Heater entry temperature | $\tilde\tau_h = \frac{\tilde\tau_e+1}{\underline{T}_h} - 1$ when $\dot{\tilde\sigma}_e<0$; $\tilde\tau_{hr} = \underline{T}_{hr}$ when $\dot{\tilde\sigma}_e>0$ |
| 8. | Cooler entry temperature | $\tilde\tau_k = \frac{\tilde\tau_c+1}{\underline{T}_k} - 1$ when $\dot{\tilde\sigma}_c<0$; $\tilde\tau_{kr} = \underline{T}_{kr}$ when $\dot{\tilde\sigma}_c>0$ |
| 9. | Expansion space Pressure | $\psi_e = \frac{\underline{T}\tilde\sigma_e(\tilde\tau_e+1)}{\underline{K}f_e(\varphi)}$ |
| 1o. | Compression space pressure | $\psi_c = \frac{\tilde\sigma_c(\tilde\tau_c+1)}{f_e(\varphi)}$. |
| 11. | Regenerator pressure | $\psi_r = \Lambda(1-\tilde\sigma_e-\tilde\sigma_c)$ |

The above eleven equations describe the cyclic variations of the eleven dependant variables as time variable non-dimensional functions of the crank angle $\varphi$ , and consequently relates them to the independent basic time variable. The nondimensional independent variable and the eleven dependent variables are presented in Tab. 2.

Tab. 2. Non-Dimensional Variables in the System of Equations

| Type of Variable | Significance | Symbol |
|---|---|---|
| Dependent | Crank angle | $\varphi$ |
| Independent | Nondimensional pressures | $\psi_e ; \psi_r ; \psi_c ;$ |
| | Standardized temperature differences | $\tau_e ; \tau_h ; \tau_{hr} ; \tau_{kr} ; \tau_k ; \tau_c ;$ |
| | Mass content ratios | $\sigma_e ; \sigma_c$ |

The input data are lumped numerical parameters which quantify all the relevant characteristics of the system, i.e. configuration, physical properties and external conditions. Consequently Finkelstein derives 14 such parameters, but in addition the volume variations and the heat-transfer area changes must be defined in terms of the independent variable $\varphi$ , together with four proportionality factors. The above briefly outlined problem definition in terms of 18 non-dimensional lumped parameters and two algebraic or Tabulated functions is shown in Tab. 3.

Tab. 3. Problem Definition in Terms of 18 Nondimensional Lumped Parameters and Two Algebraic or Tabulated Functions

| Type of Parameter | Significance | Symbol |
|---|---|---|
| Specifying Conditions | Volume Ratio | $\underline{K}$ |
| | Clearance Ratio | $\Lambda$ |
| | Time Variation of Volumes | $f_e(\varphi), f_c(\varphi)$ |
| | Heat-transfer area coefficients | $\underline{A}_e, \underline{A}_c, \underline{B}_e, \underline{B}_c$ |
| Specifying Operating Condition | Overall temperature ratio | $\underline{T}$ |
| Describing thermo-dynamic performance | Heat-transfer coefficients | $\underline{H}_e, \underline{H}_c, \underline{H}_h, \underline{H}_k$ |
| | Regenerator outlet temperature ratios | $\underline{T}_{hr}, \underline{T}_{kr}$ |
| | Heat exchanger temperature ratios | $\underline{T}_h, \underline{T}_k$ |
| | Aerodynamic friction factors | $\underline{\Phi}_e, \underline{\Phi}_c$ |
| Relating to Working fluid | Ratios of specific heats | $\tilde{\gamma}$ |

For better understanding of the regenerator process, the overall operations of the Stirling Cycle engine are indicated in terms of the energy input and relevant output quantities. The total heat transferred to the working fluid at or near the one temperature T is the sum of the heat supplied to the cylinder $Q_e$, and to the adjacent heat exchanger $Q_h$, and the heat rejection $Q_c$ and $Q_k$ respectively. Additionally the regenerator loss E is equivalent to a heat flow with the aerodynamic friction losses being $Z_e$ and $Z_c$.

Finally the mechanical work produced by the gas in the expansion space is $P_e$ and that absorbed by the compression space - $P_c$, with net work P transmitted by the output shaft being the algebraic sum of the two individual work quantities. For each completed cycle $\emptyset$ is equal $2\pi = 360^o$, and the cyclic Integration formulas for the Heat Transmission, Losses and mechanical work are listed in Tab. 4. Finkelstein's simulation model of the Stirling Cycle engine's steady-state operation under specificied conditions is fully described by these two sets of equations No. 1 $\div$ 11 and No. 12 $\div$ 20.

TAB.4. Cyclic Integration Formulas for the Heat Transmission, Losses and Mechanical Work as Derived in (4)

| Number | Significance | Equation |
| --- | --- | --- |
| 12. | Expansion space heat supply | $Q_e = \frac{\gamma WRT}{\gamma - 1} \underline{T}H_c \oint \left[ \underline{A}_c + \underline{B}_c f_c \ (\varphi) \right] \tau_c \, d\varphi$ |
| 13. | Compression space heat rejection | $Q_c = \frac{\gamma WRT}{\gamma - 1} H_c \oint \left[ \underline{A}_c + \underline{B}_c f_c \ (\varphi) \right] \tau_e \, d\varphi$ |
| 14. | Heater heat supply | $Q_h = -\frac{\gamma WRT}{\gamma - 1} \ \underline{T}T_h \oint (\tau_h - \tau_{hr}) \dot{\tilde{\sigma}}_e \, d\varphi$ |
| 15. | Cooler heat rejection | $Q_k = \frac{\gamma WRT}{\gamma - 1} \ T_k \oint (\tau_k - \tau_{kr}) \dot{\tilde{\sigma}}_c \, d\varphi$ |
| 16. | Regenerator Loss | $E_h = \frac{\gamma WRT}{\gamma - 1} \ \underline{T}T_h \oint (\tau_{hr} + 1) \dot{\tilde{\sigma}}_e \, d\varphi; \ \frac{\gamma WRT}{\gamma - 1} \underline{T}_k \oint (\tau_{kr} + 1) \dot{\tilde{\sigma}}_c \, d\varphi = E_k$ |
| 17. | Expansion work | $P_e = WRTK \oint \dot{f}_e \ (\varphi) \ \psi_e \, d\varphi$ |
| 18. | Compression work | $P_c = - WRT \oint \dot{f}_c \ (\varphi) \ \psi_c \, d\varphi$ |
| 19. | Expansion assembly flow loss | $Z_e = WRTK \oint (\psi_r - \psi_e) \ \dot{f}_e \ (\varphi) \, d\varphi$ |
| 20. | Compression assembly flow loss | $Z_c = - WRT \oint (\psi_r - \psi_c) \ \dot{f}_c \ (\varphi) \, d\varphi$ |

Finkelstein's system of equations was then represented mathematically by a set of purely analogue high-gain direct current operational amplifiers acting mainly as an "equation-solving circuit" consisting of eleven sub-circuits (equations nos. 1 ÷ 11) with all the fourteen basic numerical parameters simulated usually by means of setting the relevant rotary potentiometers. For the equations Nos. 12 ÷ 20 (see Tab. 4) a specially designed integrated circuit was provided (see Fig. 3). The complete analogue computer system for the above simulation consists of the following blocks : oscillating circuit, switching circuit, equation solving circuit, integrating circuit, numerical readout, time base recorder, co-ordinate plotter.

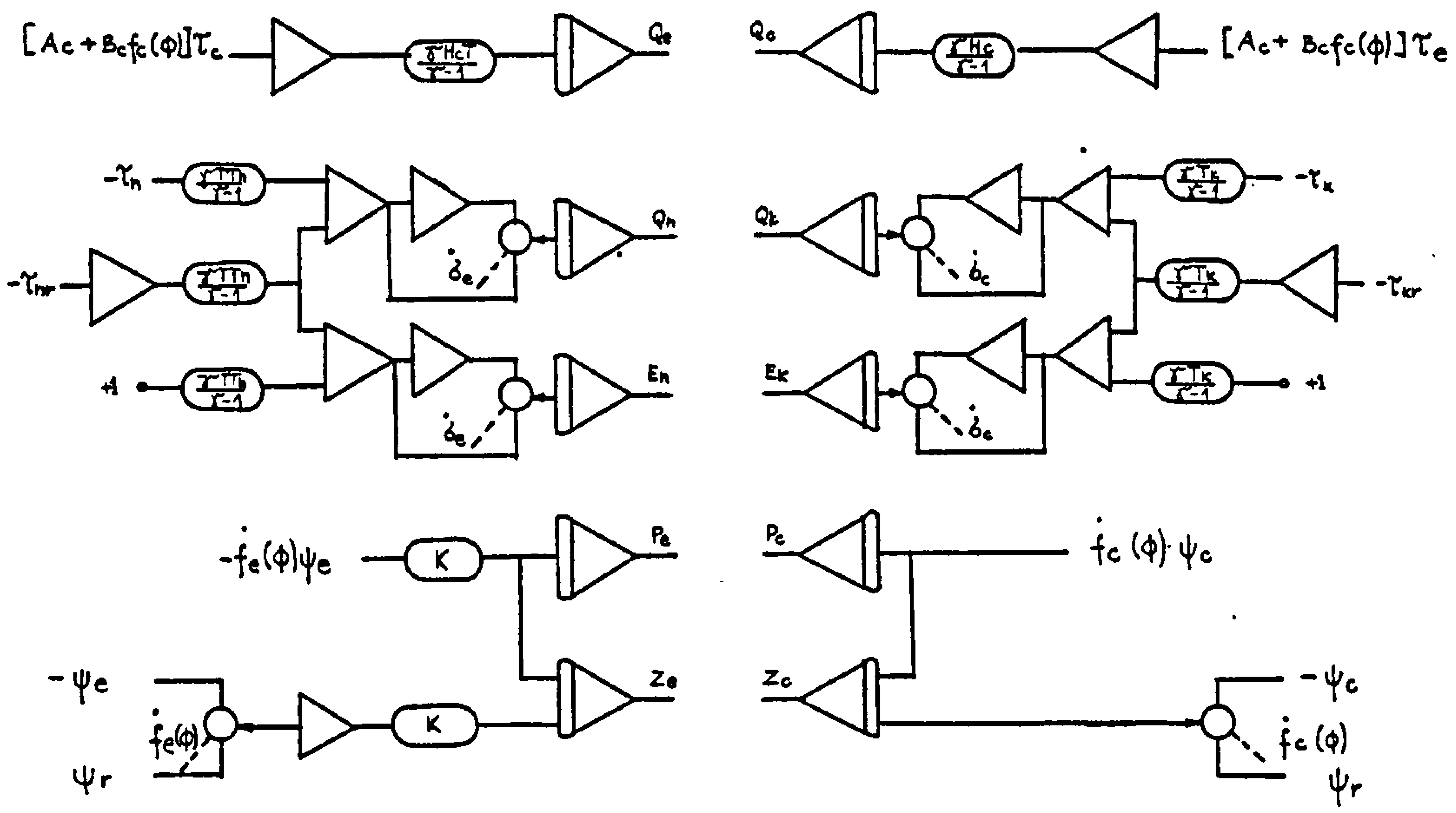


Fig. 3. Integrating Circuit, consisting of 10 subcircuits, one for each of the heat-balance equations listed in Tab. 4

The "Equation-solving circuit",consisting of 11 sub-circuits provided one for each of the basic equations (No1÷11) is shown in Fig. 4.

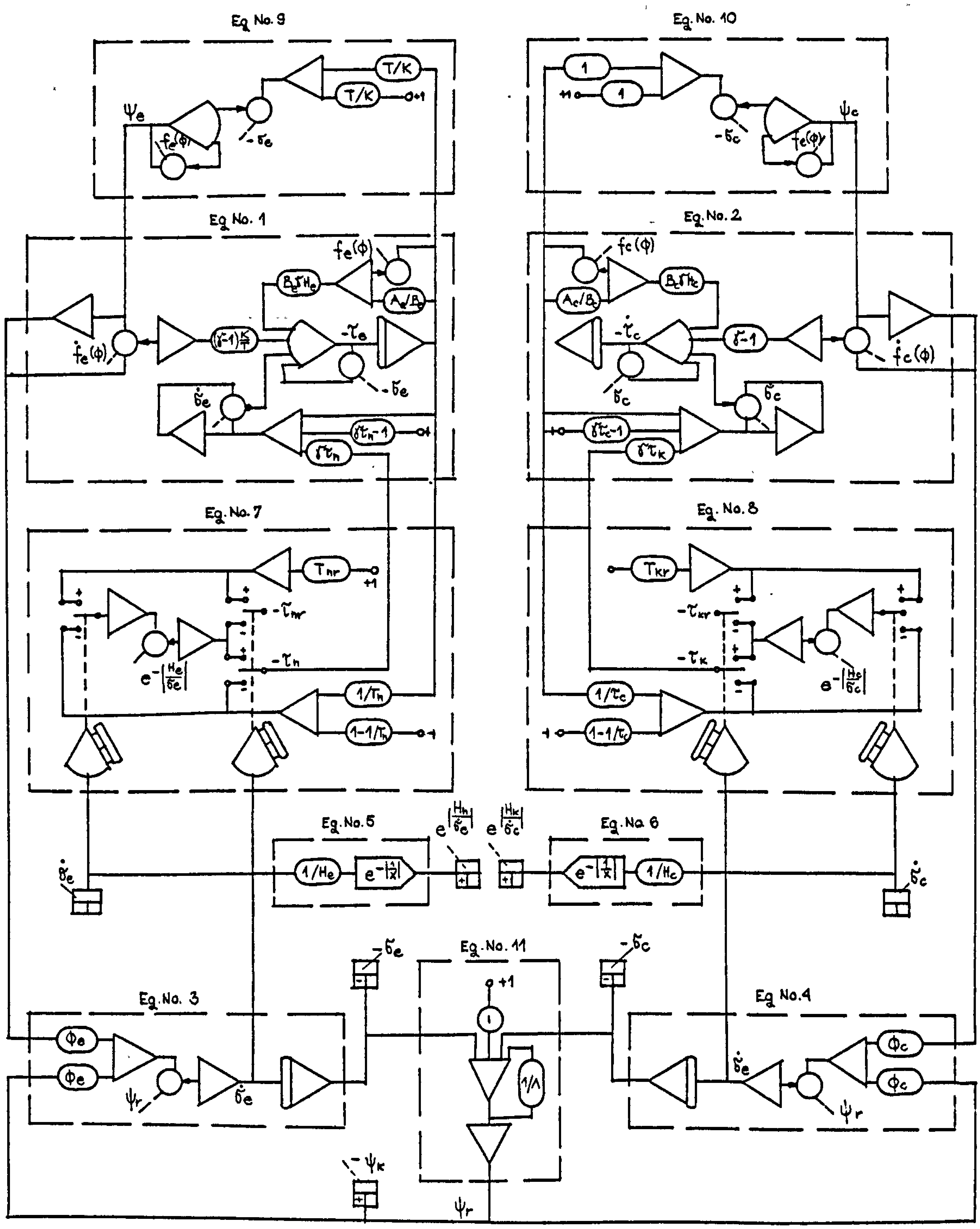


Fig.4 Equation-solving circuit, consisting of 11 subcircuits, one for each of the basic equations listed in tab.1

The "Equation-solving circuit" is kept cycling continuously, but the integrating circuit is triggered just for one cycle only.

The input signal for both "Equation-solving"as well as integrating circuits is provided for the Oscillating circuit generating periodic functions for the time dependent voltage levels representing the input functions. A block diagram of the above described system is presented in simplified form in Fig. 5.

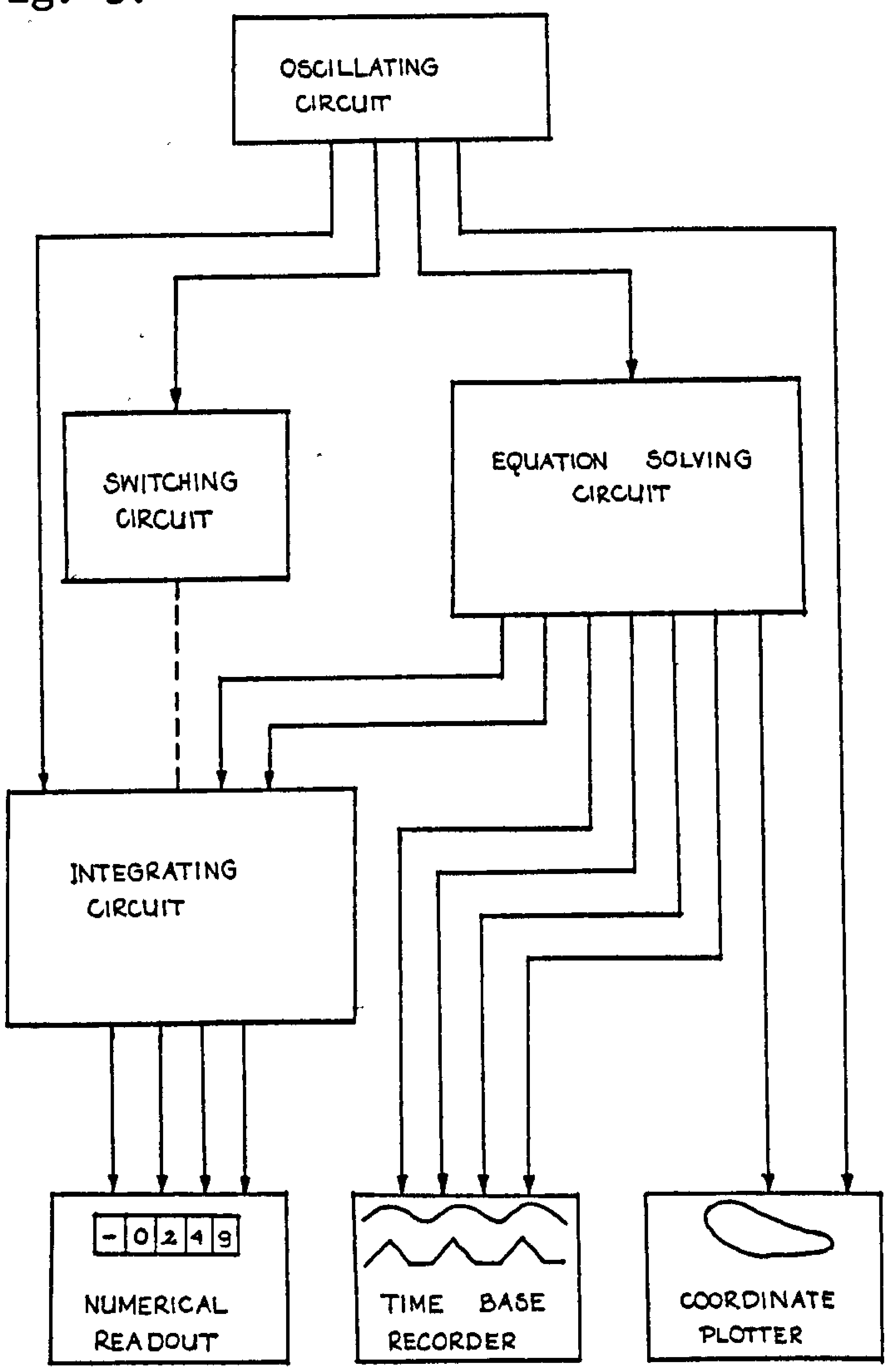


Fig. 5. Complete analog computer system for simulation

The output solution's read-out is produced in two forms: as multi-channel strip recorder and also an X - Y type plotter. Finally integrating circuit output is provided in a simple form of a usual numerical readout. The above described analog computing procedure, provides most of the necessary quantitative evaluations, important for the specific character of the Stirling Cycle process. However, there is one usual analog drawback, namely the lower accuracy of analog computing techniques when compared with the naturally more accurate digital results. A simple digital solution to one of the earlier described processes would be a simulation model based on Finkelstein's equations andprepared data, written in one of the "high-level" computer languages and run on a high-speed digital system. As an example, a mono-parametric "net heat susplied and absorbed in expansion space" process solution only is presented, based on Finkelstein's theory, (Lit. 1,2). This simulation program obviously is only a simple example of a much more sophisticated programming effort, which can be undertaken in order to simulate more complex Stirling Cycle problems, thus allowing eventually the user to omit a fairly large number of usual analog computer components such as: summers, integrators, dividers, relays and limiters, function generators etc. Net heat supplied and absorbed in the expansion space consists basically of two thermal ingredients :

1. Expansion space heat supply
2. Heater heat supply

Using the nomenclature as in Lit. 1,2, this can be expressed as:

$Q_s = Q_e + Q_h$ where: $Q_s$ - Net heat supplied (signal i/P)

$Q_e = f_1(\varphi)$ - Expansion space heat supply

$Q_h = f_2(\varphi)$ - Heater heat supply

If heat supply in the expansion is defined by :

$$Q_e = -\frac{\gamma WRT}{\gamma - 1}\, \underline{TH}_c \oint \left[\underline{A}_c + \underline{B}_c f_c(\varphi)\right] \tau_c \, d\varphi$$

and relevant heat supply is :

$$Q_h = -\frac{\gamma WRT}{\gamma - 1}\, \underline{TT}_h \oint (\tau_h - \tau_{hr})\, \dot{\sigma}_e \, d\varphi$$

Note: Small letters refer to:

c - compression space
h - heater inlet on expansion space
hr- regenerator inlet on expansion side
$\varphi$ - the crank angle of the output shaft

Adding these items together and simplifying, one obtains :

$$Q_s = -\frac{\gamma WRT}{\gamma - 1}\left\{ \underline{TH}_c \oint \left[\underline{A}_c + \underline{B}_c f_c \quad (\varphi)\right] \tau_c d\varphi + \underline{TT}_h \oint (\tau_h - \tau_{hr})\, \dot{\sigma}_e \, d\varphi \right\}$$

Compare with the "Sample Solution" (Lit.1).

$\gamma$ = 1.4
$\underline{T}$ = 2.0
$\underline{Hc}$ = 0.5
Ac = 1.0
$\underline{Bc}$ = 0.0

$f_c(\varphi) = 0.5(1-\sin\varphi) + 0.04$

$\underline{T}$ = 2.0
$\underline{T}_h$ = 1.0

and also $\tau_c = \left(\frac{tc}{T}\right) - 1$

$\tau_e = \left(\frac{te}{T}\right) - 1$

and $\tau_h$ ) are Dimensionless quantities
$\tau_{hr}$ )
$\dot{\sigma}_e$ – Mass flow rate

Therefore the $Q_s$ expression can be rewritten in the new form:

$$Q_s = \frac{1.4WRT}{1.4-1.} \left\{ 2.0 \,.\, 0.5 \oint \left[1.0+0.0 f_c\ (\varphi)\right] \tau_c d\varphi + 2.0.\ 1.0. \oint (\tau_h - \tau_{hr}) \dot{\sigma}_e d\varphi \right\} = 3.5WRT \left\{ \oint \tau_c d\varphi + 2.0 \oint (\tau_h - \tau_{hr}) \cdot \dot{\sigma}_e\, d\varphi \right\}$$

The mass flow rate may be written as :

$$\dot{\sigma}_e = \Phi_e \cdot \Psi_r \ (\Psi_r - \Psi_e)$$

Assuming (after Finkelstein Lit.1,2,) that the following system parameters are varying within the range :

| Min. Constant value | | The quantity | | Max. const. val. | Par. No |
|---|---|---|---|---|---|
| - 0.302 | $\leq$ | $\dot{\sigma}_e$ | $\leq$ | +0.198 | 1 |
| - 0.027 | $\leq$ | $\tau_c$ | $\leq$ | +0.0825 | 2 |
| - 0.052 | $\leq$ | $\tau_h$ | $\leq$ | +0.0145 | 3 |
| - 0.031 | $\leq$ | $\tau_{hr}$ | $\leq$ | +0.0027 | 4 |

Finally referring to the above data, and the equation describing the net heat supplied to the engine, this may be written as :

$$Q_s = 3.5\ WRT \left[ \tau_c \oint d\varphi + 2.0\ (\tau_h - \tau_{hr}) \cdot \dot{\sigma}_e \oint d\varphi \right] =$$

$$3.5\ WRT \left[ \tau_c + 2.0\ \ (\tau_h - \tau_{hr}) \cdot \dot{\sigma}_e \right] \oint d\varphi$$

OR:

$$\frac{Q_s}{WRT} = \frac{Q_e + Q_h}{WRT} = 3.5 \left[ \tau_c + 2.0\ (\tau_h - \tau_{hr})\ \dot{\sigma}_e \right]$$

In order to cover the whole range of the swing of the parameters No.1÷4 the following computational order is set-up (see Tab. 5)

Tab. 5

| Loop | Parameter | Variable: | Parameters | 3,2,1 - | Const. on | Min. val. |
|---|---|---|---|---|---|---|
| No.1 | 4 | | | | | |
| 2 | 3 | " | " | 4,2,1 | " | " |
| 3 | 2 | " | " | 1,3,4 | " | " |
| 4 | 1 | " | " | 2,3,4 | " | " |
| 5 | 4 | " | " | 3,2,1 | Const. on | Max. Val |
| 6 | 3 | " | " | 4,2,1 | " | " |
| 7 | 2 | " | " | 1,3,4 | " | " |
| 8 | 1 | " | " | 2,3,4 | " | " |

And a listing of the simple program written in Fortran IV language is presented in Program No.1.

```
 MINNESOTA 6600 FORTRAN COMPILER SCOPE  3.4.1 VER4.5      19/03/76  1
      PROGRAM HEATSU1(INPUT,OUTPUT,PLOT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE27=
     1PLOT)
      DIMENSION PHI(21),QS(21),ITITLE(1),IXLAB(10),IYLAB(10)
      PI=3.1415926
      READ(5,5)IXLAB(10),IYLAB(10),ITITLE(1)
    5 FORMAT(2A10,A1)
      TAUC=0.0825
      TAUH=0.0145
      TAUHR=0.0027
      CALL CAM35 MM
      CALL GRSLIDE
      CALL YAXIS(0.,2.*PI)
      CALL XAXIS(0.,2.0)
      DO 1 J=1,11
      AJ=J
      SIGEPR=-0.302+(AJ-1.0)*0.05
      DO 2 K=1,21
      AK=K
      PHI(K)=(AK-1.)*PI/10.
      QS(K)=3.5*(TAUC+2.0*(TAUH-TAUHR)*SIGEPR)*PHI(K)
    2 WRITE(6,3) SIGEPR,PHI(K),QS(K)
    3 FORMAT(3E10.3)
      CALL POLY3(QS,PHI,21)
    1 CONTINUE
      CALL GRAFDEF(ITITLE,1,IXLAB,1,IYLAB,1,0)
      CALL ENDFILM
      STOP
      END
```

Prog.1. Program HEATSU 1

The above program listing also contains graph plotting routine facilities provided for the Tektronix hard-copy plotter. Initially the program prints out the results (see Tab. 6) in a tabulated form of three columns, i.e. $\delta^{\circ}$, $\varphi$ and $Q_s$. Because of the limited space available only loop No. 8 results are presented with a corresponding plot i.e. $Q_s = f(\varphi)$ see Fig. 6. The HEATSU 1 program output plot shows graphical interpretation of the tabulated "Net heat supplied" - Q values calculated for the output crank shaft angle of rotation $\varphi$, where $0 \leqslant \varphi \leqslant 2\pi$. Ref. the loop No. 8 set-up conditions.

Tab.6. Program HEATSU 1 - Loop No. 8 results.

| | | | | | |
|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | -.152E+00 | .157E+01 | .434E+00 |
| -.302E+00 | 0 | 0 | -.152E+00 | .188E+01 | .521E+00 |
| -.302E+00 | .314E+00 | .829E-01 | -.152E+00 | .220E+01 | .607E+00 |
| -.302E+00 | .628E+00 | .166E+00 | -.152E+00 | .251E+01 | .694E+00 |
| -.302E+00 | .942E+00 | .249E+00 | -.152E+00 | .283E+01 | .781E+00 |
| -.302E+00 | .126E+01 | .332E+00 | -.152E+00 | .314E+01 | .868E+00 |
| -.302E+00 | .157E+01 | .414E+00 | -.152E+00 | .346E+01 | .954E+00 |
| -.302E+00 | .188E+01 | .497E+00 | -.152E+00 | .377E+01 | .104E+01 |
| -.302E+00 | .220E+01 | .580E+00 | -.152E+00 | .408E+01 | .113E+01 |
| -.302E+00 | .251E+01 | .663E+00 | -.152E+00 | .440E+01 | .121E+01 |
| -.302E+00 | .283E+01 | .746E+00 | -.152E+00 | .471E+01 | .130E+01 |
| -.302E+00 | .314E+01 | .829E+00 | -.152E+00 | .503E+01 | .139E+01 |
| -.302E+00 | .346E+01 | .912E+00 | -.152E+00 | .534E+01 | .148E+01 |
| -.302E+00 | .377E+01 | .995E+00 | -.152E+00 | .565E+01 | .156E+01 |
| -.302E+00 | .408E+01 | .108E+01 | -.152E+00 | .597E+01 | .165E+01 |
| -.302E+00 | .440E+01 | .116E+01 | -.152E+00 | .628E+01 | .174E+01 |
| -.302E+00 | .471E+01 | .124E+01 | -.102E+00 | 0 | 0 |
| -.302E+00 | .503E+01 | .133E+01 | -.102E+00 | .314E+00 | .881E-01 |
| -.302E+00 | .534E+01 | .141E+01 | -.102E+00 | .628E+00 | .176E+00 |
| -.302E+00 | .565E+01 | .149E+01 | -.102E+00 | .942E+00 | .264E+00 |
| -.302E+00 | .597E+01 | .157E+01 | -.102E+00 | .126E+01 | .352E+00 |
| -.302E+00 | .628E+01 | .166E+01 | -.102E+00 | .157E+01 | .440E+00 |
| -.252E+00 | 0 | 0 | -.102E+00 | .188E+01 | .528E+00 |
| -.252E+00 | .314E+00 | .842E-01 | -.102E+00 | .220E+01 | .616E+00 |
| -.252E+00 | .628E+00 | .168E+00 | -.102E+00 | .251E+01 | .705E+00 |
| -.252E+00 | .942E+00 | .253E+00 | -.102E+00 | .283E+01 | .793E+00 |
| -.252E+00 | .126E+01 | .337E+00 | -.102E+00 | .314E+01 | .881E+00 |
| -.252E+00 | .157E+01 | .421E+00 | -.102E+00 | .346E+01 | .969E+00 |
| -.252E+00 | .188E+01 | .505E+00 | -.102E+00 | .377E+01 | .106E+01 |
| -.252E+00 | .220E+01 | .589E+00 | -.102E+00 | .408E+01 | .114E+01 |
| -.252E+00 | .251E+01 | .673E+00 | -.102E+00 | .440E+01 | .123E+01 |
| -.252E+00 | .283E+01 | .758E+00 | -.102E+00 | .471E+01 | .132E+01 |
| -.252E+00 | .314E+01 | .842E+00 | -.102E+00 | .503E+01 | .141E+01 |
| -.252E+00 | .346E+01 | .926E+00 | -.102E+00 | .534E+01 | .150E+01 |
| -.252E+00 | .377E+01 | .101E+01 | -.102E+00 | .565E+01 | .159E+01 |
| -.252E+00 | .408E+01 | .109E+01 | -.102E+00 | .597E+01 | .167E+01 |
| -.252E+00 | .440E+01 | .118E+01 | -.102E+00 | .628E+01 | .176E+01 |
| -.252E+00 | .471E+01 | .126E+01 | -.520E-01 | 0 | 0 |
| -.252E+00 | .503E+01 | .135E+01 | -.520E-01 | .314E+00 | .894E-01 |
| -.252E+00 | .534E+01 | .143E+01 | -.520E-01 | .628E+00 | .179E+00 |
| -.252E+00 | .565E+01 | .152E+01 | -.520E-01 | .942E+00 | .268E+00 |
| -.252E+00 | .597E+01 | .160E+01 | -.520E-01 | .126E+01 | .357E+00 |
| -.252E+00 | .628E+01 | .168E+01 | -.520E-01 | .157E+01 | .447E+00 |
| -.202E+00 | 0 | 0 | -.520E-01 | .188E+01 | .536E+00 |
| -.202E+00 | .314E+00 | .855E-01 | -.520E-01 | .220E+01 | .626E+00 |
| -.202E+00 | .628E+00 | .171E+00 | -.520E-01 | .251E+01 | .715E+00 |
| -.202E+00 | .942E+00 | .256E+00 | -.520E-01 | .283E+01 | .804E+00 |
| -.202E+00 | .126E+01 | .342E+00 | -.520E-01 | .314E+01 | .894E+00 |
| -.202E+00 | .157E+01 | .427E+00 | -.520E-01 | .346E+01 | .983E+00 |
| -.202E+00 | .188E+01 | .513E+00 | -.520E-01 | .377E+01 | .107E+01 |
| -.202E+00 | .220E+01 | .598E+00 | -.520E-01 | .408E+01 | .116E+01 |
| -.202E+00 | .251E+01 | .684E+00 | -.520E-01 | .440E+01 | .125E+01 |
| -.202E+00 | .283E+01 | .769E+00 | -.520E-01 | .471E+01 | .134E+01 |
| -.202E+00 | .314E+01 | .855E+00 | -.520E-01 | .503E+01 | .143E+01 |
| -.202E+00 | .346E+01 | .940E+00 | -.520E-01 | .534E+01 | .152E+01 |
| -.202E+00 | .377E+01 | .103E+01 | -.520E-01 | .565E+01 | .161E+01 |
| -.202E+00 | .408E+01 | .111E+01 | -.520E-01 | .597E+01 | .170E+01 |
| -.202E+00 | .440E+01 | .120E+01 | -.520E-01 | .628E+01 | .179E+01 |
| -.202E+00 | .471E+01 | .128E+01 | -.200E-02 | 0 | 0 |
| -.202E+00 | .503E+01 | .137E+01 | -.200E-02 | .314E+00 | .907E-01 |
| -.202E+00 | .534E+01 | .145E+01 | -.200E-02 | .628E+00 | .181E+00 |
| -.202E+00 | .565E+01 | .154E+01 | -.200E-02 | .942E+00 | .272E+00 |
| -.202E+00 | .597E+01 | .162E+01 | -.200E-02 | .126E+01 | .363E+00 |
| -.202E+00 | .628E+01 | .171E+01 | -.200E-02 | .157E+01 | .453E+00 |
| -.152E+00 | 0 | 0 | -.200E-02 | .188E+01 | .544E+00 |

| | | |
|---|---|---|
| -.200E-02 | .346E+01 | .997E+00 |
| -.200E-02 | .377E+01 | .109E+01 |
| -.200E-02 | .408E+01 | .118E+01 |
| -.200E-02 | .440E+01 | .127E+01 |
| -.200E-02 | .471E+01 | .136E+01 |
| -.200E-02 | .503E+01 | .145E+01 |
| -.200E-02 | .534E+01 | .154E+01 |
| -.200E-02 | .565E+01 | .163E+01 |
| -.200E-02 | .597E+01 | .172E+01 |
| -.200E-02 | .628E+01 | .181E+01 |
| .480E-01 | 0 | 0 |
| .480E-01 | .314E+00 | .920E-01 |
| .480E-01 | .628E+00 | .184E+00 |
| .480E-01 | .942E+00 | .276E+00 |
| .480E-01 | .126E+01 | .368E+00 |
| .480E-01 | .157E+01 | .460E+00 |
| .480E-01 | .188E+01 | .552E+00 |
| .480E-01 | .220E+01 | .644E+00 |
| .480E-01 | .251E+01 | .736E+00 |
| .480E-01 | .283E+01 | .828E+00 |
| .480E-01 | .314E+01 | .920E+00 |
| .480E-01 | .346E+01 | .101E+01 |
| .480E-01 | .377E+01 | .110E+01 |
| .480E-01 | .408E+01 | .120E+01 |
| .480E-01 | .440E+01 | .129E+01 |
| .480E-01 | .471E+01 | .138E+01 |
| .480E-01 | .503E+01 | .147E+01 |
| .480E-01 | .534E+01 | .156E+01 |
| .480E-01 | .565E+01 | .166E+01 |
| .480E-01 | .597E+01 | .175E+01 |
| .480E-01 | .628E+01 | .184E+01 |
| .980E-01 | 0 | 0 |
| .980E-01 | .314E+00 | .933E-01 |
| .980E-01 | .628E+00 | .187E+00 |
| .980E-01 | .942E+00 | .280E+00 |
| .980E-01 | .126E+01 | .373E+00 |
| .980E-01 | .157E+01 | .466E+00 |
| .980E-01 | .188E+01 | .560E+00 |
| .980E-01 | .220E+01 | .653E+00 |
| .980E-01 | .251E+01 | .746E+00 |
| .980E-01 | .283E+01 | .839E+00 |
| .980E-01 | .314E+01 | .933E+00 |
| .980E-01 | .346E+01 | .103E+01 |
| .980E-01 | .377E+01 | .112E+01 |
| .980E-01 | .408E+01 | .121E+01 |
| .980E-01 | .440E+01 | .131E+01 |
| .980E-01 | .471E+01 | .140E+01 |
| .980E-01 | .503E+01 | .149E+01 |
| .980E-01 | .534E+01 | .159E+01 |
| .980E-01 | .565E+01 | .168E+01 |
| .980E-01 | .597E+01 | .177E+01 |
| .980E-01 | .628E+01 | .187E+01 |
| .148E+00 | 0 | 0 |
| .148E+00 | .314E+00 | .946E-01 |
| .148E+00 | .628E+00 | .189E+00 |
| .148E+00 | .942E+00 | .284E+00 |
| .148E+00 | .126E+01 | .378E+00 |
| .148E+00 | .157E+01 | .473E+00 |
| .148E+00 | .188E+01 | .567E+00 |
| .148E+00 | .220E+01 | .662E+00 |
| .148E+00 | .251E+01 | .756E+00 |
| .148E+00 | .283E+01 | .851E+00 |
| .148E+00 | .314E+01 | .946E+00 |
| .148E+00 | .346E+01 | .104E+01 |
| .148E+00 | .377E+01 | .113E+01 |
| .148E+00 | .408E+01 | .123E+01 |
| .148E+00 | .440E+01 | .132E+01 |
| .148E+00 | .471E+01 | .142E+01 |
| .148E+00 | .503E+01 | .151E+01 |
| .148E+00 | .534E+01 | .161E+01 |
| .148E+00 | .565E+01 | .170E+01 |
| .148E+00 | .597E+01 | .180E+01 |
| .148E+00 | .628E+01 | .189E+01 |
| .198E+00 | 0 | 0 |
| .198E+00 | .314E+00 | .959E-01 |
| .198E+00 | .628E+00 | .192E+00 |
| .198E+00 | .942E+00 | .288E+00 |
| .198E+00 | .126E+01 | .383E+00 |
| .198E+00 | .157E+01 | .479E+00 |
| .198E+00 | .188E+01 | .575E+00 |
| .198E+00 | .220E+01 | .671E+00 |
| .198E+00 | .251E+01 | .767E+00 |
| .198E+00 | .283E+01 | .863E+00 |
| .198E+00 | .314E+01 | .959E+00 |
| .198E+00 | .346E+01 | .105E+01 |
| .198E+00 | .377E+01 | .115E+01 |
| .198E+00 | .408E+01 | .125E+01 |
| .198E+00 | .440E+01 | .134E+01 |
| .198E+00 | .471E+01 | .144E+01 |
| .198E+00 | .503E+01 | .153E+01 |
| .198E+00 | .534E+01 | .163E+01 |
| .198E+00 | .565E+01 | .173E+01 |
| .198E+00 | .597E+01 | .182E+01 |
| .198E+00 | .628E+01 | .192E+01 |

4657 CHARACTERS OUTPUT

Tab. 6. Continuation

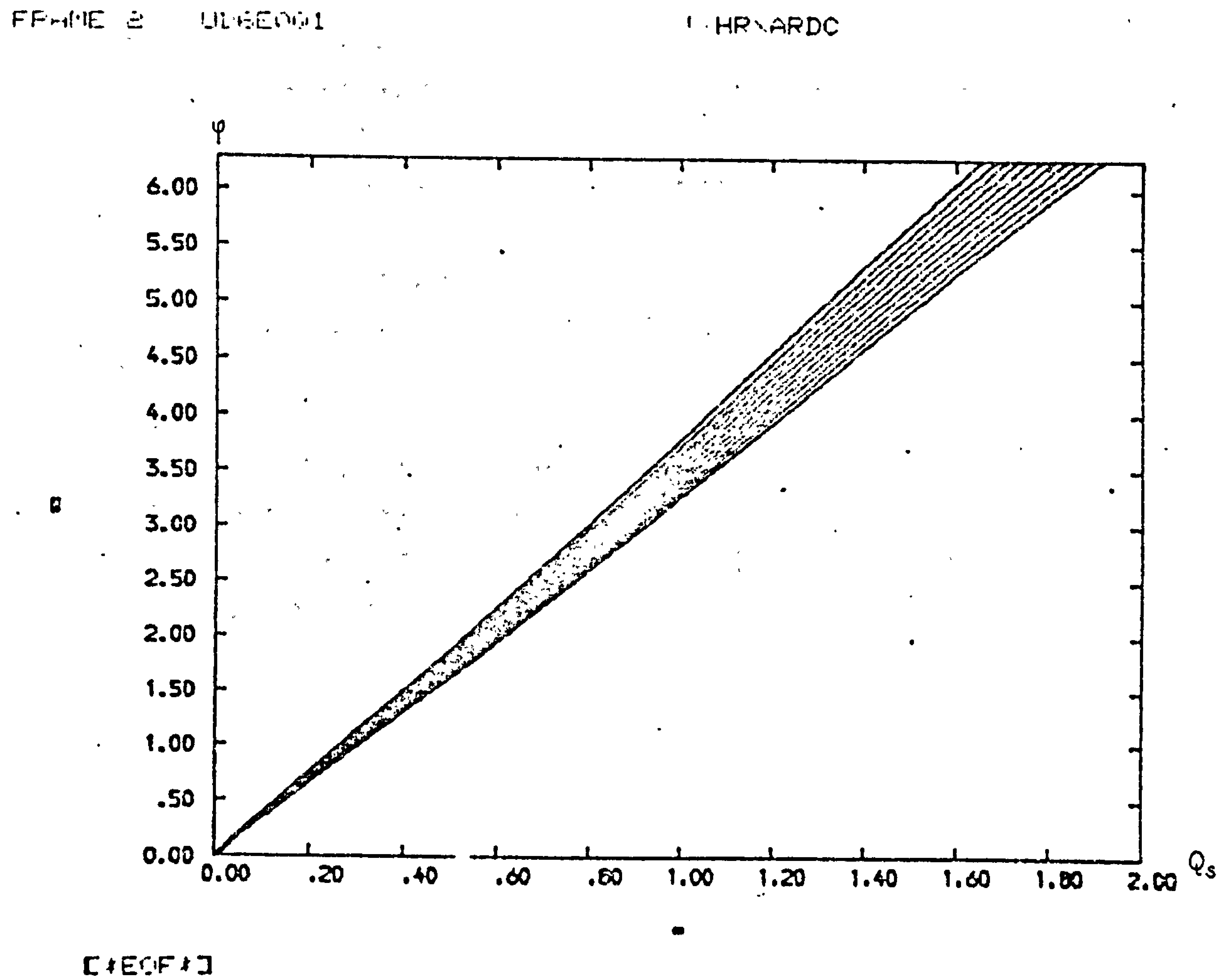


Fig. 6. Net heat supplied Qs as a function of the output crank shaft angle of rotation $0 \leqslant \phi \leqslant 2\pi$

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## 3.2. REVIEW OF VARIOUS OTHER STIRLING CYCLE MODELLING TESTS

Modelling tests and various simulations of the Stirling Cycle engine have been for some years the subject of attention by designers attempting to achieve the desired engine performance. Coming back to 1957, F.A. Creswick, (see Lit.1), wrote a digital computer program analysing the transient heating including the effects of longitudial conduction applied to the Stirling Cycle, and the same author's other papers published in subsequent years, 1962 and 1965 (see Lit. 2 and 3) are quite involved in the thermal design of Stirling Cycle machines. Around 1958, an analysis of the Stirling Cycle model by the Russian researcher, I.I. Karavensky and a co-author, show the cyclic variations of heat transfer as a function of time, unfortunately, due to the assumption of a constant temperature in the working spaces, these give only rough approximations to the performance of actual machines (see Lit.4). In July 1961, Prof. Finkelstein announced his electronic computing program developed for solving the equations earlier derived. This was done by D.E.P.I. (Differential Equations Pseudo-Code Interpretor), an interpretive routine which simulates differential analyser operation on a digital machine.D.E.P.I. was described for the DETATRON (see Lit.5) and adapted for the IBM 704 system. All the model's required integrations are done by the fourth order Runge-Kutta technique and predetermined servicing routine of the components is used.

Finally a general program based on an initially developed circuit was encoded and punched on one deck of IBM cards which established interconnections in accordance with this program. In any solution, this deck was used in conjunction with the standard D.E.P.I. subroutine on a second deck. The IBM 704 system with an 8K magnetic core memory, on-line card reader and on-line printer was used. In the print-out, being either in terms of the heat transfer and mechanical work on in forms of the basic dependent variables, each solution took about 2o minutes including the overlap necessary to ensure a steady-state solution. Seven years later, in January 1968, E.B. Quale and a co-author developed their mathematical model for steady-state operation of Stirling-type engines (See Lit.6). The complexity of the problem has been reduced by analysing the various components of the engine (heat exchangers, regenerators and cylinders), seperately for cyclically steady conditions and by selecting pressure, temperature and mass as the independent variables. As the authors claim, the theory has been carried out basically for Stirling engines, but it equally valid for refrigerators i.e. reverse cycle machines, with a minor modification only. One of the most valuable points of their work was the experimental verification conducted by the Allison Division of General Motors Corporation, where the development of the Stirling engine was part of a space-power program and several engine designs were built and tested as part of the project. The recorded performance data of two of the designs compared to the theoretical performance of Quale's model is in good agreement within the range of certain experimental errors.

The next year, 1969, brings up a very special computer simulation project - the free-piston Stirling Engine developed by Prof. W.T. Beale. This simulation based on the author's model tests of several 0.1 h.p. free piston Stirling Cycle engines, was basically an instructive isothermal computer simulation which compared reasonably well with model data and allowed investigation of parameter changes. Also some possible configurations, power transfer and control methods were considered (see Lit.7). This time a simple but highly instructive computer simulation was developed using the IBM 1130 Continuous System Modelling Program. In this simulation the following assumptions were made : - all gas processes are isothermal, the clearance volume is euqally divided between hot and cold spaces, being under equal working pressures, externally applied load and drag forces are proportional to the velocity and all the working medium in hot space is at one constant temperature and similarly in a cold space there is one fixed temperature. In the BEALE simulation, temperature, pressures, masses, geometrical characteristics, damping factors and loads were set and the resulting motions and pressures were recorded. In general this isothermal computer simulation could be a useful tool to investigate the engine response to various types of loading, mass ratios and other mechanical parameters. Recent years have shown again a tremendous interest in Stirling Cycle research and two powerful organisations i.e. U.S. Department of Energy (Division of Transportation Energy Conservation) as well as N.A.S.A. - Lewis produced a range of high-speed computer simulation programs, unfortunately, so far as is known, the full results have never appeared in open literature, mainly because of various military or commercial reasons.

An early 1978 research report was published by Dr. Organ (Dept. of Mechanical Engineering, King's College, London) under the title "Transient Response of the Miniature Reversed Stirling Cycle Cryogenic Cooling Machine - An Empirical Approach". The investigation to be discussed in this paper (see Lit.8) has been based on the thermal energy storage and release rates corresponding to the rates of change of temperature or the regenerator as well as the cylinder walls. The numerical and approximate analytical solutions for the system's first order, linear differential equations are presented in form of dynamic characteristics of the temperature·ratio and energy rate quantities. Ref. uniform angular speed. As the author claims the treatment is capable of application not only to reversed Stirling cooling machines but also to Stirling prime movers. In addition a listing of highly sophisticated Fortran program is incorporated. One of the most valuable points in this particular computer simulation lies in the fact that the data used originates from proved laboratory tests.

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by A.J. Organ, Dept. of Mech. Engineering, King's College, London. 1978

## 3.3. THE DECISION FOR FURTHER RESERACH BASED ON WALKER'S DATA

After considering various possibilities and selecting available material, a decision was made for further research on Walker's data, (see Lit.1.) on the 'ubiquitous' MP 1002 CA modified Philips Stirling Engine. There were several reasons supporting the decision in favour, i.e.

1. Highly comprehensive range of results and the satisfactory standard of their accuracy.
2. Practical confirmation of Walker's findings in at least four other Bath research reports, done by various people but still dealing with the same engine:

| | | | | |
|---|---|---|---|---|
| a) | Phillips J.B. and Reid T.J. | report | published | 1974 |
| b) | Conlin D.M. and Reed L.H.K. | " | " | 1973 |
| c) | Ward G.L. | " | " | 1972 |
| d) | Ford D.R. and Green C.F. | " | " | 1968 |

3. Fairly new publications and the authorship of the research team being a world recognised Stirling Engine authority. (Prof. G. Walker)
4. Plenty of associated literature describing the MP 1002CA Philips Stirling Engine.
5. Favourable MP 1002 CA Philips Stirling Engine properties and specifications.
6. Comprehensive manufacturer's operating literature containing a lot of practical tips and other highly valuable information.

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Paper prepared for presentation at the Conference "Stirling Engines for Automotive and other Applications" Institute of Mechanical Engineers, Reading UK. March 1977 by G.L. Ward, J. Slowley, G. Walker.

## 3.4. SUMMARY

It is evident that classical approaches for simulation and modelling start on the whole from basic steady-state considerations. Chapter three deals with a selection of available data material chosen for elementary steady-state characteristics. Experiments with Finkelstein's data include a discussion of simulation models derived from purely thermodynamical work conditions and an analog computer technique employed for solving particular sets of differential equations. Also this type of simulation can be used as a base for digital computer simulation programs, as shown in a simple example considering the amount of Net Heat Supplied values calculated in respect of the crankshaft angle of rotation and based on the above mentioned Finkelstein's equations. Also included in this chapter are various Stirling Cycle modelling tests carried out by other scientists beginning with the simple form described by Creswick (middle 50's) to the highly sophisticated simulation programs by Beale, General Motors Corporation, NASA-LEWIS etc.
The last section of Chapter 3 is devoted entirely to supporting the author's decision for further research to be based onWalker's data.

# 4. THE BASIC ENGINE SIMULATION

## 4.1. REVIEW OF COMPUTER SIMULATION METHODS

The cost of developing a high performance Stirling Cycle Engine has increased considerably in recent years, and therefore the availability of the simulation of the dynamic response prior to test bed running, can greatly improve the economic side of the engine's development program. Also any attempts to optimise the dynamic behaviour of the engine, as well as its control system on the test bed, by altering the acceleration fuel schedule, or implementing various methods of the working medium pressurisation, results in a large amount of test bed running and always creates the possibility either of damage to a valuable prototype machine, or an acceleration rate significantly lower than could be obtained, within the safety margin, assuming that more information was available earlier. The computer simulation model can be highly useful in all phases of Stirling Cycle prime-mover engineering from pure research and development work to realistic engine operation. The computer simulation in research and process development additionally explores the effects of different operating conditions for optimisation studies, and in the final engine design, allows for uncomplicated dynamic performance investigation as well as allowing one to determine the interactions of various parameters in the system and the overall performance of the Stirling Engine.

There are many simulation methods used today, however, computation of the engine response is usually carried out using either digital, analog or hybrid computers. The digital computer is by far the most widely used type and virtually most types of engine simulation can be carried out on it, mainly because of the tremendous range of values that digital computers can handle, for example of the order $10^{-300}$ to $10^{+300}$ on the C.D.C 6400 (Control Data Corporation) and also the relatively simple data and program preparation procedure which takes much less engineering time, when compared for example with analog computer methods. Once the program is working, it can be stored on cards or tape and theoretically can be run a month or year later with minimum effort. (In practice this looks somewhat different). The computation time in digital simulation is dependent upon both the complexity of the model to be studied and the mathematical efficiency of the numerical integration procedure, and the principal difficulty in digital simulation arises in integration which must be done numerically since the digital machine is not a continuous device but merely a very fast desk calculator. Digital Simulation methods are described in Lit.1,2,3,5, and the discussion of various numerical methods can be found elsewhere, for example Lit.11,12,13,14. The most significant disadvantage of the digital simulation method is the inherent remoteness of large computer systems which make it obviously very difficult for the engineer to experiment on-line with alternative philosophies and obtain a real "feel" for the engine being simulated (see Lit.4).

Another type of simulation computer is the analog machine and as its name implies, an analog computer makes use of the fact that the equations describing some electronic devices are completely analogous to the mathematical equations describing the simulated object. Analog computers are parallel devices, thus the computation time is independent of the engine complexity, but at the same time the analog accuracy cannot compete with digital. However the accuracy of modern analog devices is quite adequate for most practical engineering applications. Basically all that the analog simulation process is, is an "analog to a simulated model" circuit in the form of various operational blocks, performing the normal mathematical operations that the system requires. (see Fig.1.). In addition, analog computers can be used to design systems with non-linear components, and the standard electronic circuits are available for simulating commonly encountered non-linear effects such as saturation, dead zone, backlash, friction etc.

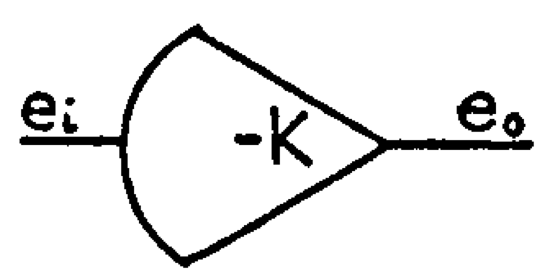
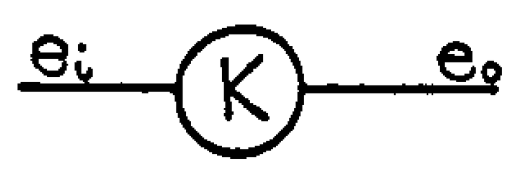
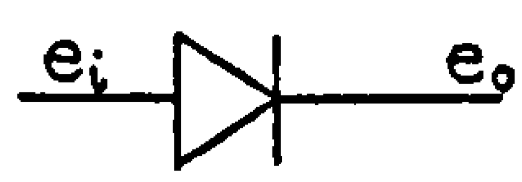
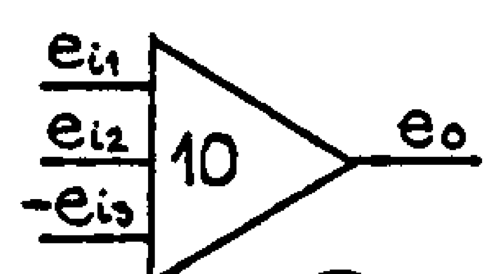
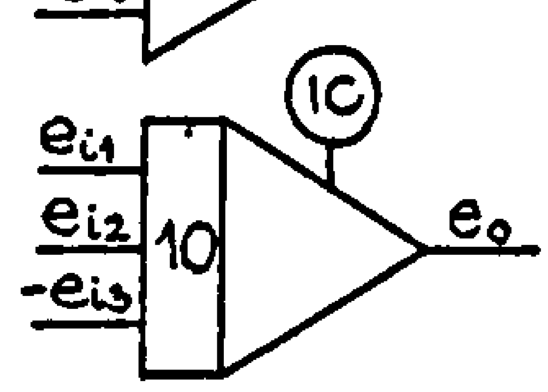
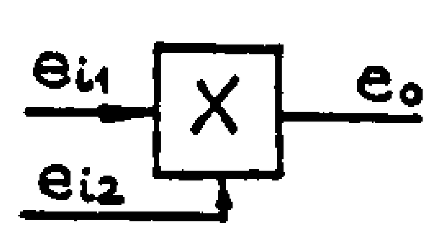
| Component or Operational Block | Symbol | Mathematical operation |
|---|---|---|
| High gain amplifier |   | $e_o = -Ke_i$ |
| Pot |   | $e_o = Ke_i, K < 1$ |
| Diode |   | $e_o = e_i, e_i > 0$<br>$= 0 \quad e_i < 0$ |
| Summer |   | $e_o = e_{i3} - e_{i1} - 10e_{i2}$ |
| Integrator |   | $e_o = IC + \int (e_{i3} - e_{i1} - 10e_{i2})dt$ |
| Multiplier |   | $e_o = -e_{i1}e_{i2}/10$ |
| Function Generator |   | $e_o = f(e_i)$ |

Fig.1. Analog Components and Operational blocks

A major advantage of the analog method of engine simulation is that an engineer can operate the analog model just as he would operate an engine on a standard test bed. Both analog and digital systems, if combining a standard analog computer with a general purpose digital machine form a balanced hybrid system, where the special interface system provides interchange of both logical and variable signals between the constituent machines. The time consuming integration routines then can be substituted by the high-performance integrating amplifiers, thus minimizing the overall computing time. Basically hybrid simulation combines the computing speed of the analog technique with the accuracy and large degree of flexibility of the digital computer and its "digital" section may always be used for various optimisation processes, which could prove to be an extremely valuable feature for future complex engine's analysis and design. With basic analog components and operational blocks as shown in Fig. 1, we can solve a simple simulation example of an engine. A usual method to simulate any physical object is to determine its overall differential equation and consequently solve this on the analog computer. The main difficulty to realise this in practice arrives when considering a real engine with its non-linear characteristics which obviously makes this method rather difficult. Let's consider the analog simulation of an engine as shown in Fig. 2. Two time lags must be introduced representing at the model "input" - the fuel supply meter valve and on the "output" - the rotational speed measuring device being usually a tachometric type of transducer. Also, since the engine possesses its own inertia, ie. TORQUE = $I \frac{dN}{dt}$ being the energy storage component, its time differential is also introduced. In practice this type of arrangement can be used both with linear and non-linear types of simulated engines steady-state characteristics, in general regarded as a graph of steady-state fuel plotted against the rotational speed measured on the output shaft of the engine.

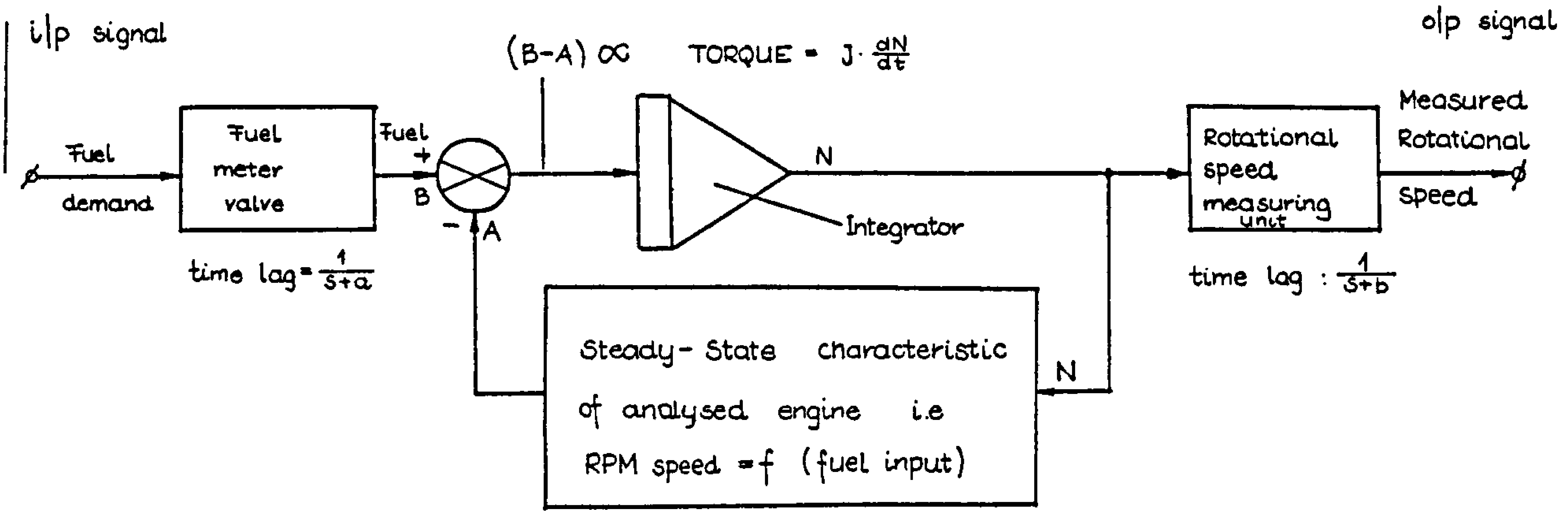



Fig. 2. <u>Electronic Analog Simulation of an Engine</u>

The analog time lag of t- seconds can be generated by the standard circuit shown in Fig. 3. and possessing the usual transfer function given by:

$$\text{Transfer Function} = \frac{\overline{Vo}/p}{\overline{Vi}/p} = \frac{\frac{R}{1+sCR}}{Ri/p} = \frac{R}{Ri/p} \cdot \frac{\frac{1}{CR}}{S + \frac{1}{CR}}$$

OR

$$\text{Transfer Function} = \frac{R}{Ri/p} \cdot \frac{\frac{1}{t}}{S + \frac{1}{t}} = \frac{a}{S + a}$$

Where : $t = C.R$
$R = R_{i/p}$
$\frac{1}{t} = a$

Where the time lag of the circuit is a strong function of the natural CR product.

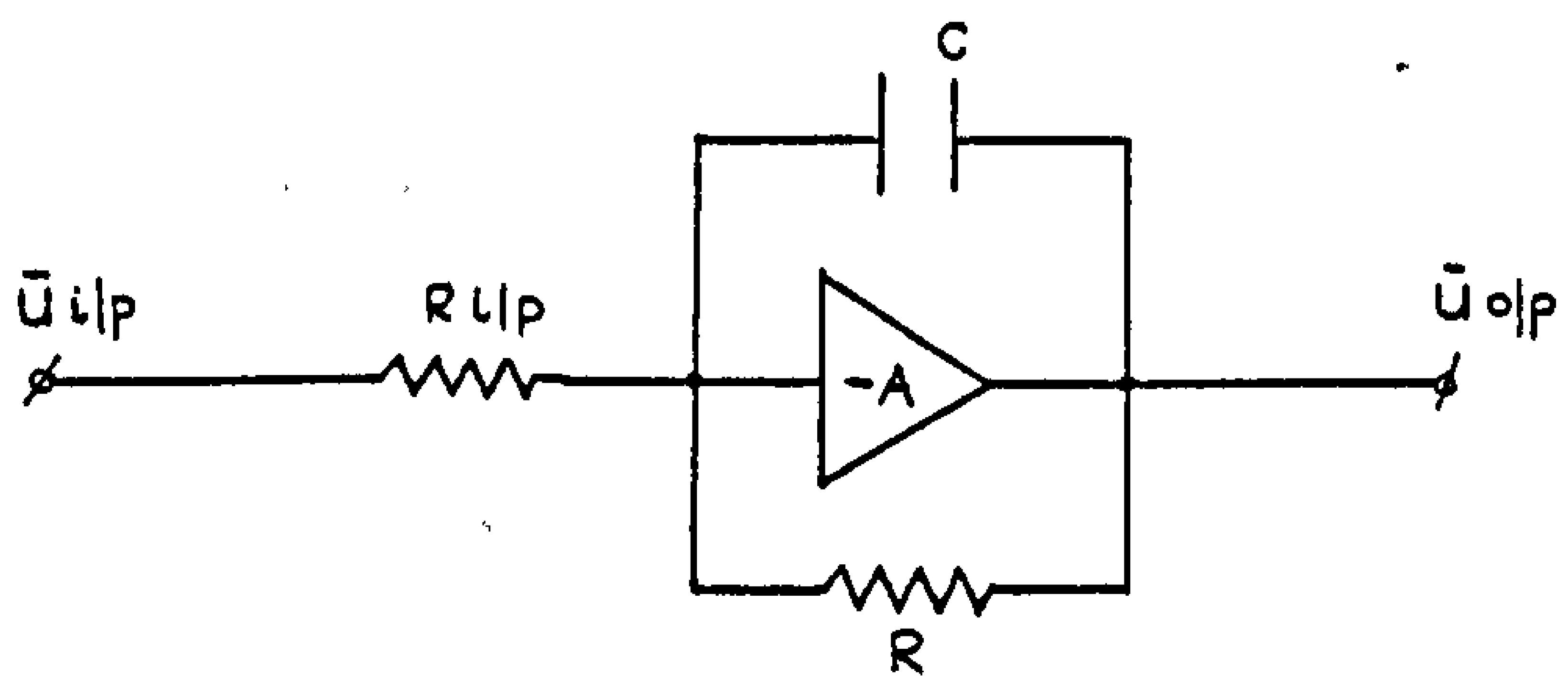



Fig. 3. Time Lag Circuit

Assuming that in general, steady state characteristic is a non-linear type, for this particular type of engine, it is presented in Fig. 4 with $G_1, G_2, G_3$ being the slopes of the approximated lines.

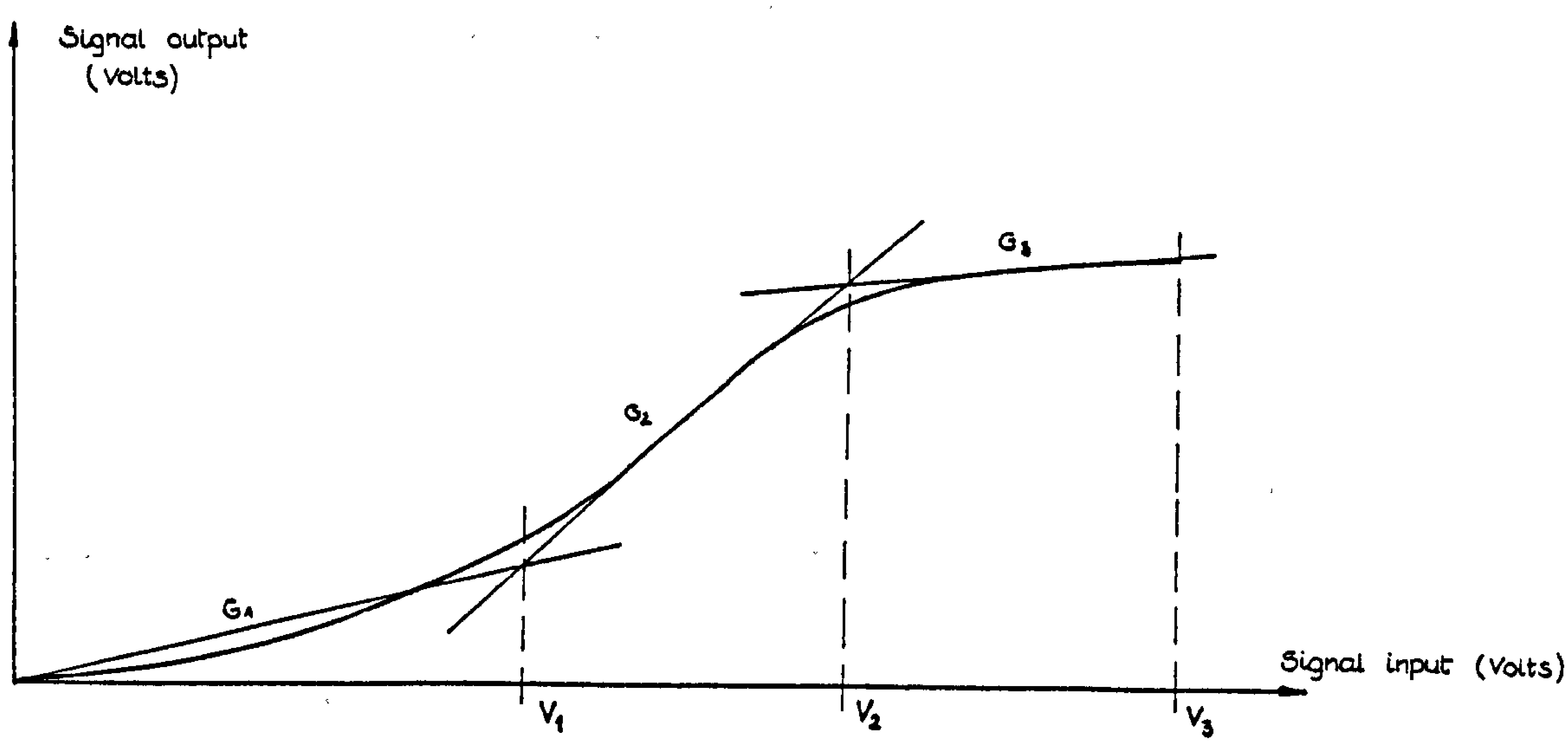



Fig. 4. Analog Approximation of Steady State Characteristic

Referring to Fig. 2., the above steady state characteristic could be simulated in form of sloping straight lines by relevant electronic circuits, or in other words this circuit should generate different gains with different input levels. A simple and reliable circuit which will operate as above described is presented in Fig. 5.

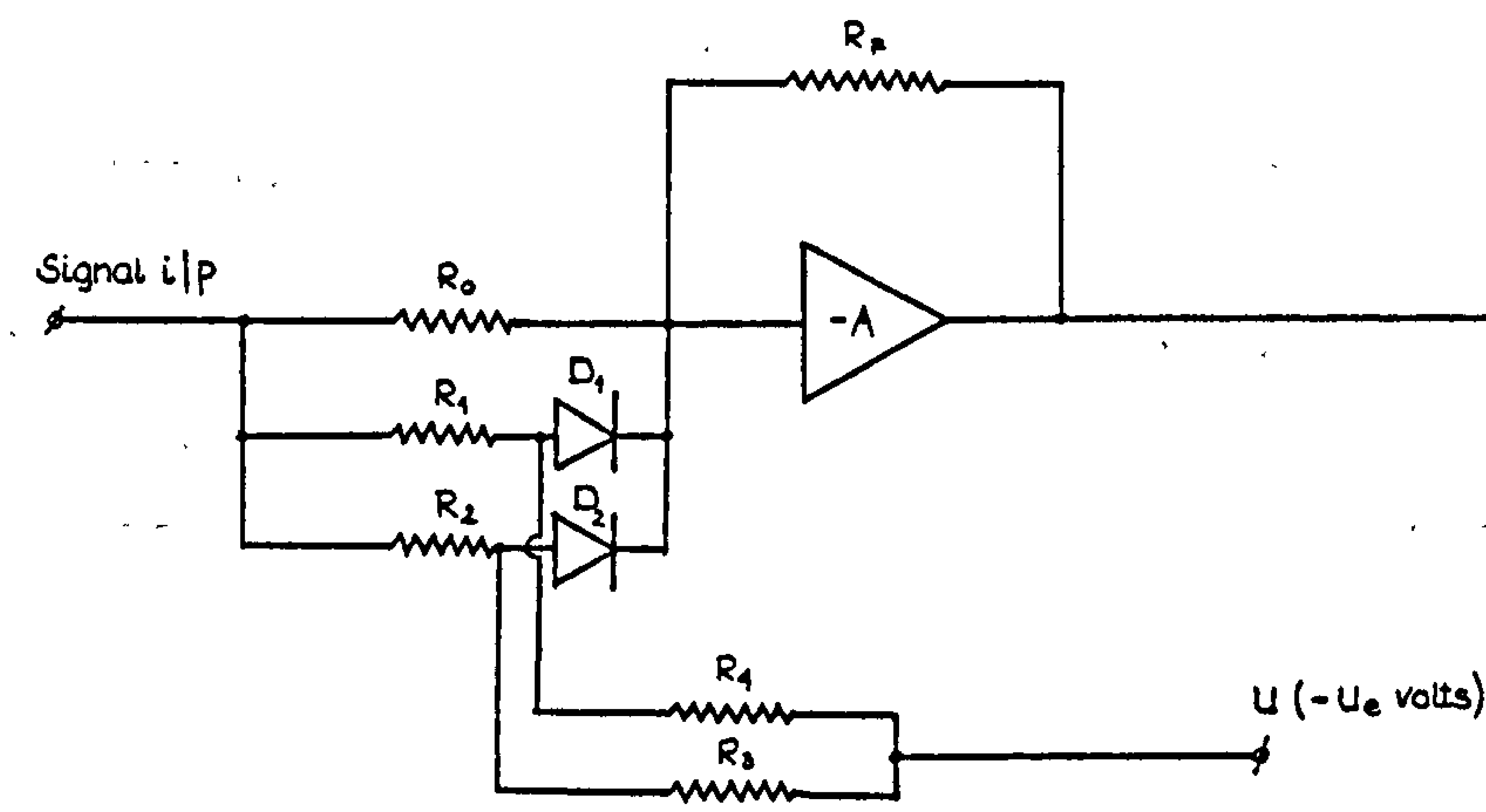



Fig. 5. <u>Circuit Generating Different Gains with Different Input Levels</u>

With small input signals, the operational amplifier acts as a standard amplifier, with gain $G1 = \frac{R_f}{R_o}$, if the input signal increases Diode D1 starts conducting, so that R1 is shunting the input resistor $R_o$ and consequently increasing the circuit gain to G2. When the input signal condition causes the diode D3 to conduct - the overall circuit gain changes to the G3 value.

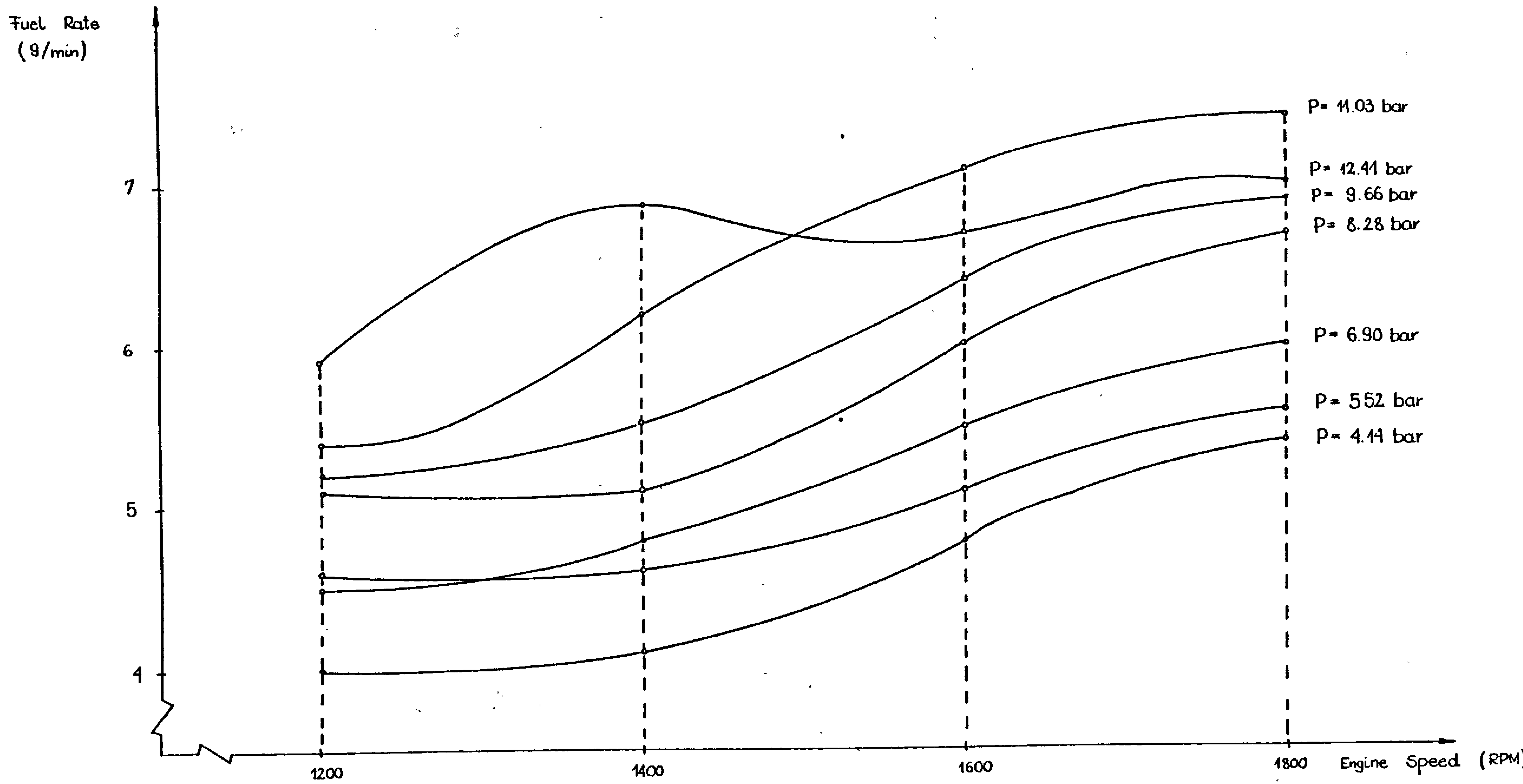
In order to specify more realistically the engine's steady state characteristic, let's consider the Philips MP 1002CA type Stirling Engine working with 800°C of the cylinder head temperature, and within the working pressure range of (4.14 - 12.41) bars. (For the results see Tab.1. in Chapter 2, Section 2.3.) The complete steady state characteristics of the Philips Mp 1002CA Stirling engine are presented in Fig. 6. An approximation gives the following data for the selected engine's runs at: 11.03bar, 9.66bar, 6.90bar, 4.14bar. (See Tab. 1 below).

| | | | |
|---|---|---|---|
| p = 11.03 bar | $G_1$=0.37647 | $G_2$ = 0.47559 | $G_3$ = 0.04076 |
| p = 9.66 bar | $G_1$=0.15100 | $G_2$ = 0.46100 | $G_3$ = 0.08500 |
| p = 6.90 bar | $G_1$=0.15566 | $G_2$ = 0.40944 | $G_3$ = 0.05928 |
| p = 4.14 bar | $G_1$=0.05419 | $G_2$ = 0.39161 | $G_3$ = 0.20912 |

Tab.1. Experimental Results based on Prof. Walker's Experiments

Also let the time constant of fuel value be approx. 5omsec , and 100 msec. for the tachometric transducer, and $U_1$ = 1.833v, $U_2$ = 4.049v (in voltage scale).

This set of date is in practice sufficient to design a complete electronic analog of a simulated engine. (See Fig.7)




<u>Fig.</u>6 The complete steady-state characteristic Ref. Cylinder Head Temp. $800^{o}C$ and pressure range 4.14- 12.41 bar.

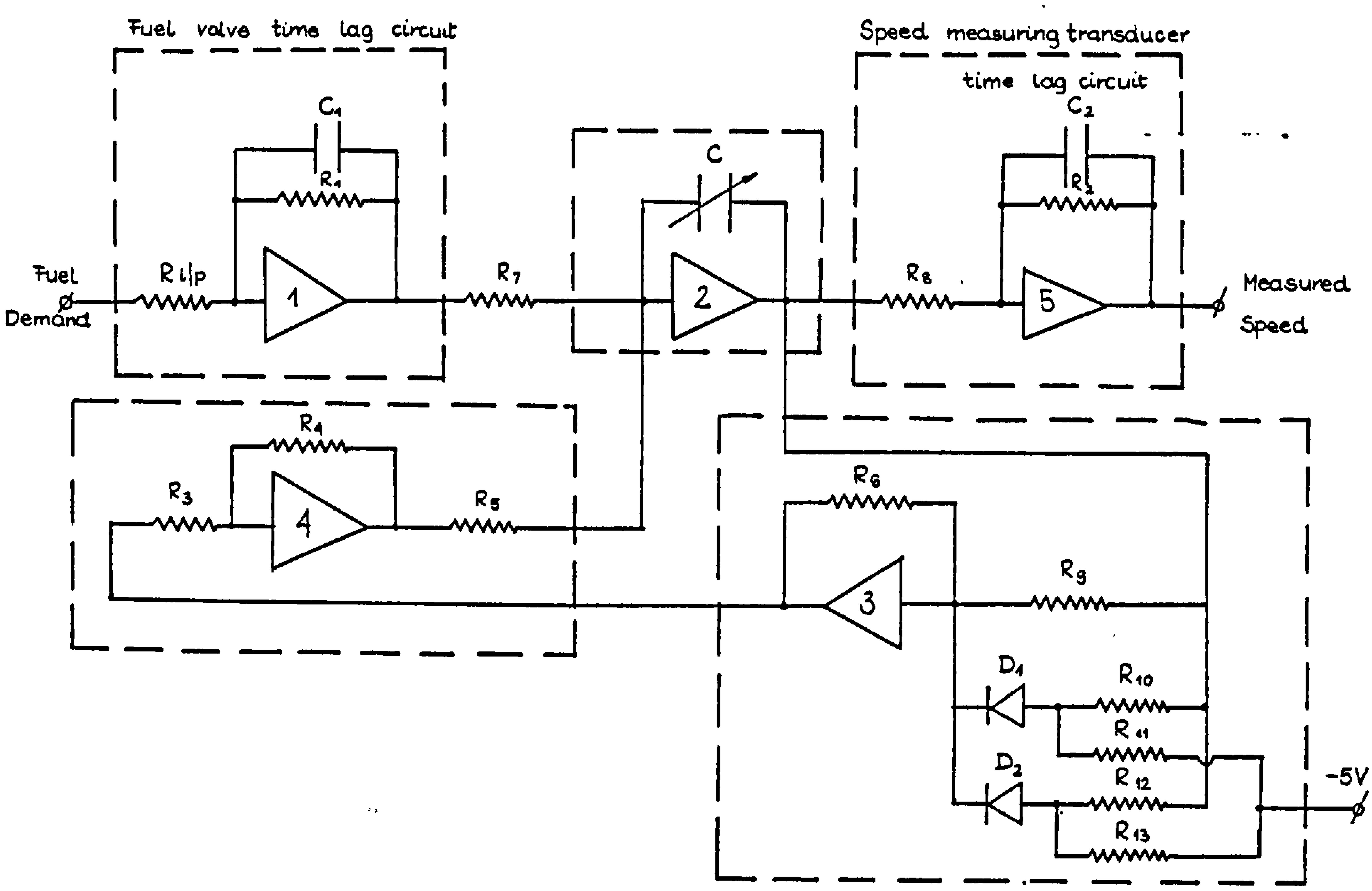



Fig. 7. Basic Engine Analog Simulation Circuit

As operational amplifiers, most of the high-performance 'op-amps' can be used for example:. 748 type, 531 type etc. and a similar electronic circuit was tested practically by the author and quite a comprehensive range of results was available.

Another way to simulate the above engine arrangement is by implementing a digital computer to a classical analysis of the system shown in Fig.8.

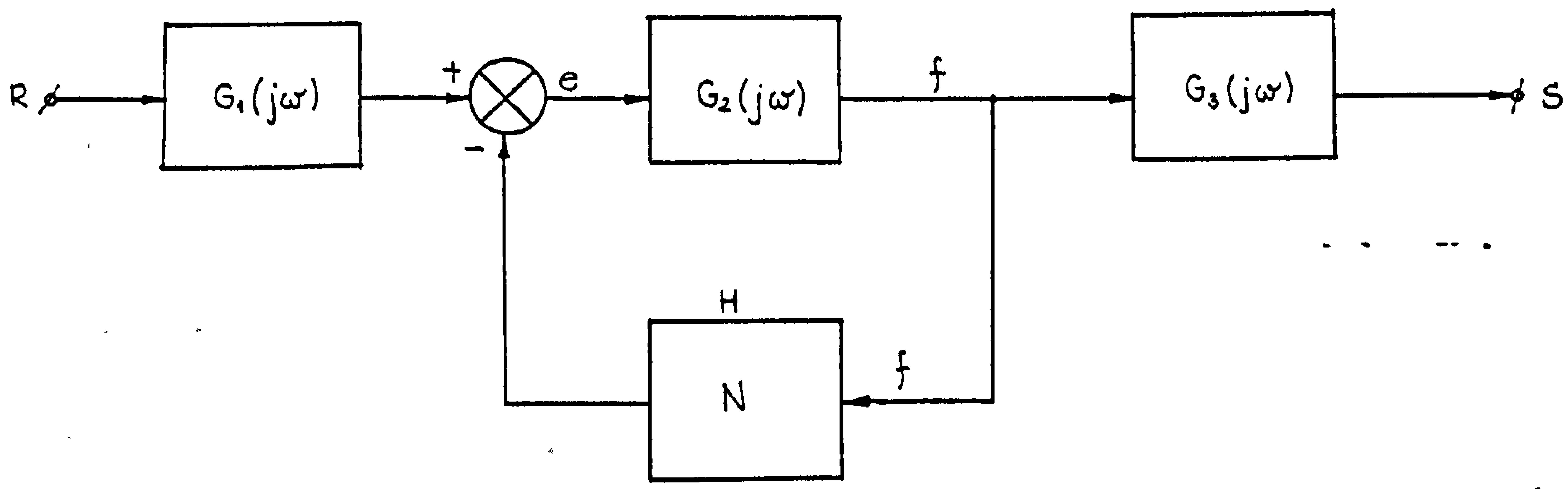



Fig. 8. A Simplified Engine Block Diagram

Where: $G_1 \quad (j\omega) = \frac{a}{j\omega + a} = \frac{a}{s + a}$ ( 50 msec. time lag)

$G_2 \quad (j\omega) = \frac{c}{j\omega} = \frac{c}{s}$ (the integrator)

$G_3 \quad (j\omega) = \frac{b}{j\omega + b} = \frac{b}{s + b}$ (100 msec. time lag)

$j\omega = S$

N = non-linear steady-state characteristic

SINCE:

$$e = G_1(s) \,.\, R - H(s) \,.f$$
$$f = G_2 . e$$
$$S = G_3 \,.f$$

Therefore :

$$e = \frac{f}{G_2(s)} = G_1(s) \,.R - H(s).f$$

$$f = G_2(s) \left[G_1(s).R - H(s).f\right] = G_1(s).G_2(s).R - G_2(s).H(s).f$$

$$f = G_1(s).G_2(s).R - G_2(s).H(s).f$$

$$f + G_2(s) \,.H(s).f = G_1(s) \,.G_2(s).R$$

$$f \left[1 + G_2(s) \,.H(s)\right] = G_1(s).G_2(s).R$$

$$f = \frac{G_1(s) \,.G_2(s).R}{1 + G_2(s).H(s)}$$

and:

$$S = f.G(s) = \frac{G_1(s).\ G_2(s)\ .\ G_3(s)}{H(s) \cdot G_2(s) + 1}\ .\ R$$

or finally:

$$\frac{S}{R} = \frac{G_1(s)\ .\ G_2(s)\ .\ G_3(s)}{1 + G_2(s)\ .\ H(s)}$$

When the input is small the approximation of the characteristic is a linear gain B i.e.

$$\frac{S\ (j\omega)}{R\ (j\omega)} = \frac{\frac{a}{s+a}\ .\ \frac{c}{s}\ .\ \frac{b}{s+b}}{1 + \frac{c}{s}\ .\ B} = \frac{a.b.c.}{(s+a)(s+b)(s+B.c)}$$

where:

$S = j\omega$

$a = \frac{1}{C_1.R_1} = \frac{1}{0.047uF.1\ Mohm} = 20$

$b = \frac{1}{C_2R_2} = \frac{1}{0.1uF.\ 1\ Mohm} = 10$

$c = \frac{1}{CR} = 10$

B1 = 0.151

B2 = 0.461

B3 = 0.085

From the above presented transfer function it is obvious that all the three polesof the system are localised on the left side of the p-plane, and the simulated system is stable.

After substituting all T.F. constants, the above form can be converted to a new form :

$$\frac{S(j\omega)}{R(j\omega)} = \frac{2000}{(s+20)(s+10)\left[(s+B.10)\right]}$$

where: B1 = 0.151
B2 = 0.461
B3 = 0.085

and its "characteristic equation" is :

$(s^2+10s+20s+200)(s+10B) =$
$s^3+10s^2+20s^2+200s+10Bs^2+100\,Bs+200\,Bs+2000\,B =$
$s^3+30s^2+200s+10Bs^2+300Bs+2000B$
$s^3+s^2(30+10B)+s\,(200+300\,B)+2000B$

Therefore:

$$\frac{S\,(j\omega)}{R\,(j\omega)} = \frac{2000}{s^3+s^2(30+10B)+s(200+300B)+2000B}$$

dividing by 2000B

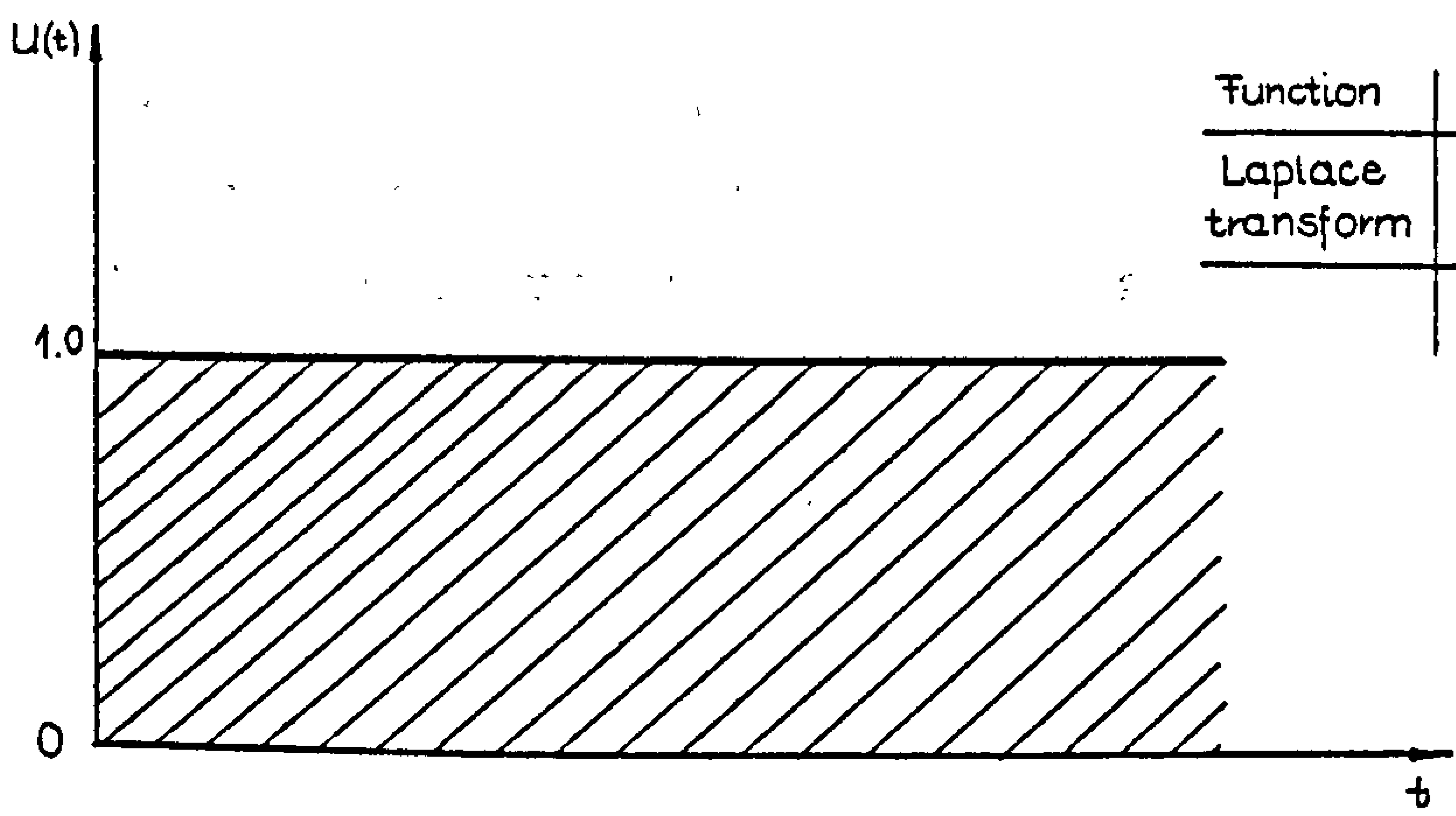
$$= \frac{\frac{1}{B}}{\frac{0.0005}{B}.S^3+\frac{0.0005\,(30+10B)}{B}.s^2+\frac{0.0005(200+300B)}{B}.S+1}$$

$$\frac{S(j\omega)}{R(j\omega)} = \frac{\frac{1}{B}}{\frac{(0.0005)}{B}S^3+\frac{(0.015+0.005B)}{B}S^2+\frac{(0.1+0.15B)}{B}S+1}$$

where : $S = j.\omega$

The above derived transfer function which describes the system earlier considered can be solved on a digital computer, however, it seems worthwhile to convert it to another possible form thus the system's computing can be carried out directly in a time domain i.e. after transforming the Laplace equation into the time domain. This sort of operation is called inversion or inverse Laplace transformation.

The principle of inverse Laplace transformation is widely used in contemporary control theory and there are several good texts (for example Lit. 9, 10) devoted to various inversion techniques.
Using the small STEP INPUT signal in the usual form as shown in Fig. 9.




| Function | F(s) | f(t) |
|---|---|---|
| Laplace transform | $\frac{1}{s}$ | u(t) for t>0 |
| | $s = j\omega$ | |

Fig. 9. The "Step Input" signal

The output form now becomes:

$$S(j\omega) = \frac{1}{j\omega}\left[\frac{2000}{(j\omega + 20)\ (j\omega + 10)\ (j\omega + 10B)}\right]$$

If converting the above expression into the time domain we can use the following formulae:

$$S(j\omega) \Longrightarrow S(t) \qquad S(t) = \mathcal{L}^{-1}(S(s))\Big|_{s = j\omega}$$

where $L^{-1} S(s)\big|_{s = j\omega}$ is the common notation for the inverse Laplace transformation. There are several ways to invert functions of $s = j\omega$ into a function of time ; Since "s" is a complex number, a contour integration in the complex "s" plane can be used (see Lit.9,1o)

$$S(t) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} e^{st} S(s)\, ds$$

or alternatively the most common inversion method, so called "partial fractions expansion". The function to be inverted S(s) is re-arranged into a series of simple functions : $S(s) = S_1(s) + S_2(s) + S_3(s) \ldots\ldots S_n(s)$

and then each of the simple terms is inverted individually. The total time dependent function is the sum of the simple time dependent functions:

$$S(t) = L^{-1}\left[S_1(s)\right] + L^{-1}\left[S_2(s)\right] + L^{-1}\left[S_3(s)\right] \ldots + L^{-1}\left[S_n(s)\right] =$$

$$= S_1(t) + S_2(t) + S_3(t) \ldots\ldots\ldots + S_n(t)$$

Coming back to our earlier form of transfer function we can express $S(s)_{\,s = j\omega}$ as a sum of four terms:

$$\frac{1}{S(S+20)(S+10)(S+10B)} = \frac{KI}{S} + \frac{K2}{(S+20)} + \frac{K3}{(S+10)} + \frac{K4}{(S+10B)}$$

where the numberators K1,K2, K3,K4 of the above terms can be evaluated and inverted individually :

$$K1 = \left[\frac{1}{(s+20)(s+10)(s+10B)}\right]_{s=0} = \frac{1}{2000B}$$

$$K2 = \left[\frac{1}{s(s+10)(s+10B)}\right]_{s=-20} = \frac{1}{2000B - 4000}$$

$$K3 = \left[\frac{1}{s(s+20)(s+10B)}\right]_{s=-10} = \frac{1}{1000 - 1000B}$$

$$K4 = \left[\frac{1}{s(s+20)(s+10)}\right]_{s=-10B} = \frac{1}{-10B(-10B+20)(-10B+10)}$$

And the final form of the output signal written in the time domain is as follows:

$$S(t) = 2000\left[K1 + K2 \cdot e^{-20t} + K3 \cdot e^{-10t} + K4 \cdot e^{-10Bt}\right]$$

where:

| | | | | |
|---|---|---|---|---|
| B | B1 | = | 0.151 | for region 1 |
| | B2 | = | 0.461 | for region 2 |
| | B3 | = | 0.085 | for region 3 |

As mentioned earlier, both forms analysed in this section can be considered for simulation model purposes, prepared for solving on a digital computer. The economic effectiveness of a computer modelling which depends on the type of problem considered, and on the purpose of the solution, therefore it is often advisable to simplify the computational material wherever possible, thus reducing not only the programming effort but also expensive machine time required to complete a given run of the solution. This applies both to a large and complex programming as well as to the less complicated initial modelling steps.

In order to solve the last derived time related "answer" of the engine's model, when the small unity step function input signal was applied and the engine was working under three speed region conditions (as specified earlier), the STYRDYN program was developed and incorporated with the usual Tektronix plotting routine. The STYRDYN program computes the values of the engine's speed output signal S(t) in terms of its exponential components derived earlier. Tabulated values of speed are presented individually for relevant regions Nos. 1,2,3.
The program printouts include seperate sets of results for the selected engine runs at:-11.03, 9.66, 6.90, 4.14 bars of the working fluid operational pressure Ref. 800°C of the cylinder head temperature and are presented in Tab. 2,3,4,5. The corresponding SCM plots, being the engine's dynamic characteristics as shown in Fig. 10, 11, 12, 13. The STYRDYN program shows an example of an economically simulated part of a simple engine model which can still be used for comparison purposes, when the engine is run under various pressure/temperature conditions.
The dynamic characteristics of an engine, presented here, can be basically plotted on one graph( which is possible with the Tektronix plotting routine) if required, with only minor modifications in the program.

```
F MINNESOTA 6600 FORTRAN COMPILER SCOPE 3.4.1 VER4.5        23/08/76

      PROGRAM STRDYN(INPUT,OUTPUT,PLOT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE27=PLOT)
      DIMENSION S1(20),TI(20),S2(20),S3(20)
      REAL K1,K2,K3,K4
      J=1
      CALL CAM35MM
      CALL GRSLIDE
      CALL XAXIS(0.0,6.0)
      CALL YAXIS(0.0,25.0)
    2 READ(5,1)B
    1 FORMAT(F10.3)
      T=0
      WRITE(6,4)B
    4 FORMAT(//3H B=,F10.4//)
      DO 3 K=1,20
      T=T+0.1*(K-1)
      S=2000.*(5.E-4/B+1./(2000.*B-4000.)*EXP(-20.*T)+1./(1000.*(1.-B)
     1*EXP(-10.*T)-1./(10.*B*(-10.*B+20.)*(-10.*B+10.))*EXP(-10.*B*T))
      WRITE(6,5) T,S
    5 FORMAT(2E15.5)
      GO TO (7,8,9),J
    7 S1(K)=S
      GO TO 10
    8 S2(K)=S
      GO TO 10
    9 S3(K)=S
   10 TI(K)=T
    3 CONTINUE
      J=J+1
      IF(J.LE.3) GO TO 2
      CALL POLY 3 (TI,S1,20)
      CALL POLY 3 (TI,S2,20)
      CALL POLY 3 (TI,S3,20)
      CALL GRAFDEF(34 H DYNAMIC CHARACTERISTIC OF THE SCM,34,4HTIME,
     14,5HSPEED,5,0)
      CALL ENDFILM
      STOP
      END
```

<u>Prog.1.</u>     Program STYRDYN

B=.0408

| | |
|---|---|
| 0 | -.22204E-12 |
| .10000E+00 | .16615E+00 |
| .30000E+00 | .15330E+01 |
| .60000E+00 | .40948E+01 |
| .10000E+01 | .71654E+01 |
| .15000E+01 | .10368E+02 |
| .21000E+01 | .13441E+02 |
| .28000E+01 | .16195E+02 |
| .36000E+01 | .18515E+02 |
| .45000E+01 | .20363E+02 |
| .55000E+01 | .21759E+02 |
| .66000E+01 | .22762E+02 |
| .78000E+01 | .23447E+02 |
| .91000E+01 | .23894E+02 |
| .10500E+02 | .24172E+02 |
| .12000E+02 | .24338E+02 |
| .13600E+02 | .24432E+02 |
| .15300E+02 | .24483E+02 |
| .17100E+02 | .24509E+02 |
| .19000E+02 | .24523E+02 |

B=.4756

| | |
|---|---|
| 0 | .27756E-13 |
| .10000E+00 | .14742E+00 |
| .30000E+00 | .10280E+01 |
| .60000E+00 | .18089E+01 |
| .10000E+01 | .20576E+01 |
| .15000E+01 | .20985E+01 |
| .21000E+01 | .21024E+01 |
| .28000E+01 | .21026E+01 |
| .36000E+01 | .21027E+01 |
| .45000E+01 | .21027E+01 |
| .55000E+01 | .21027E+01 |
| .66000E+01 | .21027E+01 |
| .78000E+01 | .21027E+01 |
| .91000E+01 | .21027E+01 |
| .10500E+02 | .21027E+01 |
| .12000E+02 | .21027E+01 |
| .13600E+02 | .21027E+01 |
| .15300E+02 | .21027E+01 |
| .17100E+02 | .21027E+01 |
| .19000E+02 | .21027E+01 |

B=.3765

| | |
|---|---|
| 0 | 0 |
| .10000E+00 | .15139E+00 |
| .30000E+00 | .11182E+01 |
| .60000E+00 | .21159E+01 |
| .10000E+01 | .25348E+01 |
| .15000E+01 | .26377E+01 |
| .21000E+01 | .26543E+01 |
| .28000E+01 | .26561E+01 |
| .36000E+01 | .26562E+01 |
| .45000E+01 | .26563E+01 |
| .55000E+01 | .26563E+01 |
| .66000E+01 | .26563E+01 |
| .78000E+01 | .26563E+01 |
| .91000E+01 | .26563E+01 |
| .10500E+02 | .26563E+01 |
| .12000E+02 | .26563E+01 |
| .13600E+02 | .26563E+01 |
| .15300E+02 | .26563E+01 |
| .17100E+02 | .26563E+01 |
| .19000E+02 | .26563E+01 |

3309 CHARACTERS OUTPUT

*TRACE* SUBPROGRAM CALLS

| ROUTINE | COUNT |
|---|---|
| GRAFDEF | 1 |
| GRSLIDE | 1 |
| POLY3 | 3 |
| CAM35MM | 1 |
| EXP | 180 |
| ENDFILM | 1 |
| XAXIS | 1 |
| YAXIS | 1 |

Tab.2. Program STYRDYN results
Run No.1
T = 800°C
P = 11.03 bar

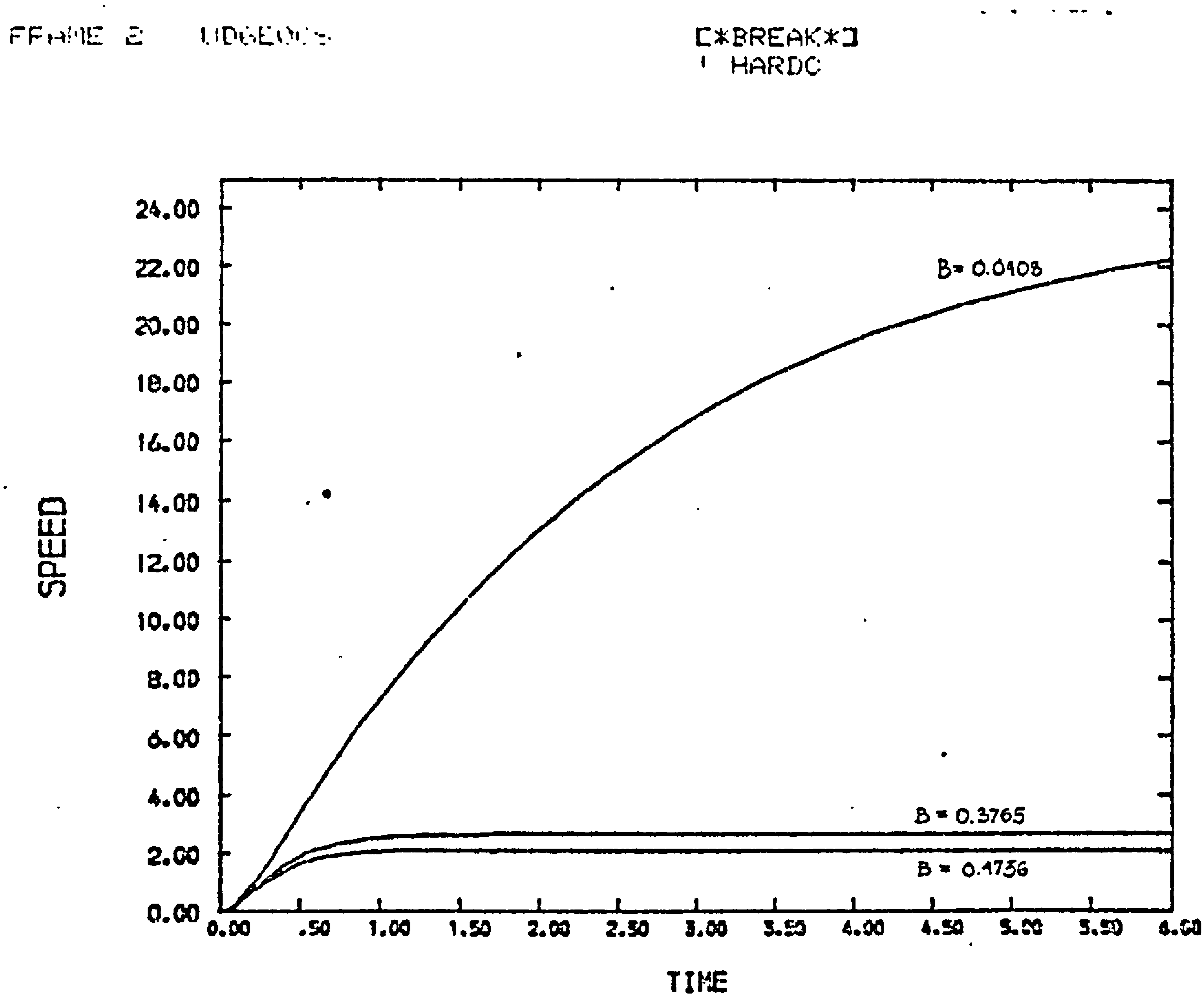



Fig.10. DYNAMIC CHARACTERISTIC OF THE SCM

Run No.1.
T. = 800°C
P = 11.03 bar

```
B=.0848

          0         .55511E-13
.10000E+00          .16409E+00
.30000E+00          .14667E+01

.60000E+00          .37079E+01
.10000E+01          .60292E+01
.15000E+01          .80199E+01
.21000E+01          .95231E+01
.28000E+01          .10537E+02
.36000E+01          .11154E+02
.45000E+01          .11493E+02
.55000E+01          .11662E+02
.66000E+01          .11738E+02
.78000E+01          .11770E+02
.91000E+01          .11782E+02
.10500E+02          .11786E+02
.12000E+02          .11788E+02
.13600E+02          .11788E+02
.15300E+02          .11788E+02
.17100E+02          .11788E+02
.19000E+02          .11788E+02

B=.4610

          0        -.55511E-13
.10000E+00          .14799E+00
.30000E+00          .10405E+01
.60000E+00          .18494E+01
.10000E+01          .21174E+01
.15000E+01          .21641E+01
.21000E+01          .21690E+01
.28000E+01          .21693E+01
.36000E+01          .21693E+01
.45000E+01          .21693E+01
.55000E+01          .21693E+01
.66000E+01          .21693E+01
.78000E+01          .21693E+01
.91000E+01          .21693E+01
.10500E+02          .21693E+01
.12000E+02          .21693E+01
.13600E+02          .21693E+01
.15300E+02          .21693E+01
.17100E+02          .21693E+01
.19000E+02          .21693E+01

B=.1510

          0                  0
.10000E+00          .16107E+00
.30000E+00          .13747E+01
.60000E+00          .32186E+01
.10000E+01          .47590E+01

.15000E+01          .57470E+01
.21000E+01          .62692E+01
.28000E+01          .65003E+01
.36000E+01          .65866E+01
.45000E+01          .66139E+01
.55000E+01          .66213E+01
.66000E+01          .66230E+01
.78000E+01          .66233E+01
.91000E+01          .66234E+01

.10500E+02          .66234E+01
.12000E+02          .66234E+01
.13600E+02          .66234E+01
.15300E+02          .66234E+01
.17100E+02          .66234E+01
.19000E+02          .66234E+01
2862 CHARACTERS OUTPUT

*TRACE* SUBPROGRAM CALLS

 ROUTINE       COUNT
 GRAFDEF           1
 GRSLIDE           1
 POLY3             3
 CAM35MM           1
 EXP             180
 ENDFILM           1
 XAXIS             1
 YAXIS             1

*TRACE* STATEMENT NUMBERS FOR
```

Tab. 3. Program STYRDYN results. Run No.2.
$T = 800^\circ C$, $P = 9.66$ bar

FRAME 2 UDGE085 [*BREAK*] HARDC

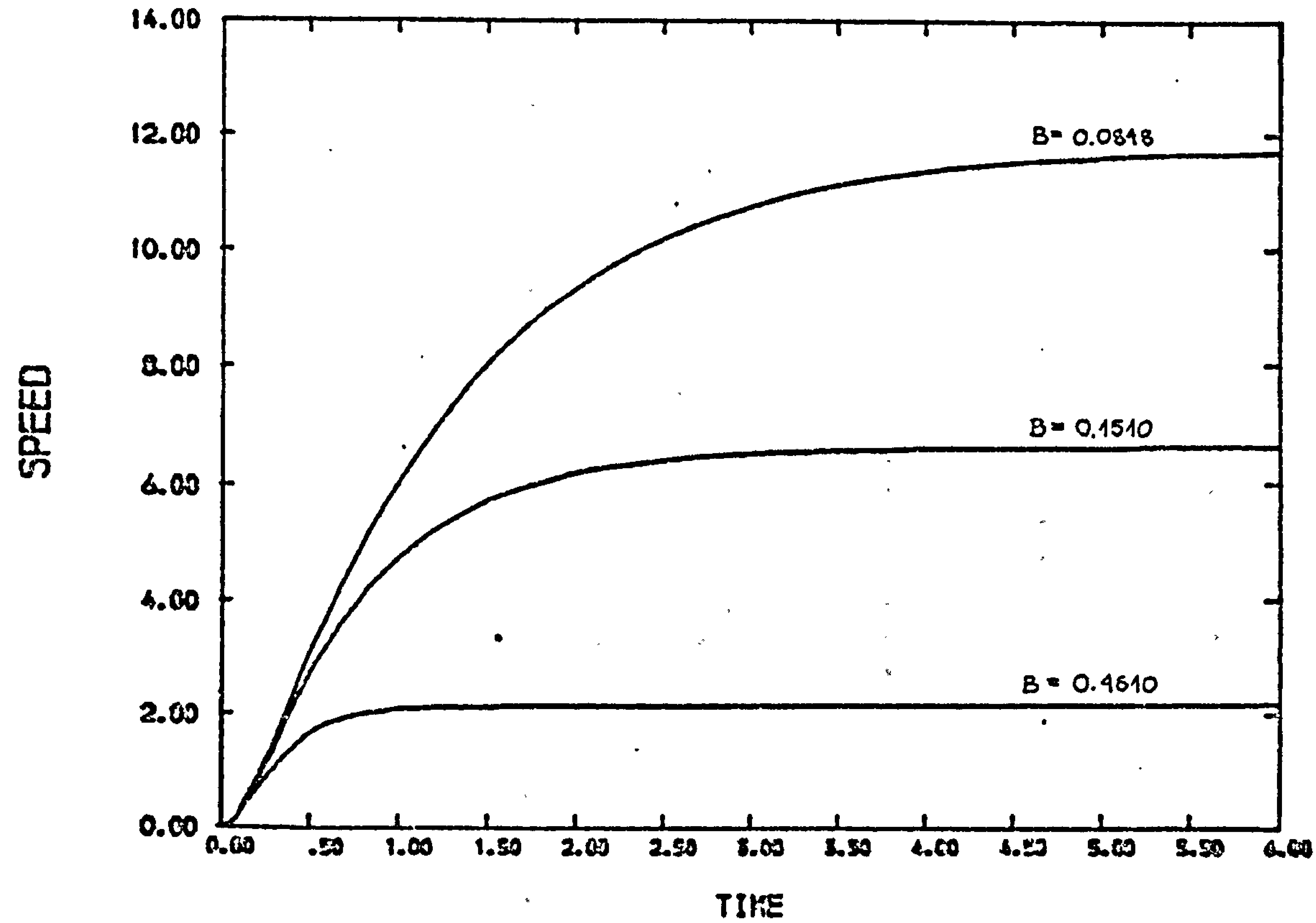



Fig.11 DYNAMIC CHARACTERISTIC OF THE SCM

Run No.2.
T = 800$^o$C
P = 9.66 bar

B-.1557

| | |
|---|---|
| 0 | 0 |
| .10000E+00 | .16086E+00 |
| .30000E+00 | .13685E+01 |
| .60000E+00 | .31876E+01 |
| .10000E+01 | .46847E+01 |
| .15000E+01 | .56254E+01 |
| .21000E+01 | .61103E+01 |
| .28000E+01 | .63187E+01 |
| .36000E+01 | .63939E+01 |
| .45000E+01 | .64168E+01 |
| .55000E+01 | .64227E+01 |
| .66000E+01 | .64240E+01 |
| .78000E+01 | .64242E+01 |
| .91000E+01 | .64243E+01 |
| .10500E+02 | .64243E+01 |
| .12000E+02 | .64243E+01 |
| .13600E+02 | .64243E+01 |
| .15300E+02 | .64243E+01 |
| .17100E+02 | .64243E+01 |
| .19000E+02 | .64243E+01 |

B-.4094

| | |
|---|---|
| 0 | 0 |
| .10000E+00 | .15005E+00 |
| .30000E+00 | .10869E+01 |
| .60000E+00 | .20050E+01 |
| .10000E+01 | .23558E+01 |
| .15000E+01 | .24312E+01 |
| .21000E+01 | .24414E+01 |
| .28000E+01 | .24423E+01 |
| .36000E+01 | .24424E+01 |
| .45000E+01 | .24424E+01 |
| .55000E+01 | .24424E+01 |
| .66000E+01 | .24424E+01 |
| .78000E+01 | .24424E+01 |
| .91000E+01 | .24424E+01 |
| .10500E+02 | .24424E+01 |
| .12000E+02 | .24424E+01 |
| .13600E+02 | .24424E+01 |
| .15300E+02 | .24424E+01 |
| .17100E+02 | .24424E+01 |
| .19000E+02 | .24424E+01 |

B-.0593

| | |
|---|---|
| 0 | .11102E-12 |
| .10000E+00 | .16528E+00 |
| .30000E+00 | .15046E+01 |
| .60000E+00 | .39256E+01 |
| .10000E+01 | .66539E+01 |
| .15000E+01 | .92742E+01 |
| .21000E+01 | .11547E+02 |
| .28000E+01 | .13355E+02 |
| .36000E+01 | .14682E+02 |
| .45000E+01 | .15586E+02 |
| .55000E+01 | .16160E+02 |
| .66000E+01 | .16500E+02 |
| .78000E+01 | .16688E+02 |
| .91000E+01 | .16785E+02 |
| .10500E+02 | .16832E+02 |
| .12000E+02 | .16854E+02 |
| .13600E+02 | .16863E+02 |
| .15300E+02 | .16867E+02 |
| .17100E+02 | .16868E+02 |
| .19000E+02 | .16869E+02 |

3051 CHARACTERS OUTPUT

*TRACE* SUBPROGRAM CALLS

| ROUTINE | COUNT |
|---|---|
| GRAFDEF | 1 |
| GRSLIDE | 1 |
| POLY3 | 3 |
| CAM35MM | 1 |
| EXP | 180 |
| ENDFILM | 1 |
| XAXIS | 1 |
| YAXIS | 1 |

*TRACE* STATEMENT NUMBERS FOR

Tab.4. Program STYRDYN results. Run No.3.
T = 800°C, P = 6.90 bar

FRAME 2 UDGEOFE HARDC

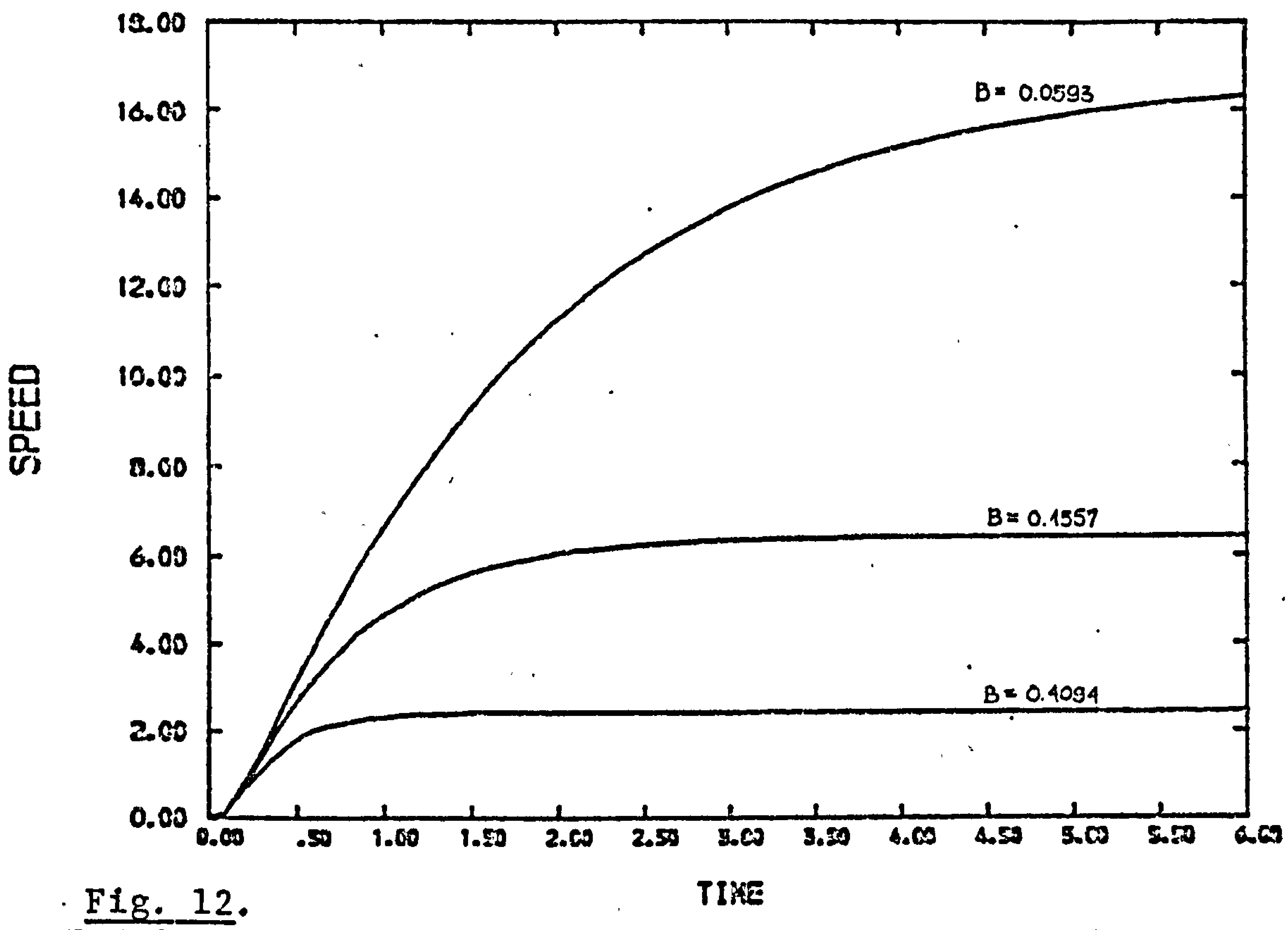



Fig. 12.
[*EOF*]

DYNAMIC CHARACTERISTIC OF THE SCM

Run No.3.
T = 800°C
P = 6.90 bar

B- .0542

| | |
|---|---|
| 0 | 0 |
| .10000E+00 | .16552E+00 |
| .30000E+00 | .15123E+01 |
| .60000E+00 | .39711E+01 |
| .10000E+01 | .67893E+01 |
| .15000E+01 | .95577E+01 |
| .21000E+01 | .12027E+02 |
| .28000E+01 | .14056E+02 |
| .36000E+01 | .15603E+02 |
| .45000E+01 | .16703E+02 |
| .55000E+01 | .17435E+02 |
| .66000E+01 | .17893E+02 |
| .78000E+01 | .18161E+02 |
| .91000E+01 | .18309E+02 |
| .10500E+02 | .18386E+02 |
| .12000E+02 | .18424E+02 |
| .13600E+02 | .18441E+02 |
| .15300E+02 | .18449E+02 |
| .17100E+02 | .18452E+02 |
| .19000E+02 | .18453E+02 |

B- .3916

| | |
|---|---|
| 0 | -.27756E-13 |
| .10000E+00 | .15077E+00 |
| .30000E+00 | .11036E+01 |
| .60000E+00 | .20638E+01 |
| .10000E+01 | .24498E+01 |
| .15000E+01 | .25389E+01 |
| .21000E+01 | .25522E+01 |
| .28000E+01 | .25535E+01 |
| .36000E+01 | .25536E+01 |
| .45000E+01 | .25536E+01 |
| .55000E+01 | .25536E+01 |
| .66000E+01 | .25536E+01 |
| .78000E+01 | .25536E+01 |
| .91000E+01 | .25536E+01 |
| .10500E+02 | .25536E+01 |
| .12000E+02 | .25536E+01 |
| .13600E+02 | .25536E+01 |
| .15300E+02 | .25536E+01 |
| .17100E+02 | .25536E+01 |
| .19000E+02 | .25536E+01 |

B- .2091

| | |
|---|---|
| 0 | .27756E-13 |
| .10000E+00 | .15848E+00 |
| .30000E+00 | .13007E+01 |
| .60000E+00 | .28627E+01 |
| .10000E+01 | .39479E+01 |
| .15000E+01 | .44887E+01 |
| .21000E+01 | .46983E+01 |
| .28000E+01 | .47626E+01 |
| .36000E+01 | .47783E+01 |
| .45000E+01 | .47814E+01 |
| .55000E+01 | .47819E+01 |
| .66000E+01 | .47819E+01 |
| .78000E+01 | .47819E+01 |
| .91000E+01 | .47819E+01 |
| .10500E+02 | .47819E+01 |
| .12000E+02 | .47819E+01 |
| .13600E+02 | .47819E+01 |
| .15300E+02 | .47819E+01 |
| .17100E+02 | .47819E+01 |
| .19000E+02 | .47819E+01 |

3138 CHARACTERS OUTPUT

*TRACE* SUBPROGRAM CALLS

| ROUTINE | COUNT |
|---|---|
| GRAFDEF | 1 |
| GRSLIDE | 1 |
| POLY3 | 3 |
| CAM35MM | 1 |
| EXP | 180 |
| ENDFILM | 1 |
| XAXIS | 1 |
| YAXIS | 1 |

*TRACE* STATEMENT NUMBERS FOR

Tab. 5. Program STYRDYN results. Run No. 4.
$T = 800^\circ C$, $P = 4.14$ bar

FRAME 2 UDGEOC3 ! HARDC

[*BREAK*]
! HARDC

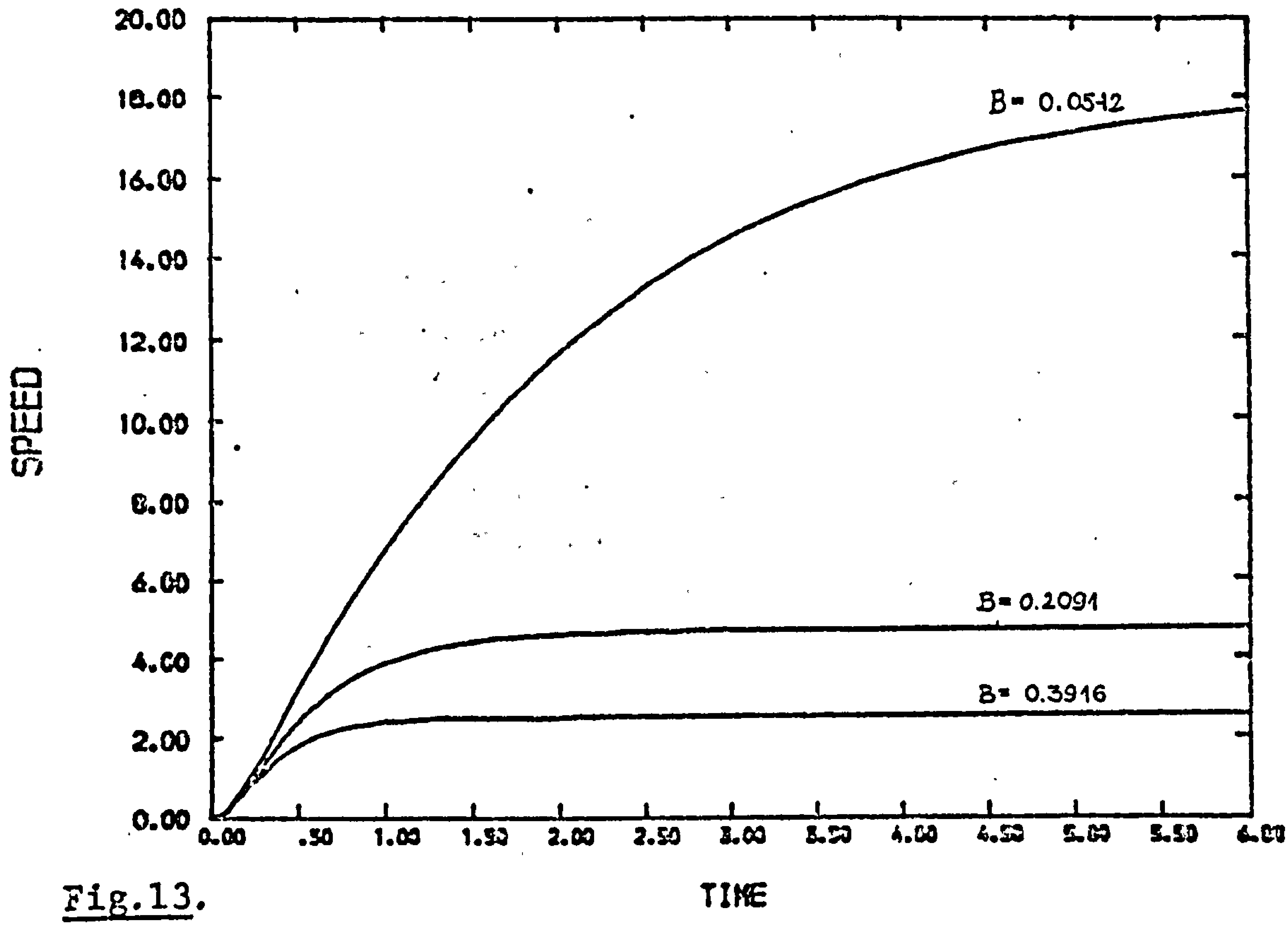



Fig.13.
[*EOF*]

DYNAMIC CHARACTERISTIC OF THE SCM

Run No.4.
T = 800°C
P = 4.14 bar

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## 4.2. THE DIGITAL ANALOG SIMULATION TECHNIQUES

Over the years, a number of digital programs have been developed which contain preprogrammed integration packages, these so called "simulation languages" relieve the engineer of knowing a great deal about numerical integration. In the majority of cases, they automatically monitor errors and stability and also adjust the integration interval or size of the step according to the relevant criteria of accuracy applied. Typical examples of simulation languages developed during the last twenty years and commonly used as digital analog simulators are : "Midas" - Modified Integrator Digital Analog Simulator (lit.1.), "MIMIC" (Lit.2), "BLODIB" (Lit.3), "LEANS" and nearly two dozen others with various levels of capability and overall effectiveness.

In practice however, these simulation languages have limited utility and very often become inefficient as the computer execution time for a realistic engineering problem when running one of these simulation languages is usually significantly longer than when run on a FORTRAN program, written just for the specific problem. However, in many engineering situations, the simulation languages are highly useful, bearing in mind important facts such as large reduction of set-up and programming times. Initially each of the earlier mentioned simulation languages was written for its special application work, i.e. "MIMIC" language provides a simple solution for the hybrid computer,as far as the simulation of sampled data systems are concerned and "BLODIB" - developed originally in the

Bell Telephone Laboratories in U.S.A. was designed for multipurpose communication as well as control systems simulation and general modelling. It is widely recognised by users that practical simulation languages provide a sort of "building block" capability, similarly as in 'orthodox' computer control systems namely: Z - transfer function, non-linear function, logic functions, time-varying controls and so on. The majority of digital analog simulation languages provide the means of programming a digital computer like a hybrid or analog computer system.

Historically, the first published work on digital analog simulation was presented by SELFRIDGE (see Lit. 4). His program was developed over twenty years ago for one of the early computers even without the advantage of automatic compilers (like FORTRAN). This particular program was an interceptive type of routine i.e. accepting and executing certain pseudo- instructions without producing a machine language translation and all computations were done in fixed point arithmetics, with problem variables being scaled to a definite maximum value. The following years brought many programming aids, automatic compilation techniques, floating-point arithmetic etc., and obviously benefited the contemporary range of simulation languages substantially, such as the above mentioned 'Mimic', 'Midas' and others like: 'Das' (Lit.6), DSL 90 (Lit 7) and the most comprehensive of them: PACTOLUS (Lit.8). The PACTOLUS simulation program exhibits a quite extraordinary innovation: MAN-MACHINE interplay (see Lit 8), being a very exciting and desirable feature but not too practical at the same time, as most professional computer systems do not allow for this type of operation.

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## 4.3. THE SYSTRAN PROGRAM

Systran is a Digital Analog simulation program of non-linear, dynamic sytems using standard Fortran language. The computer program can be obtained both from the program's author Dr. D.E. Hirst, Brunel University, Dept. of Electrical Engineering and Electronics, or from the I.E.E. Computer Program Library (see Lit.1.) and a careful program description was also published in "Electronic Letters" (see Lit.2). The original Systran users' manual for the MK 3E version was published back in April 1971, but it has since been improved and extended culminating in the MK 4A version available since July 1976. Basically Systran was developed from a simple but rather powerful Fortran integration program written by R. Cottington, J. Waters, and M. Bazen of the National Gas Turbine Establishment, Pyestock, about 1969 (see Lit.4) and further development by the original authors was carried out independently of the development of Systran. The Systran program is a set of Fortran routines, plus rules for data input, which enable a user to solve any desired set of ordinary differential equations with a minimum of programming effort. The rules for preparing jobs are such that a user acquainted with analog computer principles should not find much difficulty. The basic rules of setting up and running a system simulation in conjunction with Systran is as follows: First of all it is necessary to set up a group of equations which represent the system. These equations are called "SUBROUTINE EQNS" and are written in a standard Fortran language, describing the analysed system in terms of the interconnections between variables, selection of computations to be done, injection of disturbance etc. The above mentioned subroutine EQNS is combined with the basic SYSTRAN routine under the control of a standard FORTRAN compiler to produce a complete object program ready to solve the user's particular set of equations. During the program execution a wide variety of particular solutions may be generated, according to a "Control Sequence" specified individually in the form of a "Control Card Deck" which determines those settings and control actions which on the analog computer are carried out during problem execution. For example potentiometer settings, operate and hold push-buttons, function generator setup etc. From the user's point of view, the subroutine "EQNS" and the "Control Card Deck" can be treated independently, in particular, all the adjustments during simulations runs can be confined to the "Control Card Deck". This was found highly useful and reasonable in all situations, where it is considered uneconomic to recompile the program for a particular job at every computer run. The most efficient method of using SYSTRAN program is to keep a copy of it (in 'compiled' form) on a permanent file. It can then be used repeatedly without the necessity of referring to the original magnetic tape (U.L.C.C.) . In punched -card orientation, the following job deck is used, for putting a copy of SYSTRAN on to a permanent file:

```
JOB (UDEE 033, J6, T30, MT1)
REQUEST (TAPE, VSN = M2955M, NOLABEL, INHIBIT)
COPYBF (TAPE, OLDPL)
UNLOAD (TAPE)
UPDATE (Q, L= 1)
REQUEST  (LGO,*PF)
FTN (I = COMPILE)
CATALOG (LGO, SYSTRAN)
 ---  END OF  record card --
*  COMPILE , SYSC 4A
---   end of file card ---
```

This arrangement was used during the last three academic years on the University of London Computer (U.L.C.C.) through King's College Terminal. The following job control decks are used for this copy of Systran on all individual runs :

```
JOB (UDEE033, J6, T60)
ATTACH (LGO, SYSTRAN)
FTN (SL, B= EQNS)
LOAD (LGO)
EQNS
---End of Record Card

     ---------------Subroutine EQNS
---End of Record Card
      --------------Data for SYSTRAN
---End of File Card
```

According to the present King's College Computer Unit job arrangements, all the "permanent" files (including SYSTRAN ones) are not a 100% permanent. They must be used at least once a fortnight otherwise they will be discarded and will then have to be recreated using the job control given earlier in this chapter.

The MK 3E program was originally written for the 1900 series ICL computers, comprising approximately 1075 lines of Fortran source program (including 178 Comment lines), and when compiled from disc storage by the ICL disc Compiler XFAE on a 1903A machine (which takes about 40 seconds), the resulting program occupies approximately 21 K of core store (18.5 K if overlaid) varying obviously with the length of the user's subroutine. Each solution for the system of small to medium size takes typically between 1 - 5 minutes of compiler processing time. Systran program contains built-in limits to prevent longer processing times from occuring inadvertently and in situations where large problems require substantially longer processing times, these limits can be over-ridden. The Mark IV A Version of Systran basically supercedes MK 3E, introducing improved efficiency and facilities whilst preserving full compatibility with MK 3E. (see Lit.5). No attempt has been made to give details of the Systran facilities, as these are available from the user's manual (see Lit.3).

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## 4.4. BASIC SOLUTION FOR A SIMPLE ENGINE SIMULATION MODEL

The basic engine simulation model may be described by the block diagram shown in Fig.1. For the input signal purpose, a step disturbance STEP ($d_1$,$d_2$,t) generator is used, and $d_1$, $d_2$, t being subprogram's arguments, may be chosen as any constant, variable or even the expression of the appropriate type. The initial value of step (type Real)-$d_1$, final value of step (Type Real)- $d_2$ and the solution time beyond which step is to be applied (Type Real)describes satisfactorily the input signal. Apart from the "Step" form, other Systran " input" subprograms are available as RAMP (amplitude limited) PULSE (finite width pulse disturbance generator), but these will be applicable later on.

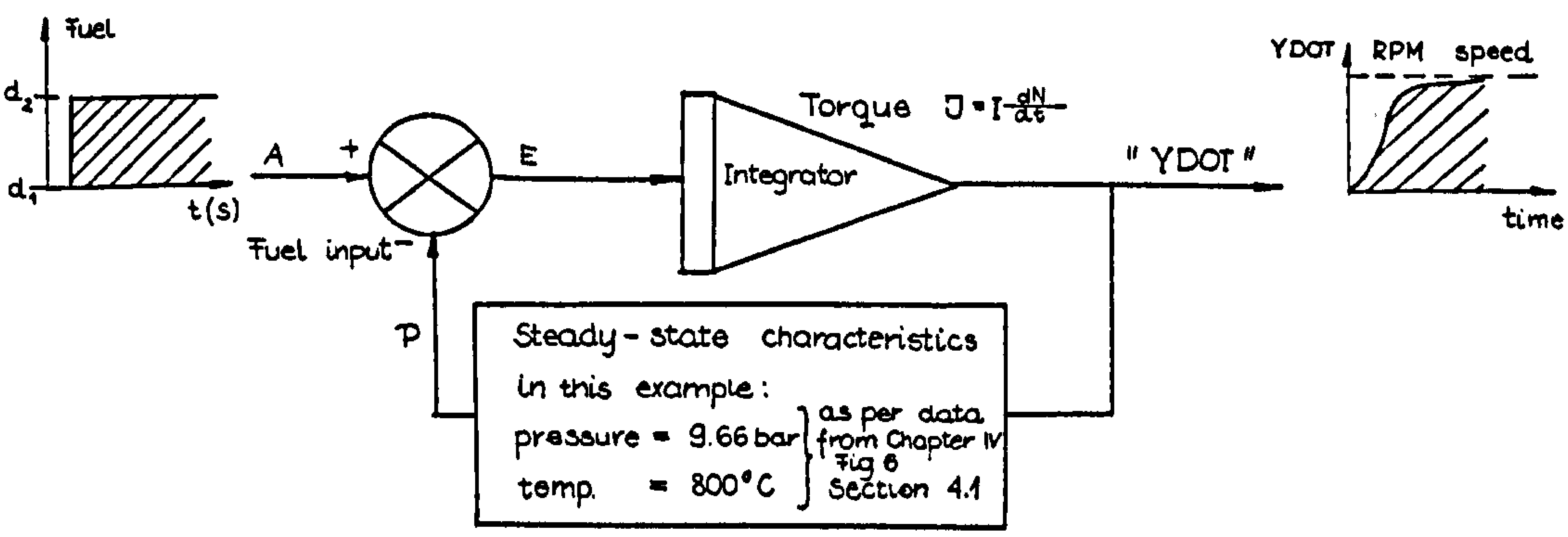



Fig. 1. The Basic Engine Simulation Model

The experimental data was chosen from "Prof. Walker's" experiment (see Fig.6; Section 4.1. Chapter 4) and stored (PROGRAM I) with the help of another Systran subprogram called FN3 and acting as the empirical function generator providing either simple linear or alternate linear/quadratic interpolation. The FN3 (n,x,c) arguments are:

n - number of functions defined on appropriate FUNCTIONS and CONSTANTS cards (Type Integer 1 n 20)

x - input value (Type Real)

c - Factor controlling length of quadratic blending segments

$c \leqslant o$ - gives simple linear interpolation

$c \geqslant 1$ - " maximum segment length

$o < c < 1$ - " blending segments of intermediate length.

FN3 works by calculating all required quadratic segment coefficients during the initiation pass through EQNS, then using these values during subsequent passes, with a specially provided 480 elements array for usual storage of these values. This was found to be quite a significant addition to the total core-store requirement of Systran. The appropriate program listing is enclosed as Program 1 which also includes the tabulated form of the output signal (Tab.1.) as well as the plot of the dynamic characteristic of the model (Fig.2). There is also a deck of four control cards i.e.

```
** LIMITS: ,, 120
**TABULATE: 1
** PLOT: 1(0.,2000)
** RUN: 0.5, 0.006
```

executed in the program at the onset of the "solution start".

```
NS          74/74    OPT=2                                    FTN 4.2+ R

      SUBROUTINE EQNS
      COMMON/VBLES/YDOT,DUM(39),E
      A=STEP(0.,6.?,0.0)
      P=0.0
      IF(YDOT.GT.1200.0) P=FN3(1,YDOT,0)
      E=(A-P)*14625.0
      RETURN
      END

 LISTING BY SYSTRAN/C PROGRAM MK.4A

 ENQUIRIES:- D.HIRST,EE DEPT,BRUNEL U.

     BEGIN READING CARDS:

  **FUN:1(1,42)        **CON:31=5.8
                                           **CON:41=6.85
  **CON:1=1200.0       **CON:11=1500.0
                                           **CON:21=1800.0
  **CON:22=5.2         **CON:32=5.9
                                           **CON:42=6.9
  **CON:2=1230.0       **CON:12=1530.0
  **CON:23=5.2         **CON:33=6.05
  **CON:3=1260.0       **CON:13=1560.0
  **CON:24=5.22        **CON:34=6.2
  **CON:4=1290.0       **CON:14=1590.0
  **CON:25=5.25        **CON:35=6.35
  **CON:5=1320.0       **CON:15=1620.0
  **CON:26=5.3         **CON:36=6.5
  **CON:6=1350.0       **CON:16=1650.0
  **CON:27=5.4         **CON:37=6.6
  **CON:7=1380.0       **CON:17=1680.0
  **CON:28=5.45        **CON:38=6.68
  **CON:8=1410.0       **CON:18=1710.0
                                           **LIMITS:,,120
  **CON:29=5.55        **CON:39=6.75
                                           **TABULATE:1
  **CON:9=1440.0       **CON:19=1740.0
                                           **PLOT:1(0.,2000)
  **CON:30=5.7         **CON:40=6.8
                                           **RUN:0.9,0.0099
  **CON:10=1470.0      **CON:20=1770.0
                                           SOLUTION START
```

Prog.1.

| TIME | R-K | N-R | WORST ERR | X | X1 |
|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. |
| 9.900E-03 | 11 | 0 | 1.301E+05 | 1 | EXCESS ERRS |
| 9.900E-03 | 11 | 0 | 1.301E+05 | 1 | 9.98709E+02 |
| 1.980E-02 | 28 | 0 | 6.537E-04 | 1 | 1.38675E+03 |
| 2.970E-02 | 32 | 0 | 2.776E-04 | 1 | 1.54414E+03 |
| 3.960E-02 | 36 | 0 | 1.698E-04 | 1 | 1.62448E+03 |
| 4.950E-02 | 39 | 0 | 1.332E-04 | 1 | 1.66913E+03 |
| 5.940E-02 | 41 | 0 | 1.041E-04 | 1 | 1.69934E+03 |
| 6.930E-02 | 43 | 0 | 6.308E-05 | 1 | 1.72111E+03 |
| 7.920E-02 | 44 | 0 | 1.381E-04 | 1 | 1.73802E+03 |
| 8.910E-02 | 45 | 0 | 1.074E-04 | 1 | 1.75131E+03 |
| 9.900E-02 | 46 | 0 | 8.376E-05 | 1 | 1.76174E+03 |
| 1.089E-01 | 47 | 0 | 6.541E-05 | 1 | 1.76994E+03 |
| 1.188E-01 | 48 | 0 | 5.116E-05 | 1 | 1.77638E+03 |
| 1.287E-01 | 49 | 0 | 4.005E-05 | 1 | 1.78145E+03 |
| 1.386E-01 | 50 | 0 | 3.137E-05 | 1 | 1.78542E+03 |
| 1.485E-01 | 51 | 0 | 2.459E-05 | 1 | 1.78855E+03 |
| 1.584E-01 | 52 | 0 | 1.929E-05 | 1 | 1.79100E+03 |
| 1.683E-01 | 53 | 0 | 1.513E-05 | 1 | 1.79293E+03 |
| 1.782E-01 | 54 | 0 | 1.188E-05 | 1 | 1.79445E+03 |
| 1.881E-01 | 55 | 0 | 9.323E-06 | 1 | 1.79564E+03 |
| 1.980E-01 | 56 | 0 | 7.320E-06 | 1 | 1.79657E+03 |
| 2.079E-01 | 57 | 0 | 5.748E-06 | 1 | 1.79731E+03 |
| 2.178E-01 | 58 | 0 | 4.514E-06 | 1 | 1.79788E+03 |
| 2.277E-01 | 59 | 0 | 3.546E-06 | 1 | 1.79834E+03 |
| 2.376E-01 | 60 | 0 | 2.785E-06 | 1 | 1.79869E+03 |
| 2.475E-01 | 61 | 0 | 2.188E-06 | 1 | 1.79897E+03 |
| 2.574E-01 | 62 | 0 | 1.718E-06 | 1 | 1.79919E+03 |
| 2.673E-01 | 63 | 0 | 1.350E-06 | 1 | 1.79937E+03 |
| 2.772E-01 | 64 | 0 | 1.060E-06 | 1 | 1.79950E+03 |
| 2.871E-01 | 65 | 0 | 8.331E-07 | 1 | 1.79961E+03 |
| 2.970E-01 | 66 | 0 | 6.545E-07 | 1 | 1.79969E+03 |
| 3.069E-01 | 67 | 0 | 5.142E-07 | 1 | 1.79976E+03 |
| 3.168E-01 | 68 | 0 | 4.040E-07 | 1 | 1.79981E+03 |
| 3.267E-01 | 69 | 0 | 3.174E-07 | 1 | 1.79985E+03 |
| 3.366E-01 | 70 | 0 | 2.493E-07 | 1 | 1.79988E+03 |
| 3.465E-01 | 71 | 0 | 1.959E-07 | 1 | 1.79991E+03 |
| 3.564E-01 | 72 | 0 | 1.539E-07 | 1 | 1.79993E+03 |
| 3.663E-01 | 73 | 0 | 1.209E-07 | 1 | 1.79994E+03 |
| 3.762E-01 | 74 | 0 | 9.500E-08 | 1 | 1.79996E+03 |
| 3.861E-01 | 75 | 0 | 7.464E-08 | 1 | 1.79996E+03 |
| 3.960E-01 | 76 | 0 | 5.864E-08 | 1 | 1.79997E+03 |
| 4.059E-01 | 77 | 0 | 4.607E-08 | 1 | 1.79998E+03 |
| 4.158E-01 | 78 | 0 | 3.620E-08 | 1 | 1.79998E+03 |
| 4.257E-01 | 79 | 0 | 2.844E-08 | 1 | 1.79999E+03 |
| 4.356E-01 | 80 | 0 | 2.234E-08 | 1 | 1.79999E+03 |
| 4.455E-01 | 81 | 0 | 1.755E-08 | 1 | 1.79999E+03 |
| 4.554E-01 | 82 | 0 | 1.379E-08 | 1 | 1.79999E+03 |
| 4.653E-01 | 83 | 0 | 1.083E-08 | 1 | 1.79999E+03 |
| 4.752E-01 | 84 | 0 | 8.513E-09 | 1 | 1.80000E+03 |
| 4.851E-01 | 85 | 0 | 6.688E-09 | 1 | 1.80000E+03 |
| 4.950E-01 | 86 | 0 | 5.255E-09 | 1 | 1.80000E+03 |
| 5.049E-01 | 87 | 0 | 4.128E-09 | 1 | 1.80000E+03 |
| 5.148E-01 | 88 | 0 | 3.243E-09 | 1 | 1.80000E+03 |
| 5.247E-01 | 89 | 0 | 2.548E-09 | 1 | 1.80000E+03 |
| 5.346E-01 | 90 | 0 | 2.002E-09 | 1 | 1.80000E+03 |
| 5.445E-01 | 91 | 0 | 1.573E-09 | 1 | 1.80000E+03 |
| 5.544E-01 | 92 | 0 | 1.236E-09 | 1 | 1.80000E+03 |
| 5.643E-01 | 93 | 0 | 9.709E-10 | 1 | 1.80000E+03 |
| 5.742E-01 | 94 | 0 | 7.628E-10 | 1 | 1.80000E+03 |
| 5.841E-01 | 95 | 0 | 5.993E-10 | 1 | 1.80000E+03 |

Tab.1. Program 1 results.

| | | | | | |
|---|---|---|---|---|---|
| 5.940E-01 | 96 | 0 | 4.709E-10 | 1 | 1.80000E+03 |
| 6.039E-01 | 97 | 0 | 3.699E-10 | 1 | 1.80000E+03 |
| 6.138E-01 | 98 | 0 | 2.906E-10 | 1 | 1.80000E+03 |
| 6.237E-01 | 99 | 0 | 2.283E-10 | 1 | 1.80000E+03 |
| 6.336E-01 | 100 | 0 | 1.794E-10 | 1 | 1.80000E+03 |
| 6.435E-01 | 101 | 0 | 1.410E-10 | 1 | 1.80000E+03 |
| 6.534E-01 | 102 | 0 | 1.107E-10 | 1 | 1.80000E+03 |
| 6.633E-01 | 103 | 0 | 8.700E-11 | 1 | 1.80000E+03 |
| 6.732E-01 | 104 | 0 | 6.836E-11 | 1 | 1.80000E+03 |
| 6.831E-01 | 105 | 0 | 5.370E-11 | 1 | 1.80000E+03 |
| 6.930E-01 | 106 | 0 | 4.219E-11 | 1 | 1.80000E+03 |
| 7.029E-01 | 107 | 0 | 3.315E-11 | 1 | 1.80000E+03 |
| 7.128E-01 | 108 | 0 | 2.605E-11 | 1 | 1.80000E+03 |
| 7.227E-01 | 109 | 0 | 2.046E-11 | 1 | 1.80000E+03 |
| 7.326E-01 | 110 | 0 | 1.608E-11 | 1 | 1.80000E+03 |
| 7.425E-01 | 111 | 0 | 1.263E-11 | 1 | 1.80000E+03 |
| 7.524E-01 | 112 | 0 | 9.923E-12 | 1 | 1.80000E+03 |
| 7.623E-01 | 113 | 0 | 7.796E-12 | 1 | 1.80000E+03 |
| 7.722E-01 | 114 | 0 | 6.125E-12 | 1 | 1.80000E+03 |
| 7.821E-01 | 115 | 0 | 4.813E-12 | 1 | 1.80000E+03 |
| 7.920E-01 | 116 | 0 | 3.781E-12 | 1 | 1.80000E+03 |
| 8.019E-01 | 117 | 0 | 2.970E-12 | 1 | 1.80000E+03 |
| 8.118E-01 | 118 | 0 | 2.334E-12 | 1 | 1.80000E+03 |
| 8.217E-01 | 119 | 0 | 1.833E-12 | 1 | 1.80000E+03 |
| 8.316E-01 | 120 | 0 | 1.441E-12 | 1 | 1.80000E+03 |
| 8.415E-01 | 121 | 0 | 1.132E-12 | 1 | 1.80000E+03 |
| 8.514E-01 | 122 | 0 | 8.896E-13 | 1 | 1.80000E+03 |
| 8.613E-01 | 123 | 0 | 6.983E-13 | 1 | 1.80000E+03 |
| 8.712E-01 | 124 | 0 | 5.488E-13 | 1 | 1.80000E+03 |
| 8.811E-01 | 125 | 0 | 4.309E-13 | 1 | 1.80000E+03 |
| 8.910E-01 | 126 | 0 | 3.388E-13 | 1 | 1.80000E+03 |
| 9.009E-01 | 127 | 0 | 2.662E-13 | 1 | 1.80000E+03 |

TIME LIMIT
ENDED THIS RUN

RUN ANALYSIS

| | R-K STEPS | | | |
|---|---|---|---|---|
| TIME | TRIED | ACCEPTED | EXC.ERRS | TRIED |
| 9.009E-01 | 127 | 121 | 1 | 0 |

| N-R STEPS | | JACN. | SOLUTION | ABORT |
|---|---|---|---|---|
| ACCEPTED | EXC.ERRS | EVALS | POINTS | CODE |
| 0 | 0 | 0 | 92 | 0 |

Program 1 results/continuation/


Fig.2. Plotted results for Program 1 example.

The "LIMITS" card is directly associated with Runge-Kutta-Mersón (B-K-M)integration process which is achieved in Systran using Merson's modification of the 4th order Runge-Kutta integration formula. As the Systran manual claims this particular formula is chosen because :

1. It can in principle be made numerically stable by choosing short enough step-lengths (i.e. small enough increments of the independent variable between solution points).

2. It permits reasonable economy of processing time by regulation of step-lengths according to integration error.

There are many varieties of Runge-Kutta algorithms, and all these are one step methods that do not require the evaluation of higher derivatives of the function, but they nevertheless produce solutions accurate to the same order of magnitude as those produced by using a corresponding number of terms in a Taylor series expansion. Merson's modification of the 4th order Runge-Kutta integration formula gives for a particular Integrator Output x:

$$X_{n+1} = X_n + \frac{1}{2} (K_1+4K_4+K_5)$$

Where: $X_n$ is the known value of x at time $t_n$.
$X_{n+1}$ is the value obtained for x at Time $t_{n+1}=t_n+h$
$h$ = time step $= t_{n+1} - t_n$

$K_1 = 1/3h \ \dot{x}\ (t_n, x_n)$

$K_2 = 1/3h \ \dot{x}\ (t_n + 1/3h,\ x_n + k_1)$

$K3 = 1/3h \ \dot{x}\ (t_n + 1/3h,\ x_n + \frac{1}{2} k_1 + \frac{1}{2} k_2)$

$K_4 = 1/3h \ \dot{x}\ (t_n + \frac{1}{2}h,\ x_n + \frac{3}{8}k_1 + 9/8k_3)$

$K_5 = 1/3h \ \dot{x}\ (t_n + h,\ x_n + 3/2\ k_1 - 9/2k_3 + 6k_4)$

$\dot{x}\ (t, a)$ is the value of $\dot{x}$ at solution time t, assuming that $x(t) = a$.

The above presented algorithm specifies the values for x and the corresponding k's, given values for $\dot{x}$ at each value of time.

As mentioned earlier most of the "simulation languages" in order to stay within some accuracy criteria, are equipped with an automatic error monitor and also adjust the integration interval or stepsize. Systran employs at the end of each attempted step the Merson truncation error formula (individually for each integrator output).

$$e_x = 1/5\ R_x \left| (k_1 - 9/2\ k_3 + 4k_4 - \tfrac{1}{2}k_5) \right|$$

Where $R_x$ is a normalising parameter whose value depends on error checking mode. Assuming that $e_x$ is within bounds for every x, the step is accepted and a next one is commenced, in situations where step is not accepted, h-value is halved and new attempt (s) is (are) made, up to the moment when a minimum specified step size is reached. (for details see Lit.1). When implicit algebraic equations are present the Newton-Raphson iteration process is applied, and also each time the integration routine calls for a new evaluation of the Systran derivatives. This is carefully described in the User's Manual (see Lit. 1. p.7.8.9.1o) and also a simplified flow diagram of combined Newton-Raphson and Runge-Kutta procedures is explained (for details see Lit.1. p.9. Fig.3).

Coming back to the earlier mentioned control deck, "LIMITS" card, controls the following operational parameters -

LIMITS : $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, d_9, d_{1o}$

Where $d_1$ = Max. no. of R-K steps to be attempted
$d_2$ = Max. no. of N-R steps to be attempted
$d_3$ = Max. no. of solution points to be generated
$d_4$ = Common error bound for all x variables
$d_5$ = minimum R-K-M step length
$d_6$ = maximum R-K-M step length
$d_7$ = minimum N-R correction step factor value
$d_8$ = maximum N-R correction step factor value
$d_9$ = Per unit test perturbation at Butter Block outputs for JACOBIAN MATRIX estimation
$d_{1o}$= Resolution limit for error control and N-R iteration.

In "LIMITS" each non-blank entry over-rides the existing value of the parameter concerned, however, any number of items may be omitted. Next card "TABULATE" specifices that subsequent solutions are to be output in tabular form, for example :

TABULATE : $h_1$ $h_2$,............$h_{k-1}$, hk

where: h is number of an x variable ( $1 \leqslant h \leqslant 40$).

Continuing with the control deck cards main objectives :

"PLOT" specifies x variables and scales for plotted output, e.g.

PLOT : $n_1$ $(s_1,t_1)$ $n_2$ $(s_2,t_2)$......$n_k$ $(s_k,t_k)$ .....

Where: n - number of an x variable ( $1 \leqslant n \leqslant 40$)
s - lower limit of scale on lineprinter plot ) $t > s$
t - upper limit of scale on lineprinter plot )

Finally "RUN" control card initiates a transient solution, using standard N-R iteration algorith (if Butter Block is present).

RUN : $d_1,d_2$

where: $d_1$ = termination value of solution time (TP)
$d_2$ = print interval for tabulation/plot

As explained earlier, a consequence of the nature of the Runge-Kutta-Merson procedure is that some of the solution time values arising in the integration process hold for more than one evaluation of the system derivatives, i.e. for more than one pass through EQNS deck. The normal solution time variable is called in Systran nomenclature TP and the second solution time TE (equal TP only when a particular integration procedure is accepted) and is representing the established value of solution time.
For the user's convenience STEP/RAMP/PULSE input disturbance generators, as mentioned previously, are incorporated with simple but very useful features allowing the delay of these particular types of input disturbances. This arrangement allows for overall synchronisation in multi-input control

```
IS         74/74    OPT=2                                  FTN 4.2+R

      SUBROUTINE EQNS
      COMMON/VBLES/YDOT,DUM(39),E
      A=STEP(0.,6.9,0.1)
      P=0.0
      IF(YDOT.GT.1200.0) P=FN3(1,YDOT,0)
      E=(A-P)*14625.0
      RETURN
      END
```

Prog.2.

```
SOLUTION OUTPUT

   TIME        R-K  N-R  WORST ERR  - X          X1

0.              0    0   0.          0    0.
 9.900E-03      1    0   0.          0    0.
 1.980E-02      2    0   0.          0    0.
 2.970E-02      3    0   0.          0    0.
 3.960E-02      4    0   0.          0    0.
 4.950E-02      5    0   0.          0    0.
 5.940E-02      6    0   0.          0    0.
 6.930E-02      7    0   0.          0    0.
 7.920E-02      8    0   0.          0    0.
 8.910E-02      9    0   0.          0    0.
 9.900E-02     10    0   0.          0    0.
 1.089E-01     42    0   6.504E+04   1     EXCESS ERRS ACCEP
 1.089E-01     42    0   6.504E+04   1    8.97895E+02
 1.188E-01     55    0   6.653E-04   1    1.36769E+03
 1.287E-01     59    0   2.677E-04   1    1.53336E+03
 1.386E-01     63    0   1.828E-04   1    1.61920E+03
 1.485E-01     66    0   1.472E-04   1    1.66574E+03
 1.584E-01     68    0   1.096E-04   1    1.69697E+03
 1.683E-01     70    0   6.666E-05   1    1.71934E+03
 1.782E-01     71    0   1.413E-04   1    1.73663E+03
 1.881E-01     72    0   1.099E-04   1    1.75021E+03
 1.980E-01     73    0   8.570E-05   1    1.76088E+03
 2.079E-01     74    0   6.692E-05   1    1.76927E+03
 2.178E-01     75    0   5.233E-05   1    1.77585E+03
 2.277E-01     76    0   4.096E-05   1    1.78103E+03
 2.376E-01     77    0   3.209E-05   1    1.78510E+03
 2.475E-01     78    0   2.515E-05   1    1.78829E+03
 2.574E-01     79    0   1.973E-05   1    1.79080E+03
 2.673E-01     80    0   1.548E-05   1    1.79277E+03
 2.772E-01     81    0   1.215E-05   1    1.79432E+03
 2.871E-01     82    0   9.534E-06   1    1.79554E+03
 2.970E-01     83    0   7.485E-06   1    1.79649E+03
 3.069E-01     84    0   5.878E-06   1    1.79725E+03
 3.168E-01     85    0   4.616E-06   1    1.79784E+03
 3.267E-01     86    0   3.625E-06   1    1.79830E+03
 3.366E-01     87    0   2.848E-06   1    1.79866E+03
 3.465E-01     88    0   2.237E-06   1    1.79895E+03
 3.564E-01     89    0   1.757E-06   1    1.79918E+03
 3.663E-01     90    0   1.380E-06   1    1.79935E+03
 3.762E-01     91    0   1.084E-06   1    1.79949E+03
 3.861E-01     92    0   8.519E-07   1    1.79960E+03
 3.960E-01     93    0   6.692E-07   1    1.79969E+03
```

Tab.2. Program 2 results.

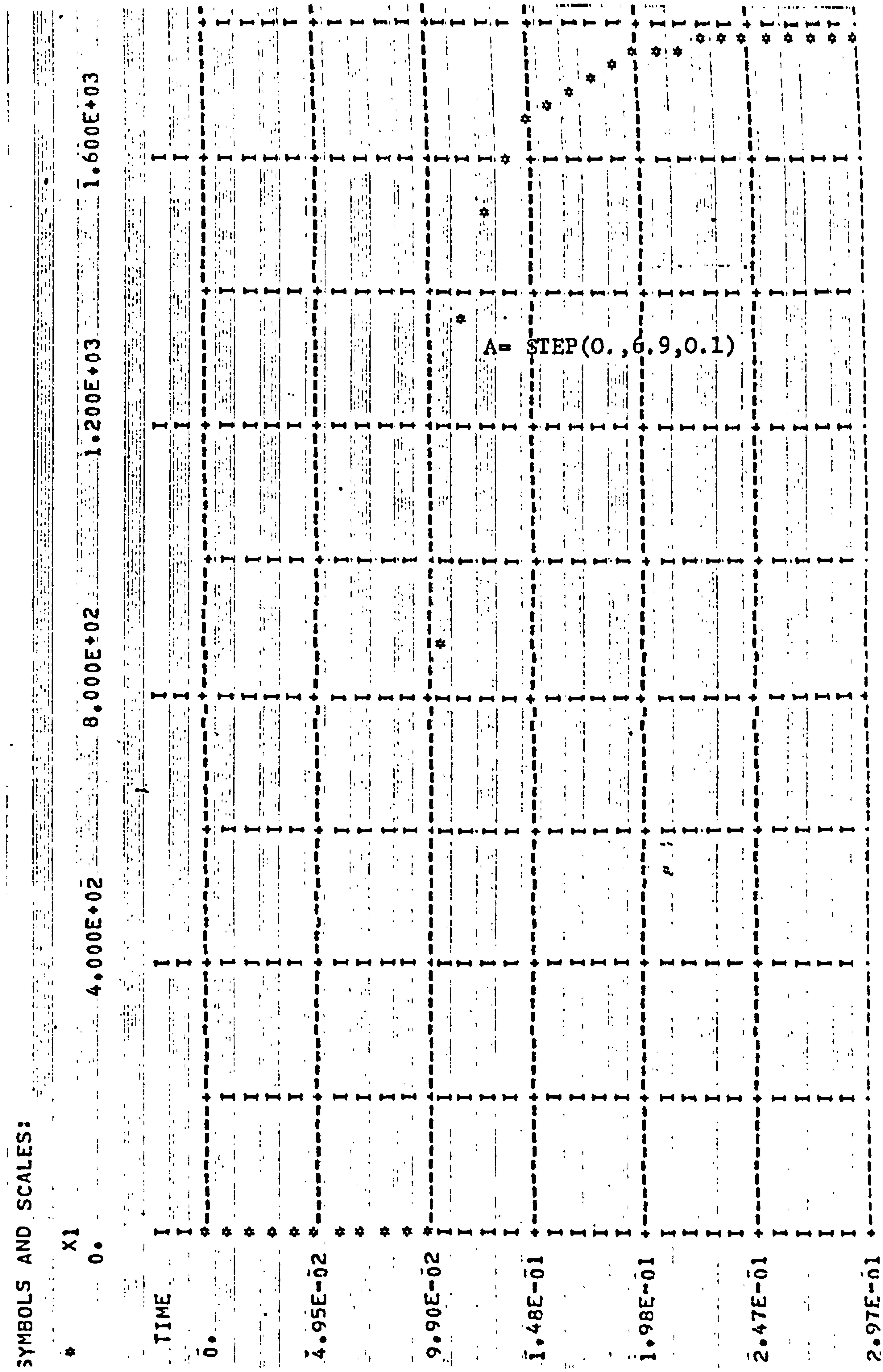



Fig. 3. Plotted results for Program 2 example.

systems as well as delaying single disturbance generators in all situations where this is required. An example of a delaying step disturbance : A = STEP (0.,6.9,0.1) is shown in Program 2. (see also Tab. 2 and Fig. 3).

The last program represents a very simple simulation model where the speed is controlled directly by the fuel input. Let us now incorporate additionally to the above, a single feedback control of the speed of the basic engine as well as two blocks representing the fuel monitoring valve and the tachometric transducer unit measuring the output speed of the engine (see Fig. 4) (Similar as Fig. 2, Section 4.1. Chapter 4).

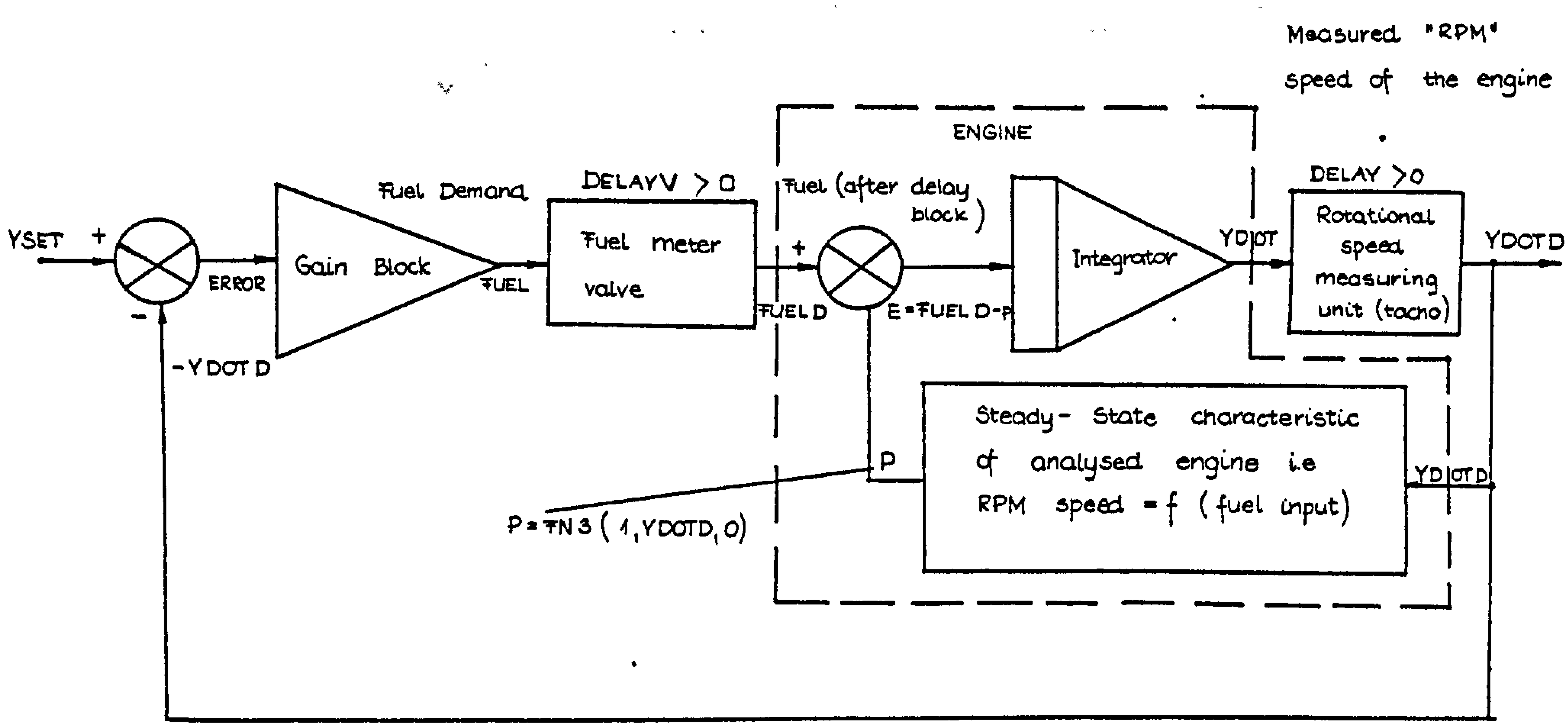



Fig. 4. A Simple Feedback Speed Control of An Engine.

Referring once again to the electronic analog circuit being a sort of equivalent of the block diagram shown in Fig. 4 and constructed using similar techniques as the analog circuit presented earlier, i.e. (Fig. 7 Section 4.1. Chapter 4) this one is shown in Fig. 5. Inverter No. 2 (Fig. 5) gives a correct sign of input voltage to the engine, and the specific resistance of the "Set Level" potentiometer is chosen such that it has very little influence on the input signal level when the switch S/W1 is in "ON" position.

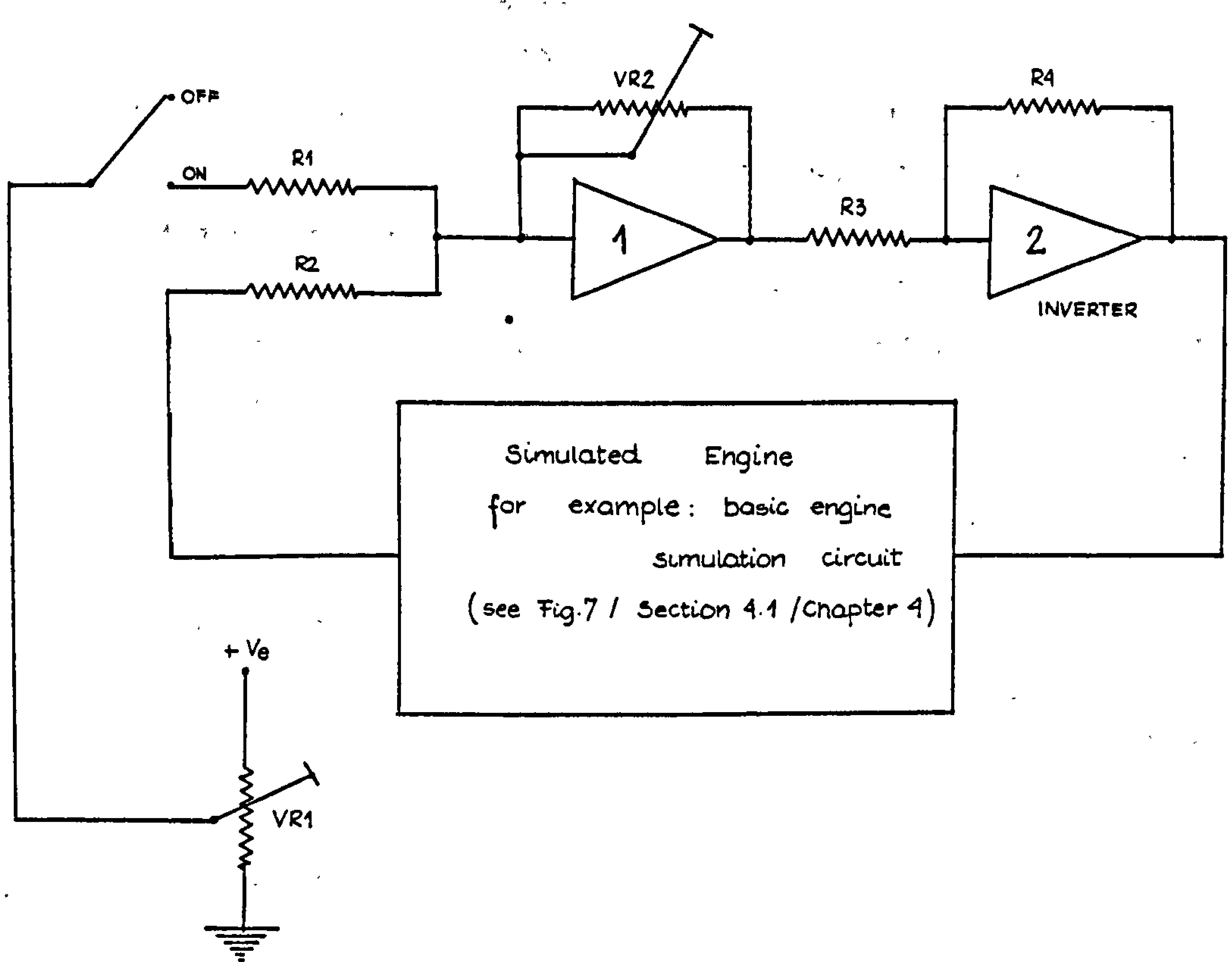



Fig. 5 Electronic Analog Realisation of a proportional Control System (zero-error seeking system).

In general, this kind of control is called "Proportional Control" and consists simply of a single feedback path (compare with Fig. 4). The step input can be supplied simply by turning the S/W1 switch into the "ON" position. From practical experiments (see Lit.7) on similar circuits it was proved that when the circuit gain was low (approx. less than 2.5) the system output did not reach the input value and when the gain was too high - output was oscillating. Also the gain margin of the system depends on the magnitude of the step input applied. The system simulation program (Program 3) for the above system (Fig.4) includes print out of the results (Tab.3) and line printer plot (Fig. 6a & b). Also this program uses the earlier described FN3 interpolation subprogram and two XD (n,d) transport delay subprogram on fuel valve FUELD = XD (2,DELAYV) and tachometric transducer respectively. YDOTD = XD (1,DELAY) The XD (n,d) subprogram generates a value equal to that of X variable n delayed by d time units. "XD" finds output values by linear interpolation in an "internal" table of one hundred past values. Various DELAY (tacho) argument values have been tested and the results show clearly that with greater DELAY time the overall speed outputs caused some dynamic overshoot effect with the initial steady-state data material (see Fig.7).

A comparison of various settings of both DELAY/DELAYV arguments is shown simultaneouly in Fig. 6 a & b and corresponds to

DELAY = 0.0055 min ) 330 msec. ) Fig.6b
DELAYV = 0.001min ) 60 msec. )

DELAY = 0.00166 min )100msec ) Fig. 6a
DELAYV = 0.000833min) 5omsec )

NS 74/74 OPT=2 FTN 4.2+ RE

```
      SUBROUTINE EQNS
      COMMON/VBLES/YDOT,FUEL,DUM(38),E
      YSET=1800.
      DELAY=0.0016666
      DELAYV=0.0008333
      YDOTD=XD(1,DELAY)
      ERROR=YSET-YDOTD
      IF(YDOTD.LT.1200.0) FUEL=6.9
      IF(YDOTD.GE.1685.0) G=3.3
      IF(YDOTD.GE.1420.0.AND.YDOTD.LT.1685.0) G=5.3
      IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1420.0) G=3.7
      IF(YDOTD.GE.1200.0) FUEL=G*ERROR/14625.0
      P=0.0
      IF(YDOTD.GT.1200.0) P=FN3(1,YDOTD,0)/14625.0
      FUELD=XD(2,DELAYV)
      E=(FUELD-P)*14625.0
      RETURN
      END
```

Prog. 3.

SOLUTION OUTPUT

| TIME | R-K | N-R | WORST ERR - X | | X1 |
|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. |
| 9.900E-03 | 1 | 0 | 2.183E-06 | 1 | 9.99034E+02 |
| 1.980E-02 | 14 | 0 | 5.881E-04 | 1 | 1.46181E+03 |
| 2.970E-02 | 18 | 0 | 9.463E-04 | 1 | 1.48195E+03 |
| 3.960E-02 | 19 | 0 | 2.042E-06 | 1 | 1.49834E+03 |
| 4.950E-02 | 20 | 0 | 1.061E-05 | 1 | 1.51390E+03 |
| 5.940E-02 | 21 | 0 | 9.828E-06 | 1 | 1.52866E+03 |
| 6.930E-02 | 22 | 0 | 9.219E-06 | 1 | 1.54266E+03 |
| 7.920E-02 | 23 | 0 | 8.662E-06 | 1 | 1.55593E+03 |
| 8.910E-02 | 24 | 0 | 8.143E-06 | 1 | 1.56851E+03 |
| 9.900E-02 | 25 | 0 | 7.659E-06 | 1 | 1.58044E+03 |
| 1.089E-01 | 26 | 0 | 7.207E-06 | 1 | 1.59175E+03 |
| 1.188E-01 | 27 | 0 | 6.785E-06 | 1 | 1.60247E+03 |
| 1.287E-01 | 28 | 0 | 6.390E-06 | 1 | 1.61264E+03 |
| 1.386E-01 | 29 | 0 | 6.020E-06 | 1 | 1.62228E+03 |
| 1.485E-01 | 30 | 0 | 5.672E-06 | 1 | 1.63142E+03 |
| 1.584E-01 | 31 | 0 | 5.348E-06 | 1 | 1.64009E+03 |
| 1.683E-01 | 32 | 0 | 5.044E-06 | 1 | 1.64831E+03 |
| 1.782E-01 | 33 | 0 | 4.759E-06 | 1 | 1.65610E+03 |
| 1.881E-01 | 34 | 0 | 4.490E-06 | 1 | 1.66349E+03 |
| 1.980E-01 | 35 | 0 | 4.238E-06 | 1 | 1.67050E+03 |
| 2.079E-01 | 36 | 0 | 4.002E-06 | 1 | 1.67714E+03 |
| 2.178E-01 | 37 | 0 | 3.779E-06 | 1 | 1.68344E+03 |
| 2.277E-01 | 38 | 0 | 2.571E-04 | 1 | 1.68783E+03 |
| 2.376E-01 | 39 | 0 | 1.025E-05 | 1 | 1.69145E+03 |
| 2.475E-01 | 40 | 0 | 1.449E-06 | 1 | 1.69490E+03 |
| 2.574E-01 | 41 | 0 | 1.212E-06 | 1 | 1.69824E+03 |
| 2.673E-01 | 42 | 0 | 1.177E-06 | 1 | 1.70147E+03 |
| 2.772E-01 | 43 | 0 | 1.136E-06 | 1 | 1.70460E+03 |
| 2.871E-01 | 44 | 0 | 1.098E-06 | 1 | 1.70763E+03 |
| 2.970E-01 | 45 | 0 | 1.060E-06 | 1 | 1.71055E+03 |
| 3.069E-01 | 46 | 0 | 1.024E-06 | 1 | 1.71339E+03 |
| 3.168E-01 | 47 | 0 | 9.889E-07 | 1 | 1.71613E+03 |
| 3.267E-01 | 48 | 0 | 9.553E-07 | 1 | 1.71878E+03 |

Tab. 3. Program 3 results.

| | | | | | |
|---|---|---|---|---|---|
| 3.366E-01 | 49 | 0 | 9.229E-07 | 1 | 1.72134E+03 |
| 3.465E-01 | 50 | 0 | 8.916E-07 | 1 | 1.72383E+03 |
| 3.564E-01 | 51 | 0 | 8.615E-07 | 1 | 1.72623E+03 |
| 3.663E-01 | 52 | 0 | 8.324E-07 | 1 | 1.72855E+03 |
| 3.762E-01 | 53 | 0 | 8.043E-07 | 1 | 1.73080E+03 |
| 3.861E-01 | 54 | 0 | 7.772E-07 | 1 | 1.73298E+03 |
| 3.960E-01 | 55 | 0 | 7.511E-07 | 1 | 1.73508E+03 |
| 4.059E-01 | 56 | 0 | 7.258E-07 | 1 | 1.73712E+03 |
| 4.158E-01 | 57 | 0 | 7.015E-07 | 1 | 1.73909E+03 |
| 4.257E-01 | 58 | 0 | 6.780E-07 | 1 | 1.74100E+03 |
| 4.356E-01 | 59 | 0 | 6.553E-07 | 1 | 1.74284E+03 |
| 4.455E-01 | 60 | 0 | 6.333E-07 | 1 | 1.74463E+03 |
| 4.554E-01 | 61 | 0 | 6.122E-07 | 1 | 1.74636E+03 |
| 4.653E-01 | 62 | 0 | 5.917E-07 | 1 | 1.74803E+03 |
| 4.752E-01 | 63 | 0 | 5.720E-07 | 1 | 1.74965E+03 |
| 4.851E-01 | 64 | 0 | 5.529E-07 | 1 | 1.75121E+03 |
| 4.950E-01 | 65 | 0 | 5.345E-07 | 1 | 1.75273E+03 |
| 5.049E-01 | 66 | 0 | 5.168E-07 | 1 | 1.75419E+03 |
| 5.148E-01 | 67 | 0 | 4.996E-07 | 1 | 1.75561E+03 |
| 5.247E-01 | 68 | 0 | 4.830E-07 | 1 | 1.75698E+03 |
| 5.346E-01 | 69 | 0 | 4.670E-07 | 1 | 1.75831E+03 |
| 5.445E-01 | 70 | 0 | 4.515E-07 | 1 | 1.75959E+03 |
| 5.544E-01 | 71 | 0 | 4.365E-07 | 1 | 1.76083E+03 |
| 5.643E-01 | 72 | 0 | 4.221E-07 | 1 | 1.76204E+03 |
| 5.742E-01 | 73 | 0 | 4.081E-07 | 1 | 1.76320E+03 |
| 5.841E-01 | 74 | 0 | 3.946E-07 | 1 | 1.76432E+03 |
| 5.940E-01 | 75 | 0 | 3.816E-07 | 1 | 1.76541E+03 |
| 6.039E-01 | 76 | 0 | 3.690E-07 | 1 | 1.76647E+03 |
| 6.138E-01 | 77 | 0 | 3.568E-07 | 1 | 1.76749E+03 |
| 6.237E-01 | 78 | 0 | 3.450E-07 | 1 | 1.76847E+03 |
| 6.336E-01 | 79 | 0 | 3.337E-07 | 1 | 1.76943E+03 |
| 6.435E-01 | 80 | 0 | 3.227E-07 | 1 | 1.77035E+03 |
| 6.534E-01 | 81 | 0 | 3.120E-07 | 1 | 1.77124E+03 |
| 6.633E-01 | 82 | 0 | 3.018E-07 | 1 | 1.77211E+03 |
| 6.732E-01 | 83 | 0 | 2.918E-07 | 1 | 1.77295E+03 |
| 6.831E-01 | 84 | 0 | 2.823E-07 | 1 | 1.77375E+03 |
| 6.930E-01 | 85 | 0 | 2.730E-07 | 1 | 1.77454E+03 |
| 7.029E-01 | 86 | 0 | 2.640E-07 | 1 | 1.77530E+03 |
| 7.128E-01 | 87 | 0 | 2.553E-07 | 1 | 1.77603E+03 |
| 7.227E-01 | 88 | 0 | 2.470E-07 | 1 | 1.77674E+03 |
| 7.326E-01 | 89 | 0 | 2.389E-07 | 1 | 1.77743E+03 |
| 7.425E-01 | 90 | 0 | 2.310E-07 | 1 | 1.77809E+03 |
| 7.524E-01 | 91 | 0 | 2.234E-07 | 1 | 1.77873E+03 |
| 7.623E-01 | 92 | 0 | 2.161E-07 | 1 | 1.77935E+03 |
| 7.722E-01 | 93 | 0 | 2.090E-07 | 1 | 1.77996E+03 |
| 7.821E-01 | 94 | 0 | 2.022E-07 | 1 | 1.78054E+03 |
| 7.920E-01 | 95 | 0 | 1.956E-07 | 1 | 1.78110E+03 |
| 8.019E-01 | 96 | 0 | 1.892E-07 | 1 | 1.78165E+03 |
| 8.118E-01 | 97 | 0 | 1.830E-07 | 1 | 1.78217E+03 |
| 8.217E-01 | 98 | 0 | 1.770E-07 | 1 | 1.78268E+03 |
| 8.316E-01 | 99 | 0 | 1.712E-07 | 1 | 1.78318E+03 |
| 8.415E-01 | 100 | 0 | 1.656E-07 | 1 | 1.78365E+03 |
| 8.514E-01 | 101 | 0 | 1.602E-07 | 1 | 1.78412E+03 |
| 8.613E-01 | 102 | 0 | 1.550E-07 | 1 | 1.78456E+03 |
| 8.712E-01 | 103 | 0 | 1.499E-07 | 1 | 1.78500E+03 |
| 8.811E-01 | 104 | 0 | 1.450E-07 | 1 | 1.78542E+03 |
| 8.910E-01 | 105 | 0 | 1.403E-07 | 1 | 1.78582E+03 |
| 9.009E-01 | 106 | 0 | 1.357E-07 | 1 | 1.78621E+03 |

TIME LIMIT
ENDED THIS RUN

Program 3 results/continuation/.

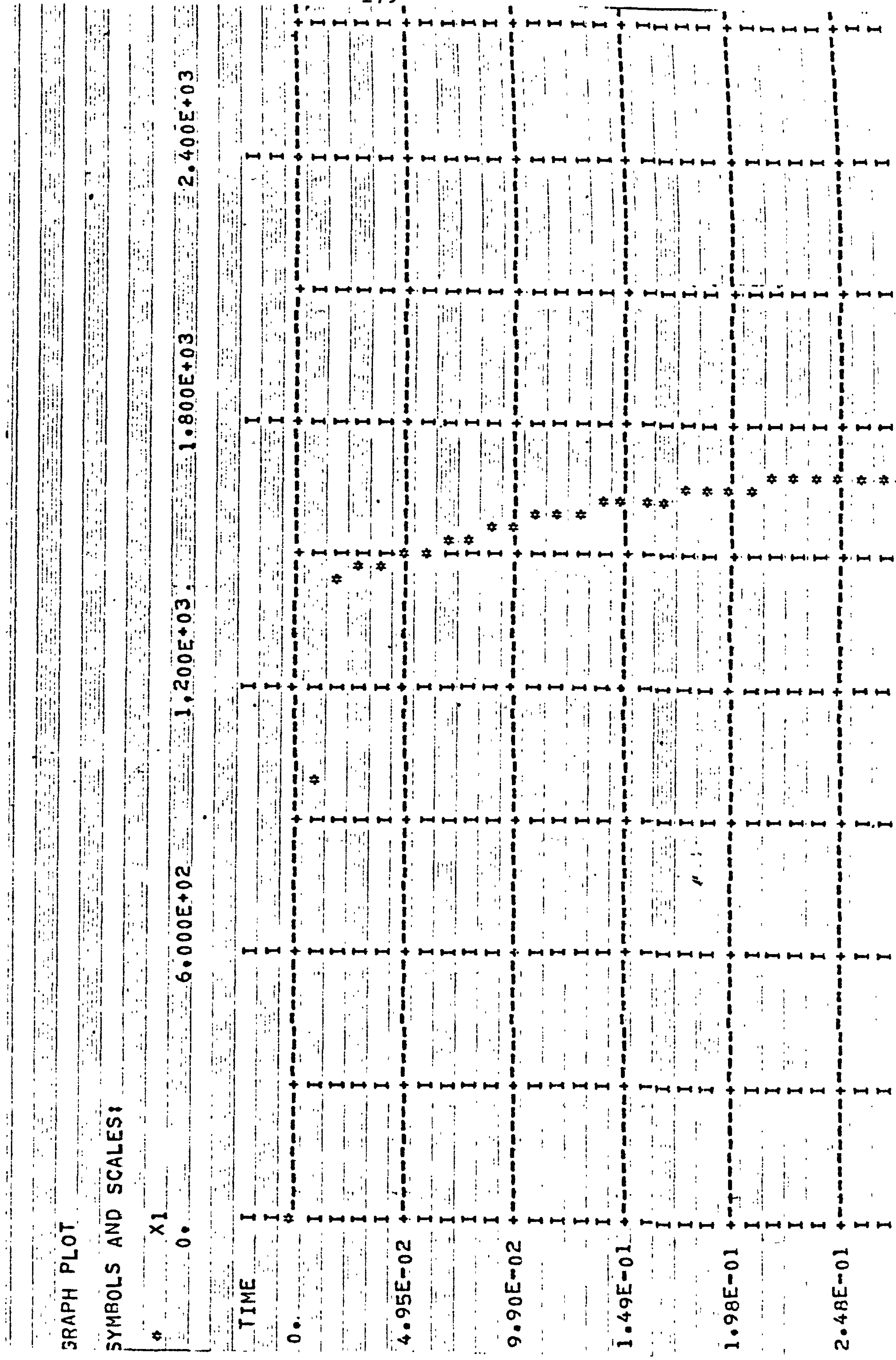



Fig 6a. Plotted results for Program3 example,Ref.
DELAY=0.0016666
DELAYV=0.0008333

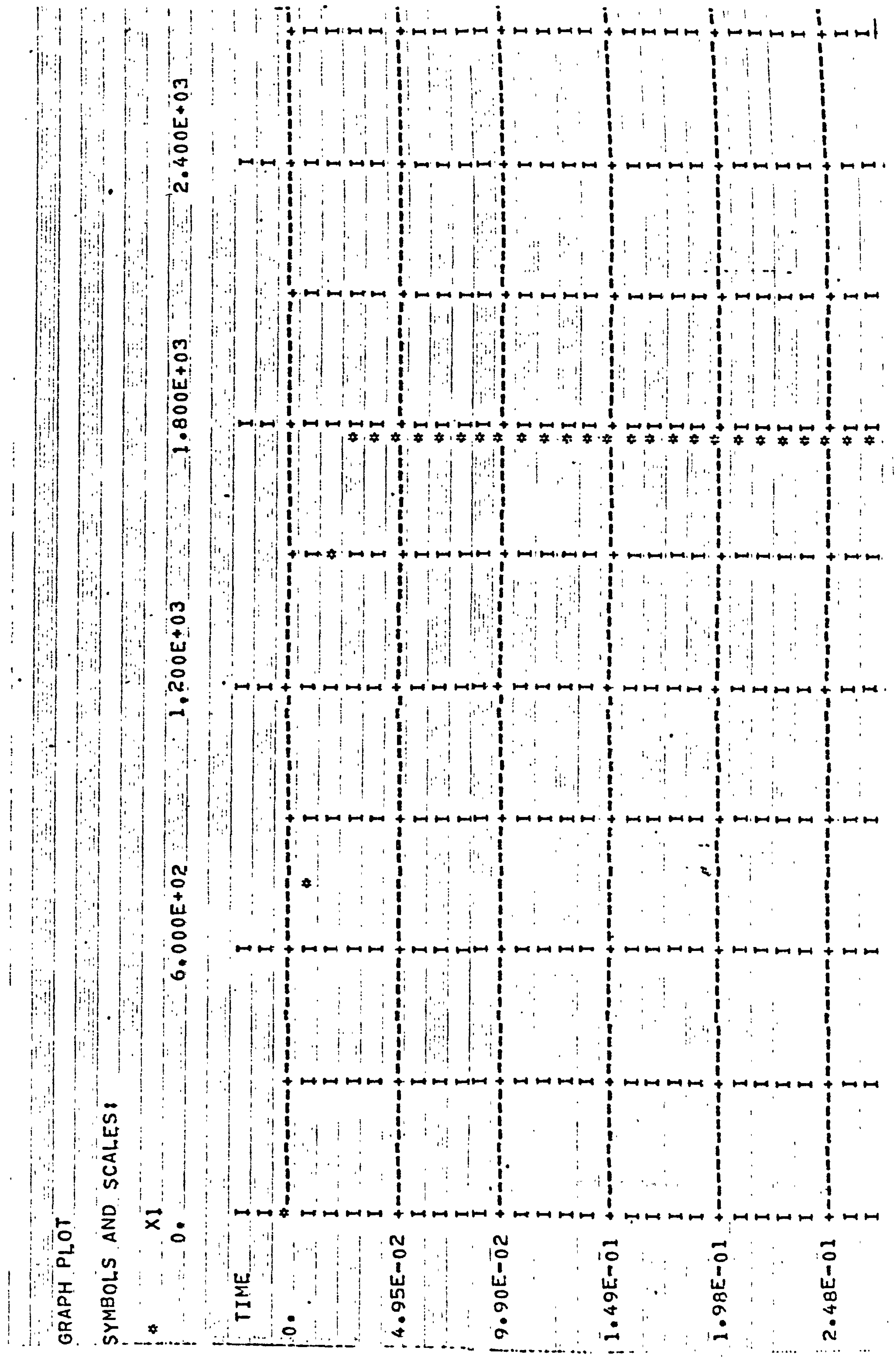



Fig. 6b. Plotted results for Program3 example,Ref.
DELAY=0.0055
DELAYV=0.001

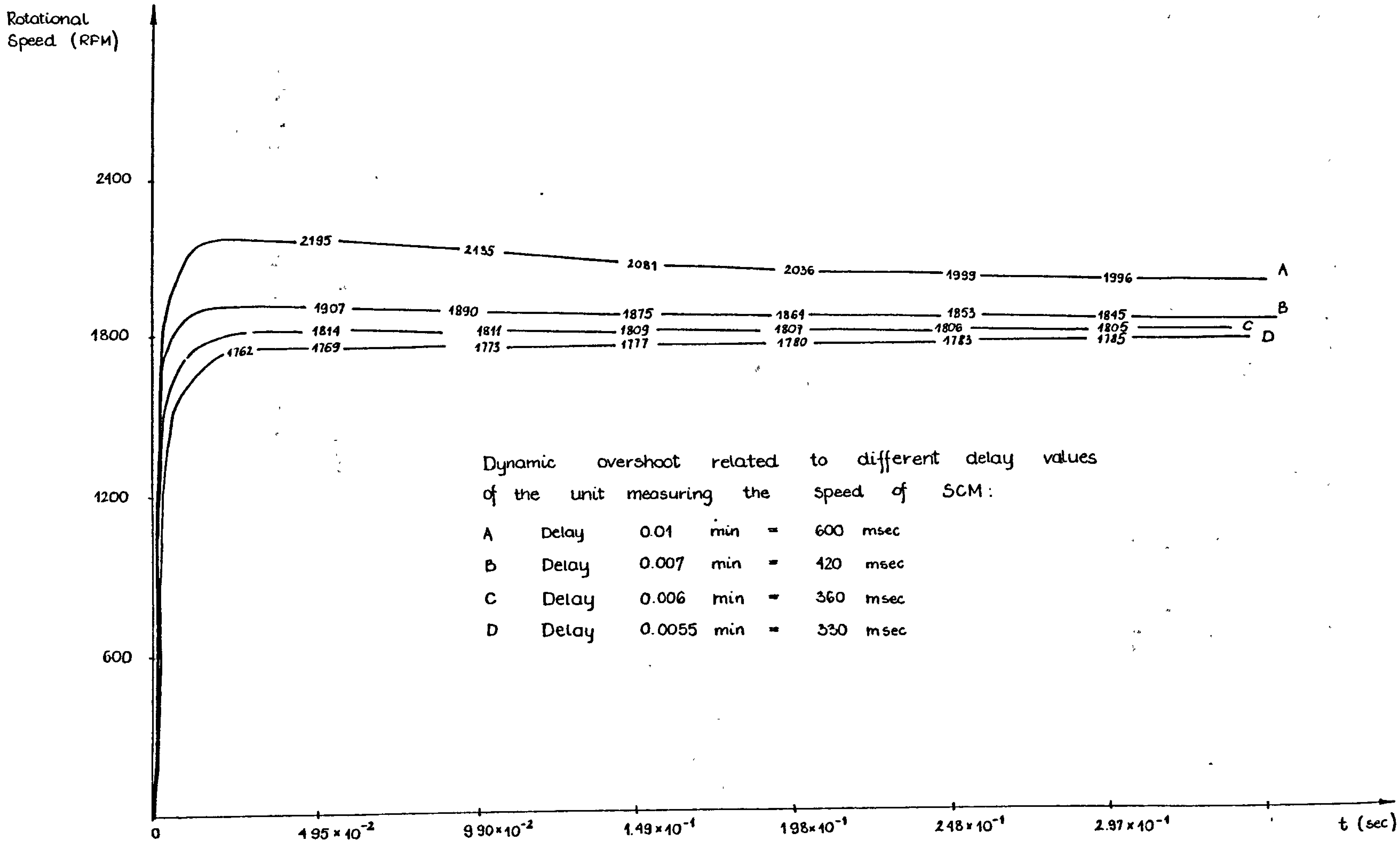



Fig. 7. Dynamic overshoot related to different delay values of the unit measuring the speed of SCM.

An investigation into the influence of an introduced gain block can lead to conclusions confirming the facts known from practical experiments (lit. 7). Let us consider again, a single feedback control of the basic engine, (see Fig. 4) as analysed earlier (section 4.1. Chapter 4) and a simplified block diagram (see Fig. 8) is presented.

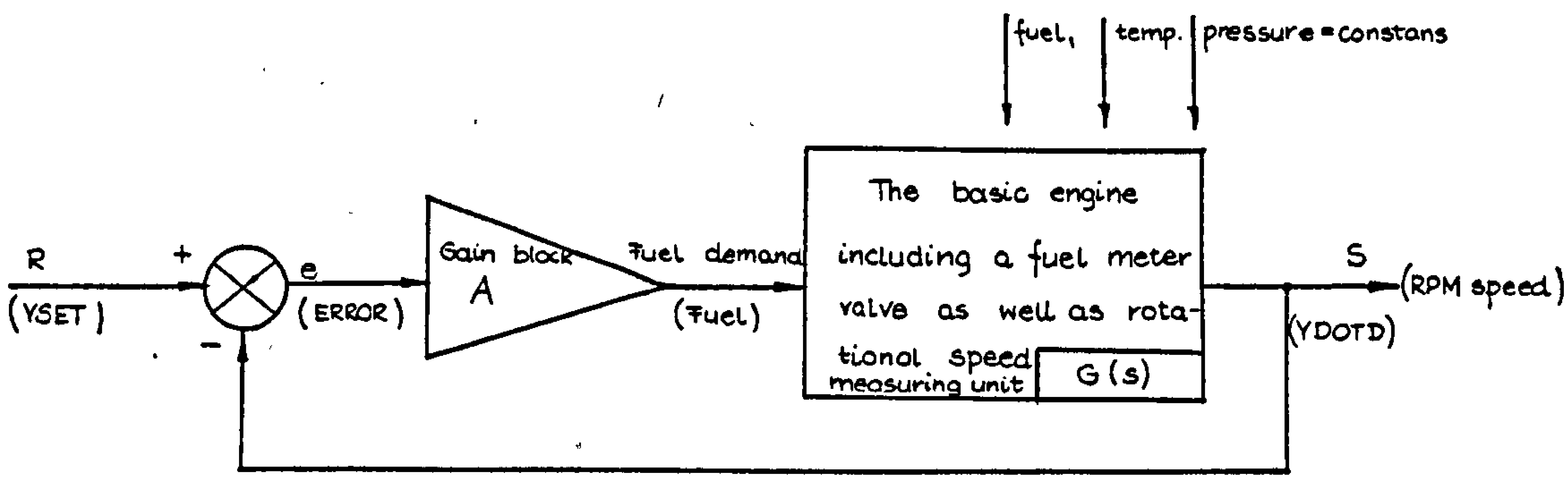



Fig. 8. Simplified Single Feedback Control of the Basic Engine

The symbols in brackets correspond to the Fig. 4 nomenclature as previously described.

And again $e = R(s) - S(s)$

$S(s) = e.A.G(s) = (R(s)-S(s)).A.G(s)$

$S(s) + S(s).A.G(s) = R(s) .A.G(s)$

Where : G(s) was derived earlier

$$G(s) = \frac{a.b.c.}{(s+a)(s+b)(s+Bc)}$$

By rearranging these expressions we may obtain the complete transfer function of the system:

$$T.F. = \frac{S(s)}{R(s)} = \frac{A.a.b.c}{(s+Bc)(s+a)(s+b)+A\ abc}$$

and its'characteristic equation'becomes

$$s^3 + (Bc+a+b)s^2 + (ab+Bac+Bbc)s + abcB + A\ abc$$

However taking into account the following:

$a = 20$ ) as explained in section 4.1.
$b = c = 10$ ) Chapter 4

$B \to B_1 = 0.151$
$B \to B_2 = 0.461$
$B \to B_3 = 0.085$

<u>Region No. 1</u>

For the engine run @ $800^{\circ}C$ of the cylinder head temperature Ref. 9.66 bar and with $1420 > \text{RPM Speed} > 1200$ $(B=B_1=0.151)$ its'characteristic equation' is now modified to:

$$S^3+(0.151x10+20+10)S^2 + (200+0.151x200+0.151x100)S+2000x0.151+Ax2000 =$$

$$\underline{\underline{S^3 + 31.51\ S^2 + 245.3\ S + (302 + 2000\ A)}}$$

Region No. 2

For $B = B_2 = 0.461$ and also : 1685 > RPM Speed > 1420

$$S^3 + (0.461x10+20+10)S^2 + (200+0.461x200+0.461x100)S + (2000x0.461+2000A)$$

$$= \underline{\underline{S^3 + 34.61S^2 + 338.3\ S + (922 + 2000A)}}$$

Region No. 3

For $B = B_3 = 0.085$

$$S^3 + (0.085x10+20+10)S^2 + (200+0.085x200+0.085x100)S + (2000x0.085+2000A)$$

$$= \underline{\underline{S^3 + 30.85S^2 + 225.5\ S + (170 + 2000A)}}$$

Applying Routh Criteria of stability :

For "characteristic function" at Reg. 1

| | |
|---|---|
| 1.0 | 245.3 |
| 31.51 | 302 +2000A |

Solving $\dfrac{(302 + 2000A) - 31.51x245.3}{31.51}$

$$302 + 2000A - 7729.403 = 0$$

$$2000A = 7427.403$$

$$\underline{\underline{A = 3.7137}}$$

To make the system stable A must be less than 3.7137 (for Reg. 1.). For "characteristic function" at Reg. 2.

| | |
|---|---|
| 1.0 | 338.3 |
| 34.61 | 922 +2000A |

solving $\dfrac{(922+2000A) - 34.61\ x338.3}{34.61}$

$$992 + 2000A - 11708.563 = 0$$

$$2000A = 10786.563$$

$$\underline{\underline{A = 5.3932}}$$

To make the system stable A must be less than 5.3932 (for Reg. 2.)

<u>For "characteristic function" at Reg. 3</u>

$$\begin{vmatrix} 1.0 & 225.5 \\ 30.85 & 170+2000A \end{vmatrix} \quad \text{solving} \quad \frac{(170+2000A)-30.85\times 225.5}{30.85}$$

$$170 + 2000A - 6956.675 = 0$$
$$2000A = 6786.675$$
$$\underline{\underline{A = 3.9333}}$$

To make the system stable A must be less than 3.9333 for Reg.3.

Therefore for system stability the following cards must be incorporated in Program No.3.

```
IF(YDOTD.GE.1685.0)                        G = 3.3   )
IF(YDOTD.GE.1420.0.AND.YDOTD.LT.1685.0)    G = 5.3)
IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1420.0)    G = 3.7)
```

Apart from using the FN3 Systran subprogram for storing data for S.C.E. steady state characteristics, there is always a possibility of computing a "minimax" polynomial fit form into a set of data points, as for example a series of Chebyshev polynomials. There are many subroutines written and available through the computer library facilities and one of the most efficient is EØ2ACF developed by N.A.G. (see Lit.8 and Appendix 1). If a set of data points $(x_i y_i)$ $i = 1 \ldots n$ is given, in the two arrays x,y, and both of dimension N, the EØ2ACF subroutine computes an Mth order polynomial.

$$P_{(x)} = a_1 + a_2 x + a_3 x^2 + \ldots\ldots\ldots + a_{m+1} x^m \qquad \text{such}$$

that $\text{Max}\left|P(x_i) - y_i\right|$ is a minimum.

With exact arithmetic the algorithm should terminate after a finite number of steps. This need not be the case with the computer arithmetic and if the routine starts cycling, the routine stops and REF (final reference deviation) is given a negative value. This is by no means an indication that a catastrophic error has occured and does not preclude useful results being obtained. The set of data points was taken from experimental work described in Chapter 2 (Section 2.3. Tab.1) for selected operations at 800$^{o}$C Ref. operational pressures @ 11.03 bar (RUN no.1), @ 9.66 bar (RUN no.2), @6.90 bar (RUN no.3) and @ 4.14 bar (RUN no.4) and the PROGRAM APPROX (Program No.4) listing as well as its results are enclosed. (see Fig.9). Program APPROX uses a 3rd order Chebyshev approximation fit and the polynomial coefficients corresponding to these four steady-state characteristics (as mentioned earlier) are shown in Tab. 4.

The next simulation attempt is basically similar to the block schematic arrangement shown in Fig.2 (Section 4.1 Chapter 4) as both "XD" Systran routines and most of the schematic diagram are the same. The new feature is the feedback loop which this time contains a steady-state characteristic of the analysed engine in Chebyshev 3rd order polynomial coded form. (see Program No.5). The results are printed out in Tab.5 also a line printer plot is enclosed (see Fig.10). Accuracy of the last described method depends to a large extent on the accuracy as well as operation quality of the Chebyshev polynomial fit facing a given set of data points.

```
28/01/76 UNIVERSITY OF MINNESOTA 6600 FORTRAN COMPILER SCOPE 3.4.1 VER4.5   18/10/76  18.11.47.
         MNF(CB=LFN)
0000008        PROGRAM APPROX(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
              1TAPE27=PLOT)
0041528        DIMENSION X(21),Y(21),A(6),S(21)
0041528        Y(1)=5.20
0042578        Y(2)=5.20
0042608        Y(3)=5.22
0042618        Y(4)=5.25
0042638        Y(5)=5.30
0042648        Y(6)=5.40
0042668        Y(7)=5.45
0042678        Y(8)=5.55
0042718        Y(9)=5.70
0042728        Y(10)=5.80
0042748        Y(11)=5.90
0042758        Y(12)=6.05
0042778        Y(13)=6.20
0043008        Y(14)=6.35
0043028        Y(15)=6.50
0043038        Y(16)=6.60
0043058        Y(17)=6.68
0043068        Y(18)=6.75
0043108        Y(19)=6.80
0043118        Y(20)=6.85
0043138        Y(21)=6.90
0043148        X(1)=1200.
0043168        X(2)=1230.
0043178        X(3)=1260.
0043218        X(4)=1290.
0043228        X(5)=1320.
0043248        X(6)=1350.
0043258        X(7)=1380.
0043278        X(8)=1410.
0043308        X(9)=1440.
0043328        X(10)=1470.
0043338        X(11)=1500.
0043358        X(12)=1530.
0043368        X(13)=1560.
0043408        X(14)=1590.
0043418        X(15)=1620.
0043438        X(16)=1650.
0043448        X(17)=1680.
0043468        X(18)=1710.
0043478        X(19)=1740.
0043518        X(20)=1770.
0043528        X(21)=1800.
0043548        N=21
0043558        M1=4
0043568        CALL E02ACF(X,Y,21,A,M1,REF)
0043618        WRITE(6,200)
0043648    200 FORMAT(1H ,25H POLYNOMIAL COEFFECIENTS//)
0043648        WRITE(6,201) (A(I),I=1,M1)
0043748    201 FORMAT(1H,5X,1PE20.8)
0043748        DO10 I=1,21
0043758        D=A(1)+A(2)*X(I)+A(3)*X(I)*X(I)+A(4)*X(I)*X(I)*X(I)
0044028        S(I)=D
0044038        WRITE(6,202) D
0044118    202 FORMAT (1H , F10.4)
0044118     10 CONTINUE
0044138        CALL CAM35MM
0044158        CALL GRSLIDE
0044178        CALL XAXIS(1800.,1800.)
0044218        CALL YAXIS(4.0,7.5)
0044238        CALL POLY3(X,Y,21)
0044258        CALL DOT
0044278        CALL POLY3(X,S,21)
0044318        CALL DASHCFF
0044338        CALL LXTICK
0044358        CALL LYTICK
0044378        CALL LXVAL
0044418        CALL LYVAL
0044438        CALL GRFRAME
0044458        CALL ENDFILM
0044478        STOP
0044508        END

 POLYNOMIAL COEFFICIENTS

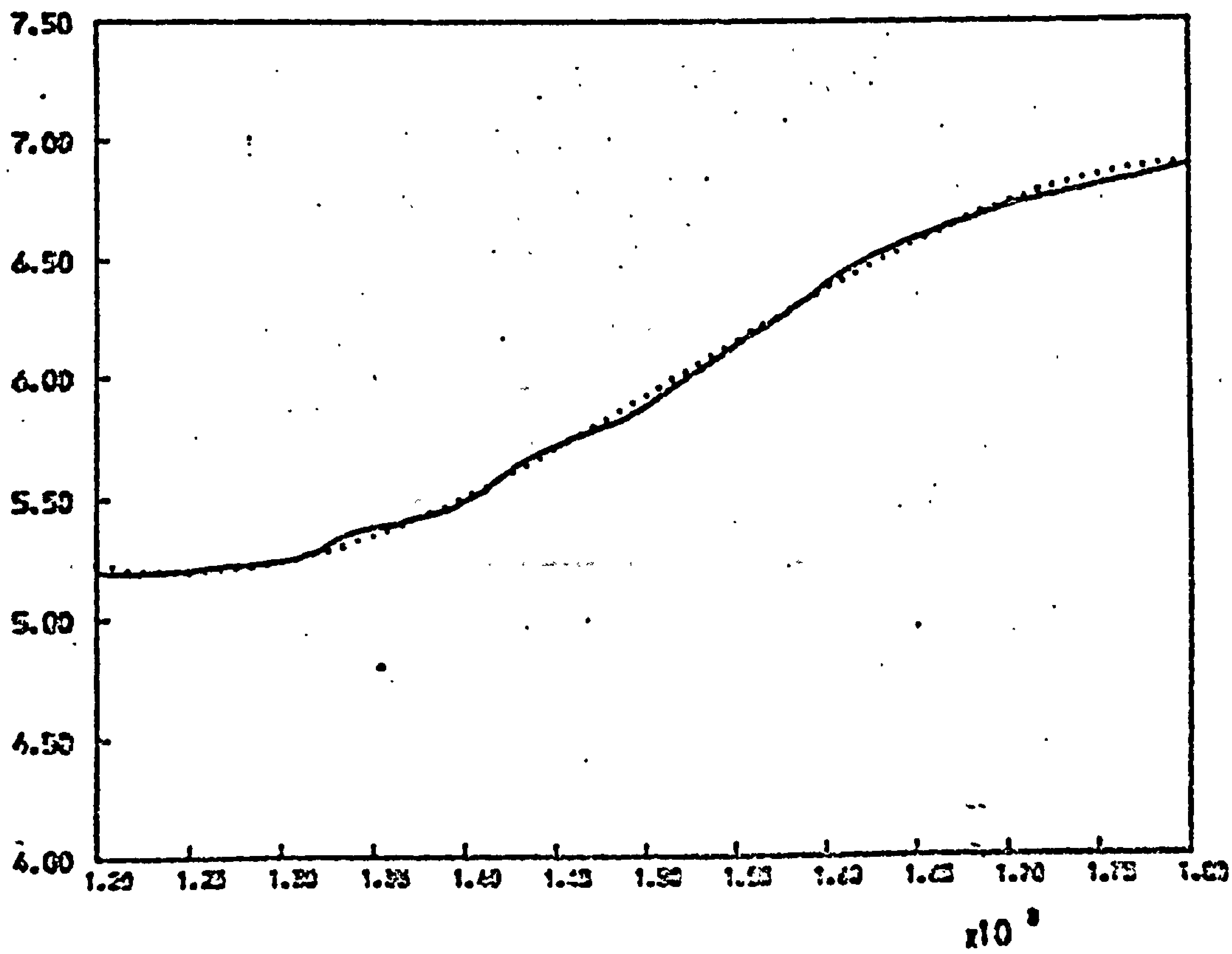
            6.60572404E+01
           -1.26946418E-01
            8.61187007E-05
           -1.88052351E-08
    5.2370
    5.2081
    5.2093
    5.2375
    5.2898
    5.3630
    5.4541
    5.5602
    5.6780
    5.8046
    5.9370
    6.0721
    6.2068
    6.3381
    6.4630
    6.5783
    6.6812
    6.7684
    6.8376
    6.8839
    6.9061
 2406 CHARACTERS OUTPUT
```

Prog.4. Program APPROX.

ᖴRAME 2 UD3E0JJ

!·\HARDC

[*BREAK*]
!·HARDC




[*EOF*]

Fig.9. Plotted results for Program APPROX example.

TAB. 4.

| | | | | |
|---|---|---|---|---|
| A1 | $A_1(1) = 4.50587139E +0.1$ | $A_1(2) = 5.62669026E +01$ | $A_1(3) = 6.60572404E +01$ | $A_1(4) = 4.55904173E +01$ |
| P | $P_4 = 4.14$ | $P_3 = 6.90$ | $P_2 = 9.66$ | $P_1 = 11.03$ |
| A2 | $A_2(1) = -8.28963675E -02$ | $A_2(2) = -1.07338348E-01$ | $A_2(3) = 1.26946418E-01$ | $A_2(4) = 9.17195990E -02$ |
| P | $P_4 = 4.14$ | $P_3 = 6.90$ | $P_2 = 9.66$ | $P_1 = 11.03$ |
| A3 | $A_3(1) = 5.41221510E-05$ | $A_3(2) = 7.22997835E -05$ | $A_3(3) = 8.61187007E -05$ | $A_3(4) = 6.72223615E -05$ |
| P | $P_4 = 4.14$ | $P_3 = 6.90$ | $P_2 = 9.66$ | $P_1 = 11.03$ |
| A4 | $A_4(1) = -1.12773029E-08$ | $A_4(2) = -1.56525573E-08$ | $A_4(3) = -1.88052351E-08$ | $A_4(4) = -1.5589907E -08$ |
| P | $P_4 = 4.14$ | $P_3 = 6.90$ | $P_2 = 9.66$ | $P_1 = 11.03$ |

NS 74/74 OPT=2 FTN 4.2+ REL

```
      SUBROUTINE EQNS
      COMMON/VBLES/YDOT,FUEL,DUM(38),E
      DIMENSION A(4)
      FUEL=6.9
      DELAY=0.00478
      DELAYV=0.001
      A(1)=6.60572404E+01
      A(2)=-1.26946418E-01
      A(3)=8.61187007E-05
      A(4)=-1.88052351E-08
      YDOTD=XD(1,DELAY)
      SP=YDOTD
      S=0.0
      IF(YDOTD.GT.1200.0) S=A(1)+A(2)*SP+A(3)*SP*SP+A(4)*SP*SP*SP
      FUELD=XD(2,DELAYV)
      E=(FUELD-S)*14625.0
      RETURN
      END
```

Prog.5.

```
**LIMITS:,,120
**TABULATE:1
**PLOT:1(0.,3000)
**RUN:0.9,0.0099
   SOLUTION STARTED

SOLUTION OUTPUT
```

| TIME | R-K | N-R | WORST ERR | X | $X_1$ |
|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. |
| 9.900E-03 | 1 | 0 | 2.183E-06 | 1 | 9.99034E+02 |
| 1.980E-02 | 11 | 0 | 9.721E-04 | 1 | 1.75071E+03 |
| 2.970E-02 | 14 | 0 | 3.942E-05 | 1 | 1.79983E+03 |
| 3.960E-02 | 15 | 0 | 1.014E-05 | 1 | 1.79933E+03 |
| 4.950E-02 | 16 | 0 | 1.176E-07 | 1 | 1.79847E+03 |
| 5.940E-02 | 17 | 0 | 1.580E-08 | 1 | 1.79765E+03 |
| 6.930E-02 | 18 | 0 | 1.792E-08 | 1 | 1.79687E+03 |
| 7.920E-02 | 19 | 0 | 1.927E-08 | 1 | 1.79614E+03 |
| 8.910E-02 | 20 | 0 | 2.035E-08 | 1 | 1.79545E+03 |
| 9.900E-02 | 21 | 0 | 2.116E-08 | 1 | 1.79480E+03 |
| 1.089E-01 | 22 | 0 | 2.171E-08 | 1 | 1.79419E+03 |
| 1.188E-01 | 23 | 0 | 2.199E-08 | 1 | 1.79362E+03 |
| 1.287E-01 | 24 | 0 | 2.203E-08 | 1 | 1.79309E+03 |
| 1.386E-01 | 25 | 0 | 2.186E-08 | 1 | 1.79260E+03 |
| 1.485E-01 | 26 | 0 | 2.148E-08 | 1 | 1.79215E+03 |
| 1.584E-01 | 27 | 0 | 2.095E-08 | 1 | 1.79173E+03 |
| 1.683E-01 | 28 | 0 | 2.028E-08 | 1 | 1.79134E+03 |
| 1.782E-01 | 29 | 0 | 1.949E-08 | 1 | 1.79099E+03 |
| 1.881E-01 | 30 | 0 | 1.863E-08 | 1 | 1.79066E+03 |
| 1.980E-01 | 31 | 0 | 1.771E-08 | 1 | 1.79036E+03 |
| 2.079E-01 | 32 | 0 | 1.675E-08 | 1 | 1.79009E+03 |

Tab.5. Program 5 results.


Fig.10. Plotted results for Program 5 example.

The two usual ways to improve the accuracy of this approximation ( and consequently minimise the fit error) were tried i.e. more data points were generated and larger orders of Chebyshev polynomiel power were requested. The first method works according to expectations, but the second was not completely successful as the higher order of power created other computational problems especially when associated with larger simulated objects.

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## 4.5. SUMMARY

This chapter is divided into four basic sections. The first section deals with the review of Computer Simulation methods. The second discusses Digital Analog Simulation techniques, followed by a description of the SYSTRAN program - a digital-analog simulation program of non-linear dynamic systems. Systran routines use standard Fortran language and the Systran program has a set of Fortran routines, plus rules for data input which enable a user to solve any desired set of ordinary differential equations with a minimum of programming effort. Chapter 4 is completed by a section presenting some basic solutions for a simplified Stirling engine simulation model. The experimental data was chosen from Walker's experiment described previously. All the example's modelling refinements presented are performed in a time domain. Probably the most important reason for this, is that the time domain performance of most systems usually is of the ultimate concern. One way of investigating time domain performance is through analytical study of system equations. This is however not discussed in this chapter, as Part 2 of this thesis enlarges on this in detail. Since computer simulation is as much an art as it is a science, the emphasis of the last section of this chapter will not be upon the many techniques for simulating the various features of the Stirling engine but upon the fundamental aspects of Systran simulation. The scope of the following discussion will be broadened to include an advanced SYSTRAN Stirling Engine model conception which will be presented in Part 2 of this thesis.

# 5. ADVANCED SYSTRAN STIRLING ENGINE MODEL CONCEPTION

## 5.1. A THEORETICAL DYNAMIC MODEL BASED ON WALKER'S DATA USING SYSTRAN

Nearly all the simple simulation examples considered so far have been feedback control devices; a deviation of an output variable from a set point was detected, this error signal was fed into a feedback controller that changed the manipulative variable. The controller did not make use of any information about the engine cylinder head temperature, working medium operational pressure, etc. In order to cover the whole range of changes of parameters describing "Prof. Walker's" engine, a theoretical model was developed using SYSTRAN and shown as a simplified block diagram in Fig.1. Basically proportionally controlled, the modelling program PCS consists of three interpolative subprograms i.e. SUBROUTINE EQNS, SUBROUTINE POWTOR, and SUBROUTINE SLGAIN, which will be described in detail later on in this chapter. Apart from this, the simulation program uses the external computer library SOFTWARE namely EØ1 ACF as well as EØ1 AAF which are NAG (Nottingham Algorithms Group) interpolative packages, (see Appendix No.3 and No.4), written in FORTRAN IV and are available through King's College Computer Library facilities as well as the ULCC Computer Centre. As Program PCS works in close conjunction with a Systran file (VCN= M2955M), two types of Control Cards are used : firstly, the Main Program Control Card Deck and secondly, special SYSTRAN Control Cards. The data files for all three PCS program subprograms are provided seperately and additionally the post-processing data file is generated acting as an input storage medium for program PLT - the graph-plotter routine, working together with a "TEKTRONIX" plotting package, required for plotting any of the two

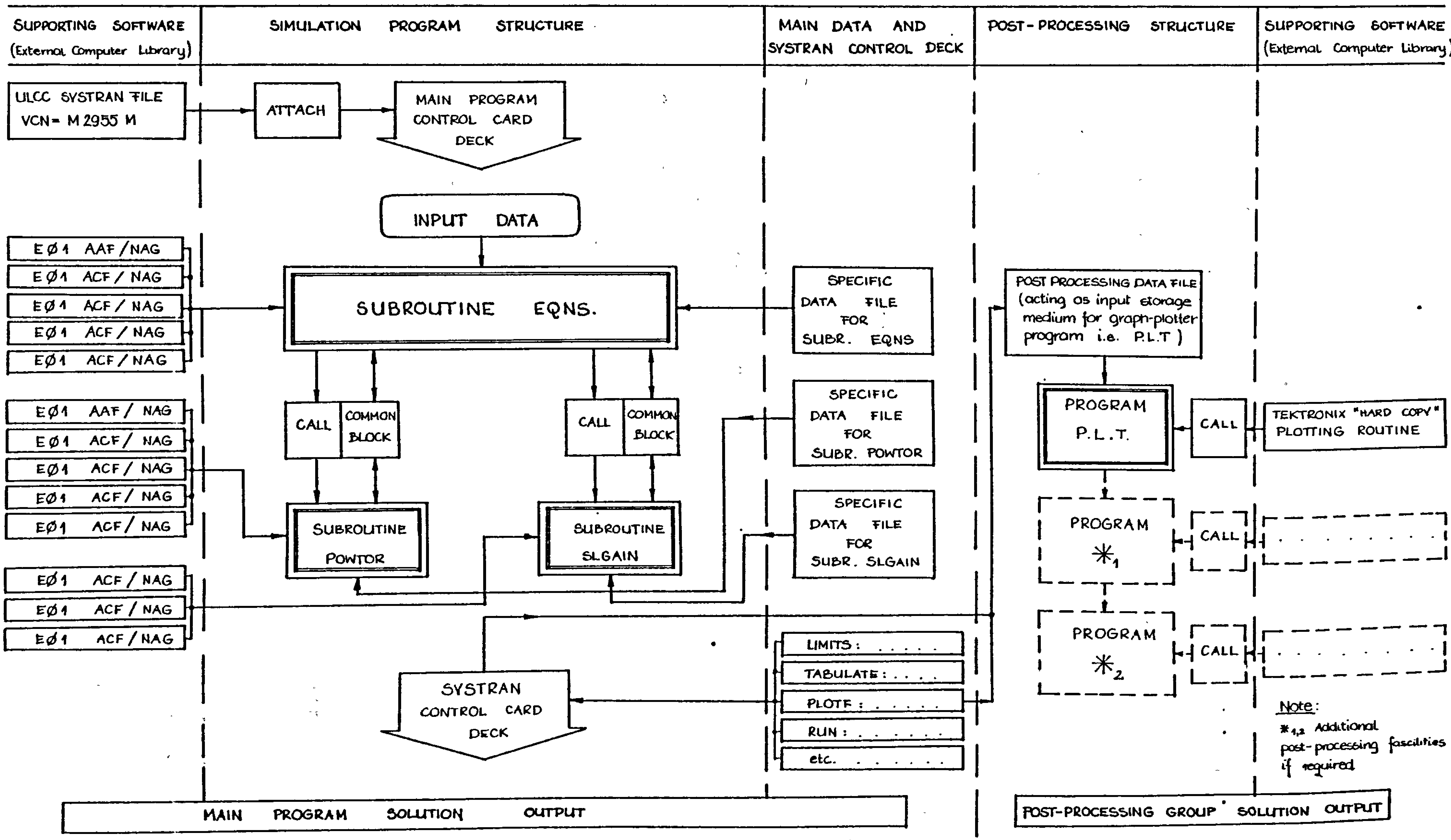


**Fig.1 Simplified functional block diagram of program PCS.**

x variables, tabulated after the main process is completed. The post-processing data file which is specially generated in program PCS can be used with any additional post-processing facility, i.e. additional processing, system optimisation, etc. For simplicity and economy, especially during all editing job runs, the main Input Data Deck is incorporated in the front end of the Subroutine EQNS following only the necessary COMMON/VBLES/, COMMON/CONTROL/,DIMENSION and DATA cards.

A combination of COMMON and CALL statements allow for individual compilation of all the PCS subprograms independently of the main program in which they are used. This set-up "communication" between a main program and subprograms thus simplifies the above mentioned program compilation and allows for substantial flexibility when writing these programs. All three subprogram's specific data file organisation is based on a standard READ/PRINT statement with the usual data set reference number as well as an array storage facility, seperate for each subprogram. Referring back to program No. 3 discussed in section 4.4/Chapter 4, the parameters such as temperature, pressure and specific gain, $G_1$, $G_2$,$G_3$ were basically assumed as constants since the program was not intended to cover the whole range of this particular experiment. Program PCS allows for virtually any selection of operating parameters within a reasonably selected operating area. The statement CALL POWTOR in EQNS is used only to send the appropriate Input Data and allows for POWTOR subprogram execution. Similar order applies to CALL SLGAIN. In addition, subroutine POWTOR receives a complete array of previously calculated (in EQNS) values of rotational speed, tabulated in the same order as transient solution in the main print-out. Both print interval and integration step are discriminated by RUN and LIMITS control cards. One of the main reasons for creating the post-processing data file through a modified PLOT statement (PLOTF was introduced in SYSTRAN MK4A Version) is to plot one of the Stirling "automotive" characteristics i.e. Engine's torque, expressed as a function of rotational speed. In previous Systran versions this would normally have been impossible, as the solutions obtained from the computer for the system concerned were always related to time. The MK 4A version introduces facilities for selective generation during Systran runs of a backing-store file containing complete tabulations of results, intended as SYSTRAN MK 4A Supplement claims (Lit.2) for additional graph-plotting operations. Finally the SUBROUTINE SLGAIN evaluates the three values of gain (within a range of stability conditions) as requested by EQNS for relevant fuel rate calculations, and this is then transferred to EQNS through a COMMON/DFHK/·block. The gain values are also printed-out locally in SLGAIN following the SLGAIN data deck. Fig.2. shows a general form of the program PCS Solution output comprising both Main Program and post-processing group Solution output. Both "outputs" contain tabulated results, graph-plots. overall program diagnostics and a RUN analysis for the Main program execution run. Naturally the Main Program's graph plot is achieved from the line printer following the usual Systran tabulation of results and RUN analysis, whilst post-processing graph-plots are obtained from a teletype "TEKTRONIX" Plotter.

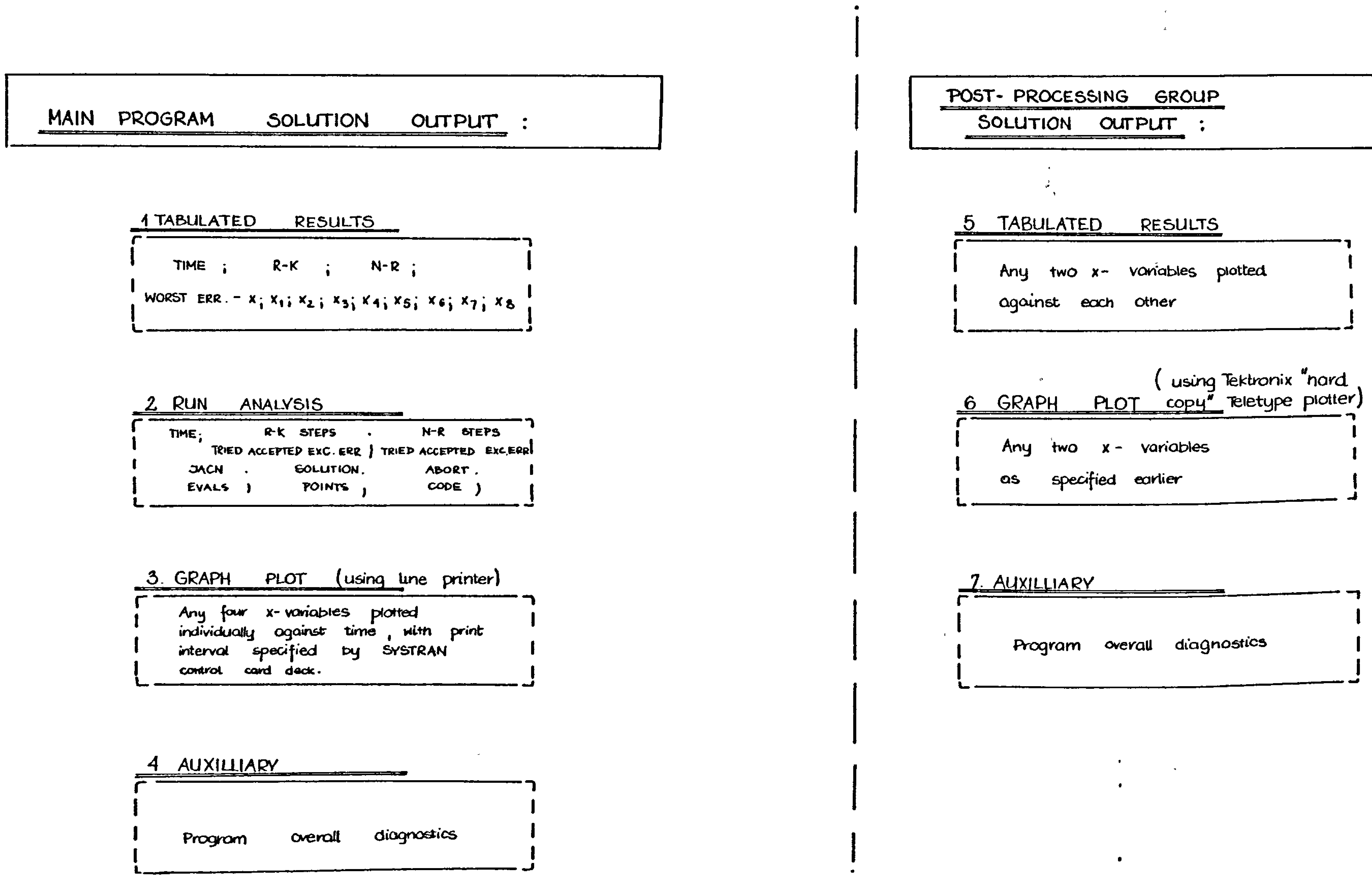


Fig.2 PROGRAM PCS Solution Output.

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## 5.2. SUBROUTINE EQNS

A Subroutine EQNS program listing is presented as Program No.1., comprising approx. 115 lines of FORTRAN source program including comment lines. Using a similar nomenclature as program No.3, discussed in section 4.4/ Chapter 4, EQNS subprogram COMMON/VBLES/-block includes the following engine parameters: rotational speed, fuel, temperature, operating pressure, control loop error signal, power and torque. For program abbreviation details see Tab.1.

| Abbreviation | Meaning |
|---|---|
| YDOT | Speed signal |
| FUEL * | Fuel Rate |
| TEMPER * | Cylinder Head Temperature |
| PRSSUR * | Operational Pressure |
| ERROR | Control Loop Error Signal |
| YDOTD | Measured Rotational Speed |
| POWER | Specific Power Output |
| TORQUE | Engine's Specific Torque |
| FUELD | Measured Fuel |
| E | Fuel Signal Error |
| DFUEL | Equal to Zero Parameter for YDOTD $\leqslant$ 1200 RPM |

* Note: Are adjustable input quantities.

TABLE. 1. COMMON/VBLES/-Block parameters

Additionally part of the adjustable input quantities group (except for those listed in Tab.1.) act as a sort of control disturbance and are presented in TAB.2.

```
      SUBROUTINE EQNS
CVERSION
CVERSION                THIS VERSION   23/02/77
CVERSION
      COMMON /VALES/ YDOT,FUEL,TEMPER,PRSSUR,ERROR,YDOTD,POWER,TORQUE,
     1FUELD,DUM(31),E,DFUEL
      COMMON /CONTRL/TP,TE,TF,ICON
      COMMON/DFHK/ G1,G2,G3
      DIMENSION PRESS(10),FUELS(10),C(25),TEMP(10),SPEEDS(25),CURR(10)
     *  ,FEED31(21,6),FEED32(21,6),FEED33(21,6),FEED34(21,6),DTEMP(10),
     *  WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4)
      DATA N,NT,N2,NP,NS,IG1,IFAIL/3,4,6,6,21,21,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      Y=100.
      V=50.
      Z=14625.0
      DELAY=Y/60000.
      DELAYV=V/60000.
      YDOT=0.
      TEMPER=900.
      YSET=1440.
      PRSSUR=6.90
      CALL SLGAIN (PRSSUR,TEMPER,DELAY,DELAYV,ICON)
      FUEL=5.4
      DFUEL=0.0
C
C
      READ 5001,(TEMP(I),I=1,NT)
      PRINT 6003,(TEMP(I),I=1,NT)
      READ 5001,(PRESS(I),I=1,NP)
      PRINT 6004,(PRESS(I),I=1,NP)
      READ 5001,(SPEEDS(I),I=1,NS)
      PRINT 6005,(SPEEDS(I),I=1,NS)
      DO 1 J=1,NP
1     READ 5001,(FEED31(K,J),K=1,NS)
      DO 5 J=1,NP
5     READ 5001,(FEED32(K,J),K=1,NS)
      DO 6 J=1,NP
6     READ 5001,(FEED33(K,J),K=1,NS)
      DO 7 J=1,NP
7     READ 5001,(FEED34(K,J),K=1,NS)
      PRINT 6006,((FEED31(I,J),I=1,NS),J=1,NP)
      PRINT 6007,((FEED32(I,J),I=1,NS),J=1,NP)
      PRINT 6008,((FEED33(I,J),I=1,NS),J=1,NP)
      PRINT 6009,((FEED34(I,J),I=1,NS),J=1,NP)
      PRINT 6001,PRSSUR,TEMPER,FUEL
C
C
C
  100 CONTINUE
      YDOTD=XD(J,DELAY)
      IF(YDOTD.LT.1200..OR.YDOTD.GT.1800.) GO TO 4
C
```

Progr .1  Subroutine EQNS.

```
C
C
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB1,VAL1(1),VAL2(1),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB2,VAL1(2),VAL2(2),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB3,VAL1(3),VAL2(3),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB4,VAL1(4),VAL2(4),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      DO 2 K=1,NT
      DTEMP(K)=TEMP(K)
      V1=VAL1(K)
      V2=VAL2(K)
      IF(ABS(V1-V2).GT.0.001*ABS(V1)) PRINT 6002,YDOTD,PRSSUR,TEMP(K),
     *   V1,V2
2     CURR(K)=0.5*(V1+V2)
      CALL E01AAF (DTEMP,CURR,C,NT,N2,N,TEMPER)
      FBFUEL=C(N2)
C
C
C
4     CONTINUE
      CALL POWTOR (PRSSUR,TEMPER,YDOT,ICON,POWER,TORQUE)
      FUELD=XD(2,DELAYV)
      ERROR=YSET-YDOTD
      IF(YDOTD.LT.1200.0)  DFUEL=0
      IF(YDOTD.GE.1600.0)  G=G3
      IF(YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0)  G=G2
      IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)  G=G1
      IF(YDOTD.GE.1200.0)  DFUEL=G*ERROR/14625.0
      IF(YDOTD.GT.1800.)  FBFUEL=FUEL+1.
      IF(YDOTD.LT.1200.)  FBFUEL=FUEL-1.
      FBFUEL=AMAX1(FBFUEL,0.0)
      E=(FUELD-FBFUEL)*Z
C     PRINT 555,TP,TE,YDOT,YDOTD,FBFUEL,FUEL
      RETURN
5001  FORMAT(16F5.0)
6001  FORMAT(/32H INITIAL PRESSURE, TEMPERATURE =,F7.3,1H,,F7.1,8X,
     *  11HFUEL STEP =,F7.3//4X,4HTIME,6X,3HR-K,2X,3HN-R,2X,13HWORST
     *RR - X,12X,2HX1,12X,2HX2,12X,2HX3,12X,2HX4/)
6002  FORMAT(35H INTERPOLATION ERROR - PARAMETERS =,3F7.2,5X,
     *    20HINTERPOLATED FUELS =,2F7.3)
6003  FORMAT(/(13H TEMPERATURES,5X,16F7.1))
6004  FORMAT(/(10H PRESSURES,8X,16F7.2))
6005  FORMAT(/(7H SPEEDS,11X,16F7.0))
6006  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAG
     *) AND SPEED (ACROSS) AT T = 600  /(21F6.2))
6007  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAG
     *) AND SPEED (ACROSS) AT T = 700  /(21F6.2))
6008  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAG
     *) AND SPEED (ACROSS) AT T = 800  /(21F6.2))
6009  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAG
     *) AND SPEED (ACROSS) AT T = 900  /(21F6.2))
555   FORMAT(1P6G20.6)
      END
```

**Progr.1** continuation

| Abbreviation | Meaning |
|---|---|
| Y | Tachometric Generator Time Lag |
| V | Fuel Monitoring Valve —"— —"— |
| Z | Time Adjustement |
| DELAY | Program Time Lag |
| DELAYV | Program Time Lag |
| YSET | Set Speed Level |

Table. 2 Adjustable Input Quantities

It is possible at this stage to draw a complete SUBROUTINE EQNS functional circuit, including the practical interrelation with associated POWTOR/SLGAIN subroutines (see Fig.1.).
The whole system can be divided into the following group categories :

1. Adjustable Input Quantities Group
2. Output Group Components
3. BASIC ENGINE SIMULATION CIRCUIT
4. SUBROUTINE EQNS
5. Associated SUBPROGRAMS i.e. POWTOR, SLGAIN
6. SYSTRAN PERIPHERALS i.e. Program XD
7. DATA Files

All these groups are interlinked and each part fulfils a specific role in the system. As the basic philosophy of functioning of this type of control model has already been discussed (Chapter 4), this section will deal mainly with new features. The starting point is the steady-state characteristic block and as explained earlier available data material (see Lit.1.) allows one to draw a family of steady-state characteristics grouped at a constant temperature range of $600^{o}C$ - $800^{o}C$. For the purpose of the PCS program, the pressure range was selected from 4.14 bar to 11.03 bar, Ref. Specific fuel rate (g/min), and the engine speed scale was 1200 - 1800 RPM. A complete set of steady

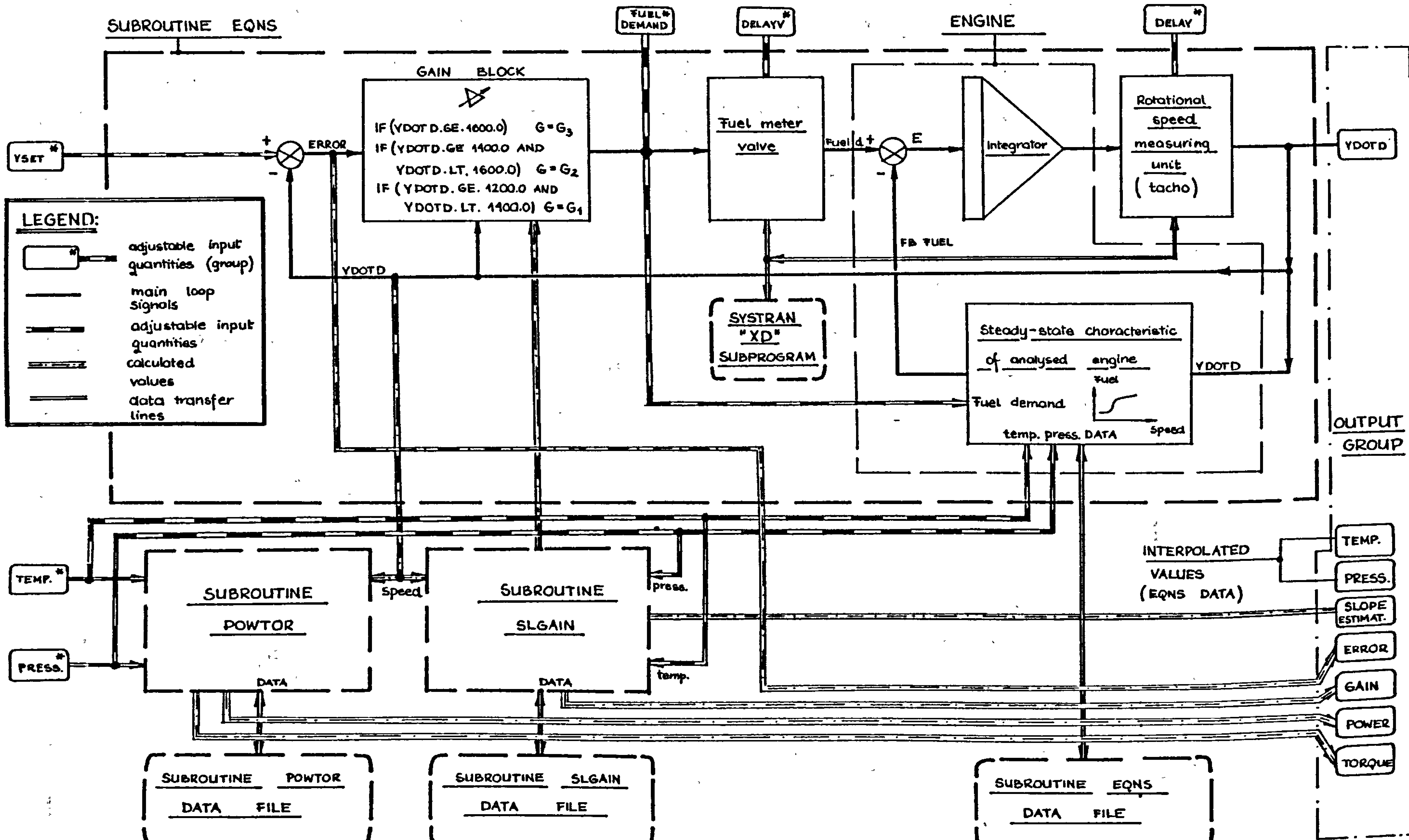


Fig.1 Subroutine EQNS. functional circuit and interrelations with POWTOR/SLGAIN subprograms.

state characteristics used by EQNS is shown in Fig. 2 and a tabulated form of EQNS DATA governed by the relevant EQNS format deck is given in Tab. 3. In Order to choose any particular work conditions for the engine, which correspond to appropriate steady-state curves, engine's head temperature, operational pressure and fuel must be selected and this information is sent to the Steady- State block. The main problem at this stage is lack of enough information as Walker's Data list only refers to particular pressure/ temperature etc. values, and levels between these should somehow be generated in order to achieve a data continuity line. After careful analysis of available data points and various methods, a special system was set-up allowing for a combinative interpolation network, using the external NAG program software. The structure of this interpolation as used in the EQNS subroutine is shown in Fig.4 as applied to one of the possible data interpretation techniques. (Fig.3).

Referring back to section 5.1. where the EØ1 ACF/EØ1AAF NAG packages had been introduced, these offered the user the following features :

| | |
|---|---|
| AØ1ACF : | 2 Dimensional interpolation at a given point of a grid by fitting bi-cubic spline function. |
| AØ1AAF : | 3 Dimensional interpolation of a given point in form of a table of values, from function values evaluated at non-equidistant or equidistant points on a line, using AITKEN's technique known also as SUCCESSIVE Linear interpolation. |

Naturally in situation where selected run parameters are choosen as data points, the above system will give the same

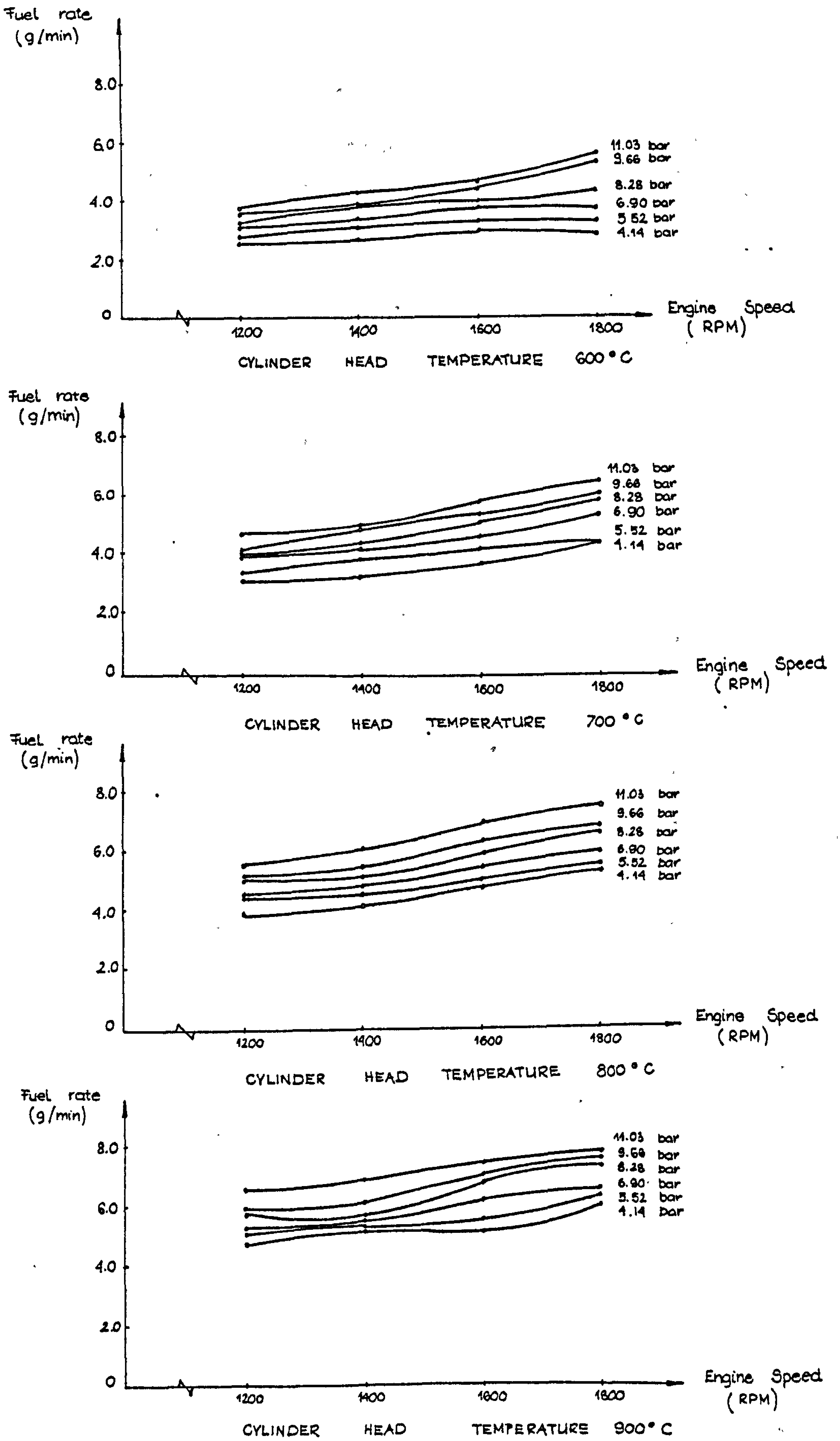


Fig.2 Complete set of steady-state characteristics /based on Walker's data/.

```
TEMPERATURES        600.0   700.0   800.0   900.0

PRESSURES           4.14    5.52    6.90    8.28    9.66    11.03

SPEEDS              1200.   1230.   1260.   1290.   1320.   1350.   1380.
   1410.   1440.   1470.   1500.   1530.   1560.   1590.   1620.   1650.
                                    1680.   1710.   1740.   1770.   1800.
FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE) AND SPEED
 2.50  2.50  2.50  2.50  2.50  2.52  2.55  2.62  2.70  2.79  2.82
 2.70  2.71  2.72  2.77  2.80  2.90  2.98  3.07  3.10  3.15  3.18
 3.00  3.00  3.02  3.05  3.10  3.15  3.23  3.30  3.40  3.50  3.55
 3.10  3.30  3.45  3.55  3.65  3.72  3.78  3.80  3.80  3.80  3.80
 3.40  3.50  3.55  3.60  3.67  3.71  3.79  3.80  3.84  3.90  3.94
 3.70  3.78  3.87  3.96  4.03  4.10  4.17  4.20  4.23  4.24  4.30
       (ACROSS) AT T = 600
       2.87  2.90  2.90  2.90  2.90  2.90  2.89  2.87  2.85  2.80
       3.19  3.20  3.20  3.20  3.20  3.20  3.20  3.20  3.20  3.20
       3.60  3.65  3.69  3.70  3.70  3.70  3.70  3.70  3.70  3.70
       3.80  3.80  3.80  3.85  3.91  4.00  4.08  4.20  4.30  4.30
       4.05  4.15  4.24  4.36  4.51  4.70  4.85  5.02  5.18  5.30
       4.31  4.35  4.39  4.50  4.60  4.80  4.95  5.12  5.30  5.50
FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE) AND SPEED
 3.10  3.10  3.10  3.11  3.12  3.13  3.15  3.20  3.23  3.30  3.35
 3.40  3.42  3.46  3.51  3.60  3.67  3.73  3.87  3.90  3.95  4.00
 3.90  3.95  4.00  4.07  4.10  4.15  4.18  4.20  4.22  4.29  4.32
 4.00  4.02  4.03  4.12  4.20  4.26  4.33  4.40  4.46  4.52  4.6[illegible]
 4.10  4.24  4.36  4.50  4.60  4.70  4.75  4.80  4.86  4.90  5.00
 4.70  4.70  4.71  4.72  4.75  4.80  4.85  4.90  5.00  5.07  5.14
       (ACROSS) AT T = 700
       3.45  3.55  3.66  3.80  3.90  4.02  4.10  4.20  4.25  4.30
       4.01  4.05  4.10  4.15  4.20  4.26  4.28  4.29  4.30  4.30
       4.40  4.48  4.56  4.68  4.80  4.90  5.04  5.15  5.24  5.30
       4.72  4.82  4.96  5.16  5.34  5.50  5.63  5.72  5.79  5.80
       5.10  5.18  5.28  5.40  5.55  5.70  5.80  5.90  5.95  6.00
       5.24  5.35  5.45  5.60  5.72  5.89  6.03  6.16  6.29  6.40
FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE) AND SPEED
 4.00  4.00  4.00  4.01  4.02  4.05  4.10  4.16  4.21  4.30  4.40
 4.60  4.60  4.60  4.60  4.60  4.60  4.60  4.60  4.66  4.70  4.7[illegible]
 4.50  4.50  4.51  4.53  4.58  4.63  4.70  4.80  4.87  4.99  5.0[illegible]
 5.10  5.10  5.10  5.10  5.10  5.10  5.10  5.13  5.21  5.34  5.45
 5.20  5.20  5.22  5.25  5.30  5.40  5.45  5.55  5.70  5.80  5.90
 5.40  5.49  5.57  5.68  5.80  5.90  6.08  6.20  6.38  6.50  6.67
       (ACROSS) AT T = 800
       4.52  4.65  4.78  4.89  5.00  5.10  5.20  5.28  5.35  5.40
       4.88  4.98  5.10  5.21  5.30  5.40  5.47  5.52  5.59  5.6[illegible]
       5.20  5.32  5.47  5.60  5.70  5.80  5.85  5.91  5.98  6.00
       5.60  5.74  5.93  6.10  6.25  6.38  6.50  6.60  6.68  6.70
       6.05  6.20  6.35  6.50  6.60  6.68  6.75  6.80  6.85  6.90
       6.80  6.93  7.01  7.15  7.23  7.30  7.36  7.39  7.40  7.40
FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE) AND SPEED
 4.80  4.90  4.96  5.00  5.05  5.10  5.14  5.19  5.20  5.20  5.2[illegible]
 5.20  5.20  5.20  5.23  5.24  5.25  5.29  5.30  5.33  5.37  5.4[illegible]
 5.40  5.40  5.41  5.41  5.42  5.45  5.49  5.53  5.60  5.70  5.8[illegible]
 5.80  5.75  5.70  5.69  5.68  5.69  5.70  5.73  5.80  5.92  6.12
 6.00  6.00  6.01  6.03  6.05  6.10  6.13  6.27  6.35  6.44  6.55
 6.70  6.70  6.71  6.73  6.80  6.82  6.89  6.92  7.00  7.09  7.15
       (ACROSS) AT T = 900
       5.20  5.20  5.20  5.25  5.32  5.42  5.55  5.70  5.82  6.00
       5.46  5.52  5.60  5.71  5.84  5.96  6.05  6.15  6.20  6.30
       5.90  6.02  6.15  6.25  6.35  6.40  6.44  6.49  6.50  6.5[illegible]
       6.36  6.60  6.77  6.91  7.00  7.11  7.20  7.27  7.29  7.30
       6.70  6.81  6.98  7.12  7.28  7.40  7.50  7.62  7.75  7.8[illegible]
       7.25  7.31  7.40  7.50  7.55  7.60  7.65  7.69  7.70  7.70
```

Tab.3 Numerical data for Subroutine EQNS.

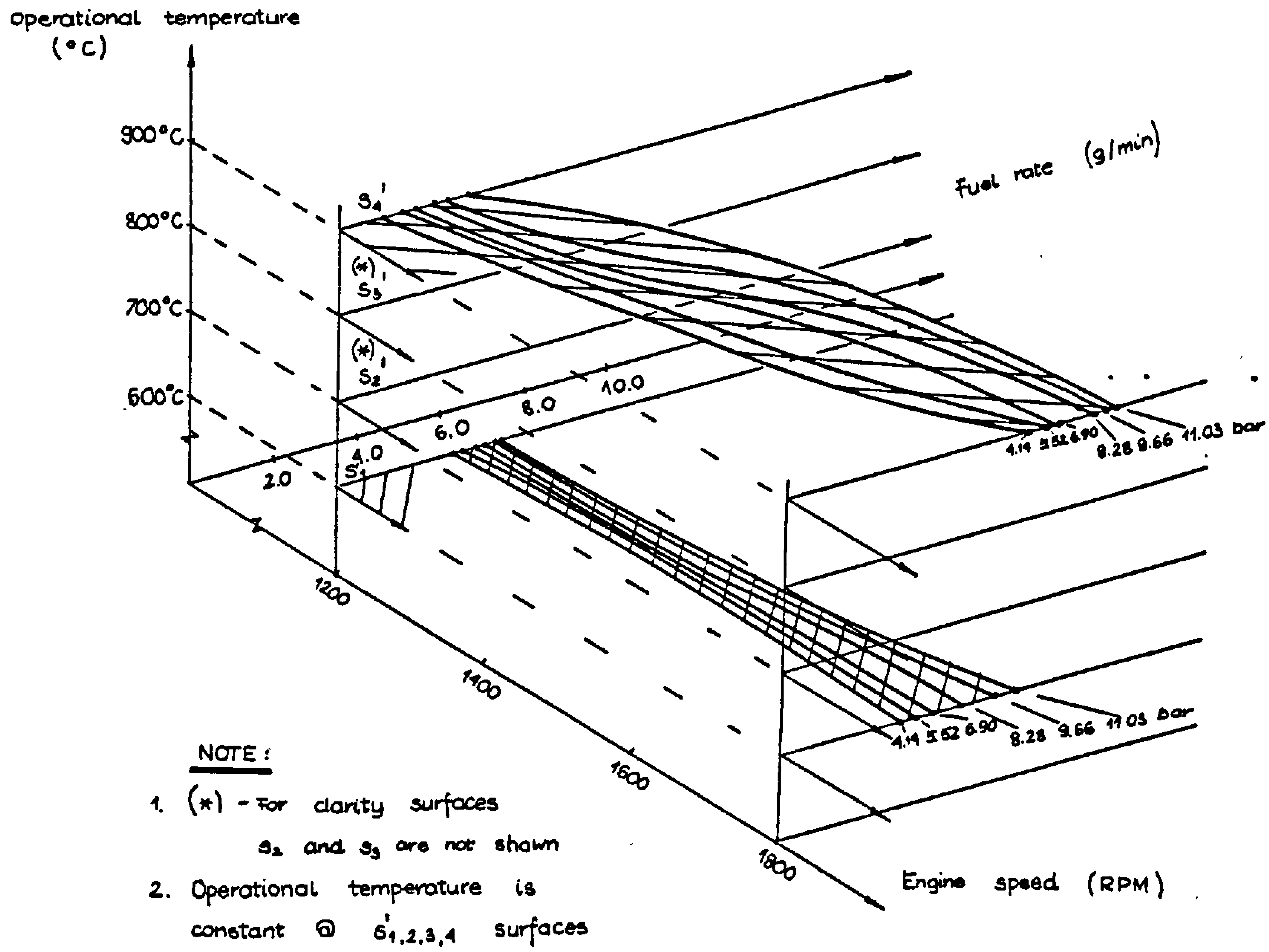


**Fig.3 One of the possible data interpretation techniques as used in EQNS. subroutine/steady-state block/.**

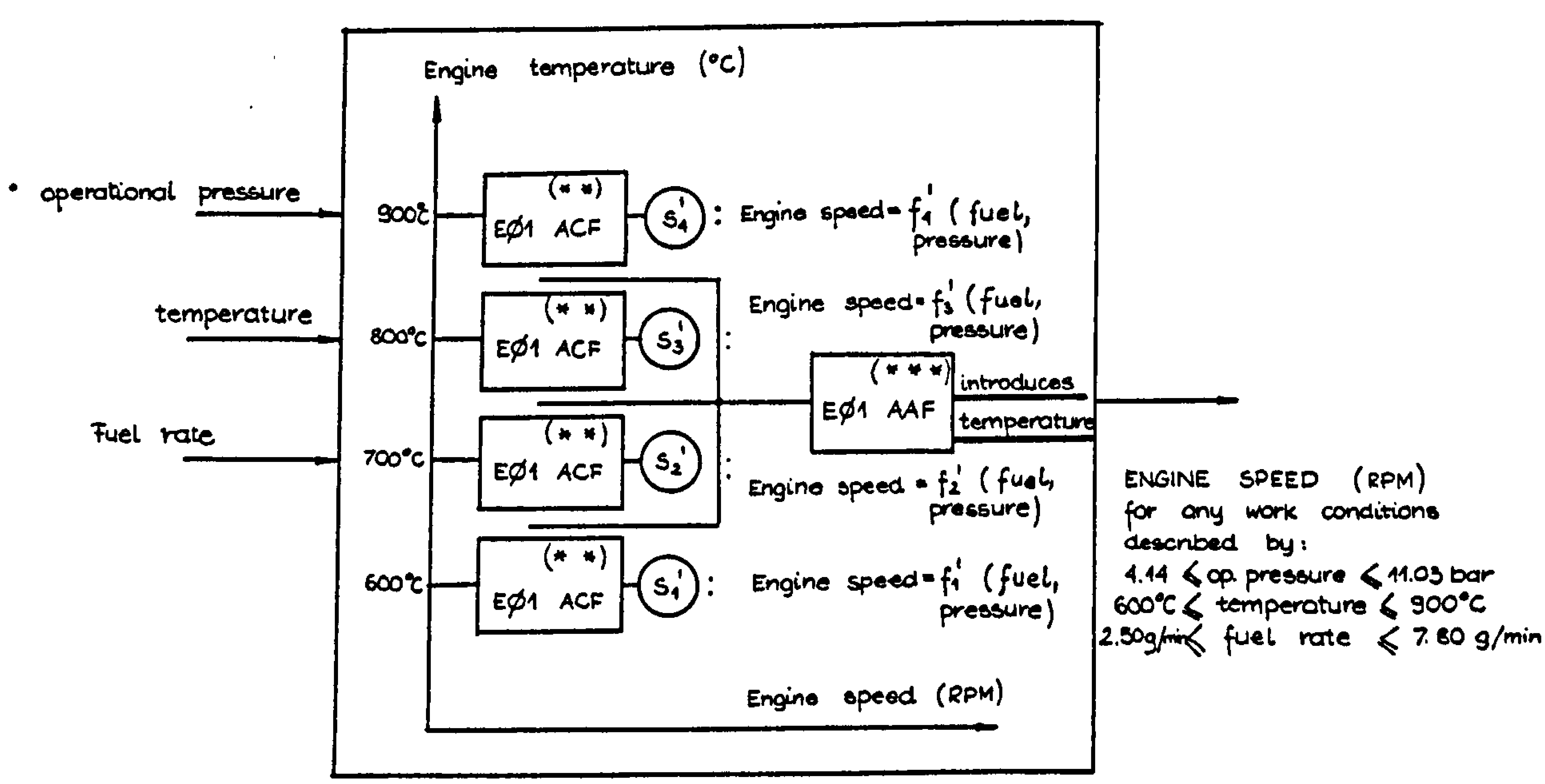


**Fig.4 Steady-State block structure of interpolation ie.4xEØ1ACF& 1xEØ1AAF.**

(* *) - 2-dimensional INTERPOLATION SURFACES ($S'_{4,3,2,1}$) where EØ1 ACF interpolates at a given point on a surface by fitting bi-cubic spline functions

(* * *) - 3- dimensional interpolation at a given point in form at a table of values, from function values evaluated at non-equidistant or equidistant points on a line, using AITKEN'S technique of SUCCESSIVE linear interpolation.

values but in all other situations linear interpolation is used, which, bearing in mind the character of the considered steady-state material, seems quite a sensible date assumption. As indicated earlier, the provided data applied to this interpolation technique set-up, allows practically any reasonable combination of pressure, temperature or fuel. Summing up, EØ1 ACF is called in EQNS four times and operates initially on $S_1', S_2', S_3', S_4'$, surfaces whilst EØ1 AAF is called in one time only for 3 dimensional operations allowing for the introduction of cylinder head temperature to the analysed model. One calculated fuel value (FBFUEL) is then compared with another (FUELD) - (Ref. Fuel meter valve) and this is then fed into the actual engine block. The calculated output of rotational speed being the main output group parameter, (see Fig.1.) apart from its initial role as the main loop feedback component, (proportional control system), provides quantitive information to the above STEADY-STATE block as well as to the POWTOR/SLGAIN SUBROUTINES which shall be dealt with in the following sections. Also SYSTRAN XD subprogram - a transport delay program is used for Fuel meter valve and rotational speed measuring unit simulation, with DELAY/DELAYV statements acting as corresponding time lags, adjusted externally and independently. Details concerning the use of XD subprograms can be found in Lit. 2.

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## 5.3. SUBROUTINE POWTOR

Subroutine POWTOR with approximately 80 lines of FORTRAN source program makes use of the engine's power output data (Lit.1.) - extremely valuable from the point of view of the model's experimental material. The POWTOR listing is presented as Program No.1. and its data set is given in a computer tabulation form in Tab.1. Similarly, as previously discussed in EQNS, this data is plotted in a family of four engine characteristics (Fig.1.) i.e. Engine power (KW) which is expressed as a function of operational pressure (bars). Each of the constant engine's (RPM) speed runs consists of four constant temperature (cylinder head temperature) curves, all very regular and recognised as monothonic functions related to the theoretical engine's approaches. Simultaneously to cover all the operations of specified temperature, engine speed and operational pressure, and interpolate the values "in between" these experimental data points, one must achieve an image of continuity, therefore an additional interpolative set-up was developed named Subroutine POWTOR which calculates, firstly, the values of engine power output and secondly, converts this to another "mechanical" quantity - engine-Torque, by simply using the following conversion formulae:

$$\frac{\text{Engine's O/P Power}}{\text{Engine's O/P Speed}} = \text{Engine's Torque} * K$$

Where: K is a dimensionless factor equal to $2\pi * \frac{1}{60}$ =

= 9.549296586

```
      SUBROUTINE POWTOR (PRSSUR,TEMPER,YDOT,ICON,POWER,TORQUE)
CVERSION
CVERSION              THIS VERSION  17/06/77
CVERSION
      COMMON/ABC/ POWER1(4,6),POWER2(4,6),POWER3(4,6),POWER4(4,6),
      1DTEMP(10),WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4
      COMMON/ABC/ TEMP(10),PRESS(10),SPEEDS(10),CURR(10),C(25)
      DATA N,NT,N2,NP,NS,IG1,IFAIL/3,4,6,6,4,6,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      READ 5001,(TEMP(I),I=1,NT)
      PRINT 6003,(TEMP(I),I=1,NT)
      READ 5001,(PRESS(I),I=1,NP)
      PRINT 6004,(PRESS(I),I=1,NP)
      READ 5001,(SPEEDS(I),I=1,NS)
      PRINT 6005,(SPEEDS(I),I=1,NS)
      DO 1 J=1,NT
1     READ 5002,(POWER1(J,K),K=1,NP)
      DO 5 J=1,NT
5     READ 5002,(POWER2(J,K),K=1,NP)
      DO 6 J=1,NT
6     READ 5002,(POWER3(J,K),K=1,NP)
      DO 7 J=1,NT
7     READ 5002,(POWER4(J,K),K=1,NP)
      PRINT 6006,((POWER1(J,I),I=1,NP),J=1,NT)
      PRINT 6007,((POWER2(J,I),I=1,NP),J=1,NT)
      PRINT 6008,((POWER3(J,I),I=1,NP),J=1,NT)
      PRINT 6009,((POWER4(J,I),I=1,NP),J=1,NT)
      PRINT 6001,PRSSUR,TEMPER
C
C
C
100   CONTINUE
      POWER=0.0
      TORQUE=0.0
      IF(YDOT.LT.1200.) GO TO 4
      YD=YDOT
      IF(YDOT.GT.1800.0) YD=1800.0
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,POWER1,VAL1(1),VAL2(1),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      CALL E01ACF(TEMPER,PRSSUR,TEMP,PRESS,POWER2,VAL1(2),VAL2(2),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,POWER3,VAL1(3),VAL2(3),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,POWER4,VAL1(4),VAL2(4),
     *            IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      DO 2 K=1,NS
      DTEMP(K)=SPEEDS(K)
      V1=VAL1(K)
      V2=VAL2(K)
      IF(ABS(V1-V2).GT.0.001*ABS(V1)) PRINT 6002,YDOT,PRSSUR,TEMP(K),
     *   V1,V2
2     CURR(K)=0.5*(V1+V2)
      CALL E01AAF (DTEMP,CURR,C,NS,N2,N,YD)
```

Progr.1  Subroutine  POWTOR

```
 POWTOR      74/74    OPT=2                                              FTN 4.6+428

      POWER=C(N2)
      TORQUE=9.549296586*POWER/YD
C
C
C
4     CONTINUE
      RETURN
5001  FORMAT(16F5.0)
5002  FORMAT(10F6.0)
6001  FORMAT(/32H INITIAL PRESSURE, TEMPERATURE =,F7.3,1H,,F7.1,8X)
6002  FORMAT(35H INTERPOLATION ERROR - PARAMETERS =,3F7.2,5X)
6003  FORMAT(/(13H TEMPERATURES,5X,16F7.1))
6004  FORMAT(/(10H PRESSURES,8X,16F7.2))
6005  FORMAT(/(7H SPEEDS,11X,16F7.0))
6006  FORMAT(/90H     POWER VALUES AS FUNCTION OF TEMPER    (DOWN THE PAGE
     *) AND PRESSURE (ACROSS) AT S=1200  /(4(6F10.3/)))
6007  FORMAT(/90H     POWER VALUES AS FUNCTION OF TEMPER    (DOWN THE PAGE
     *) AND PRESSURE (ACROSS) AT S=1400  /(4(6F10.3/)))
6008  FORMAT(/90H     POWER VALUES AS FUNCTION OF TEMPER    (DOWN THE PAGE
     *) AND PRESSURE (ACROSS) AT S=1600  /(4(6F10.3/)))
6009  FORMAT(/90H     POWER VALUES AS FUNCTION OF TEMPER    (DOWN THE PAGE
     *) AND PRESSURE (ACROSS) AT S=1800  /(4(6F10.3/)))
      END
```

**Progr.1 continuation.**

TEMPERATURES 600.0 700.0 800.0 900.0

PRESSURES 4.14 5.52 6.90 8.28 9.66 11.03

SPEEDS 1200. 1400. 1600. 1800.

POWER VALUES AS FUNCTION OF TEMPER (DOWN THE PAGE)

AND PRESSURE (ACROSS) AT S=1200

| | | | | | |
|---|---|---|---|---|---|
| 71.600 | 103.000 | 138.200 | 163.300 | 179.700 | 199.000 |
| 125.600 | 174.600 | 216.000 | 253.000 | 231.400 | 231.400 |
| 148.300 | 199.800 | 255.000 | 304.000 | 335.700 | 363.100 |
| 170.900 | 240.000 | 299.000 | 344.300 | 403.300 | 447.300 |

POWER VALUES AS FUNCTION OF TEMPER (DOWN THE PAGE)

AND PRESSURE (ACROSS) AT S=1400

| | | | | | |
|---|---|---|---|---|---|
| 77.700 | 117.300 | 149.[illegible] | 165.500 | 189.100 | 199.300 |
| 115.800 | 167.100 | 215.500 | 247.700 | 278.500 | 293.500 |
| 149.500 | 228.700 | 285.000 | 331.300 | 348.900 | 365.000 |
| 190.600 | 247.700 | 328.300 | 367.900 | 406.000 | 463.200 |

POWER VALUES AS FUNCTION OF TEMPER (DOWN THE PAGE)

AND PRESSURE (ACROSS) AT S=1600

| | | | | | |
|---|---|---|---|---|---|
| 50.300 | 103.900 | 124.000 | 149.100 | 142.400 | 139.700 |
| 95.500 | 160.300 | 132.500 | 251.300 | 254.600 | 279.000 |
| 152.400 | 216.100 | 276.400 | 337.000 | 356.300 | 393.700 |
| 191.000 | 271.400 | 350.100 | 338.600 | 455.700 | 443.900 |

POWER VALUES AS FUNCTION OF TEMPER (DOWN THE PAGE)

AND PRESSURE (ACROSS) AT S=1800

| | | | | | |
|---|---|---|---|---|---|
| 35.800 | 73.500 | 101.[illegible] | 107.400 | 154.300 | 143.200 |
| 92.300 | 148.900 | 292.200 | 222.400 | 235.600 | 248.600 |
| 137.600 | 214.800 | 269.500 | 318.500 | 350.500 | 376.300 |
| 171.500 | 265.700 | 324.200 | 388.200 | 442.900 | 442.900 |

Tab.1 Numerical data for Subroutine POWTOR.

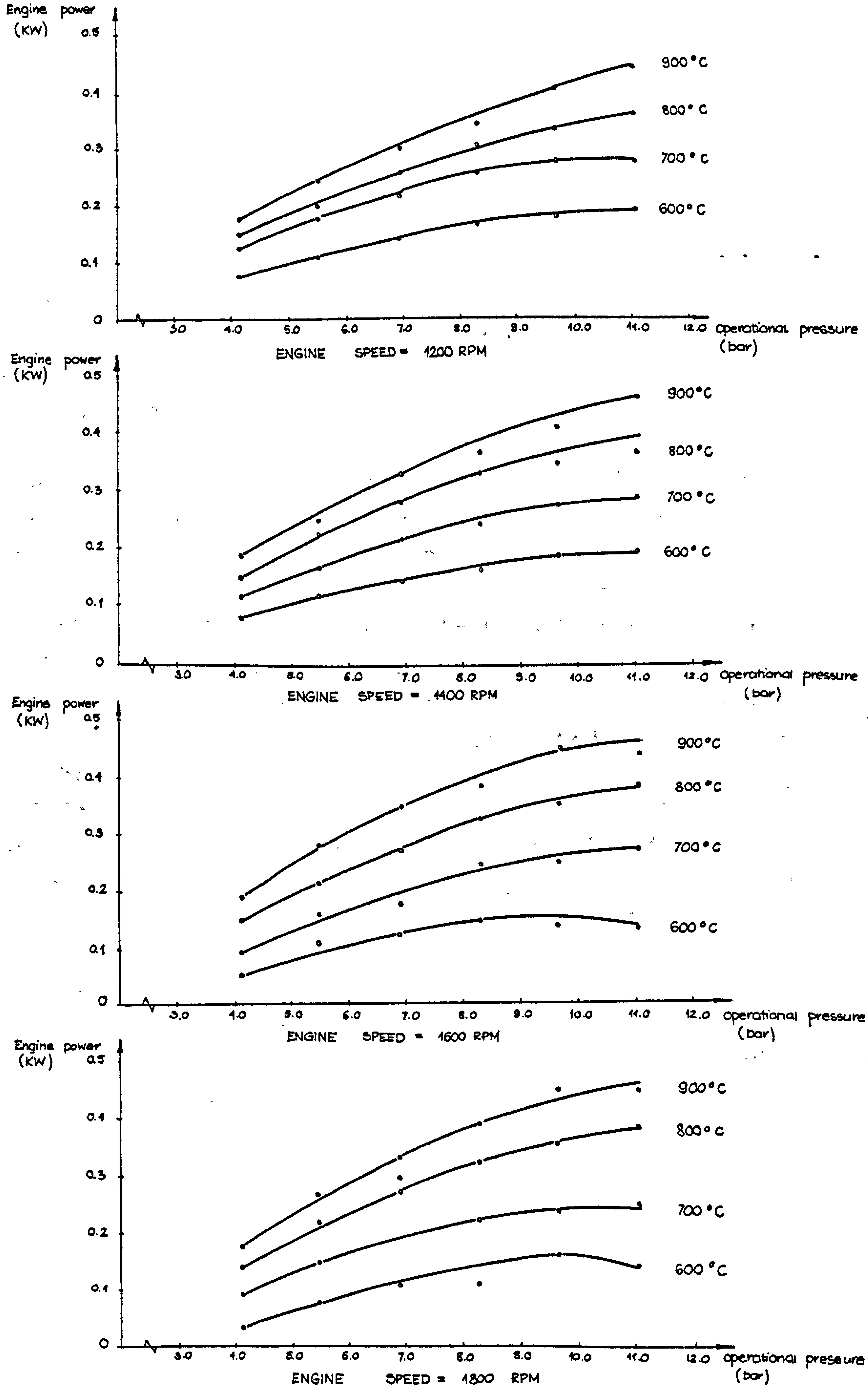



Fig.1 Engine Power vs operational pressure @ constant temperature and speed.

The above is valid only in situations where power is expressed as (Watts), speed as (RPMs) and Torque as (N·m). The interpolative routine POWTOR shown schematically in Fig.3. employs an exact number and type of external SOFTWARES eg. EQNS, and its structure is also explained in Fig.2. On the "input" side - pressure , temperature and speed, where the first two are adjustable input quantities, and where speed is calculated in EQNS and consequently transferred for additional processing in POWTOR.
Referring back to section 5.2. of this chapter, subroutine POWTOR DATA file format organisation is presented as shown in Tab.1. i.e. engine's power values are printed out as a function of cylinder head temperature - listed downwards, whilst the operational pressure is listed across for each of the constant speed runs.
All four main interpolation surfaces $S_1''$, $S_2''$, $S_3''$, $S_4''$, are surfaces formed from previously mentioned characteristics (Fig.1.) and similarly as in EQNS, EØ1 AAF introduces engine's speed (in EQNS - temperature) and EØ1AAF interpolation procedure is based on AITKEN's technique of successive linear interpolation.
Subroutine POWTOR calculations of Power and Torque are tabulated in the main program print-out with the number of points generated, depending on SYSTRAN'S RUN statement setting.

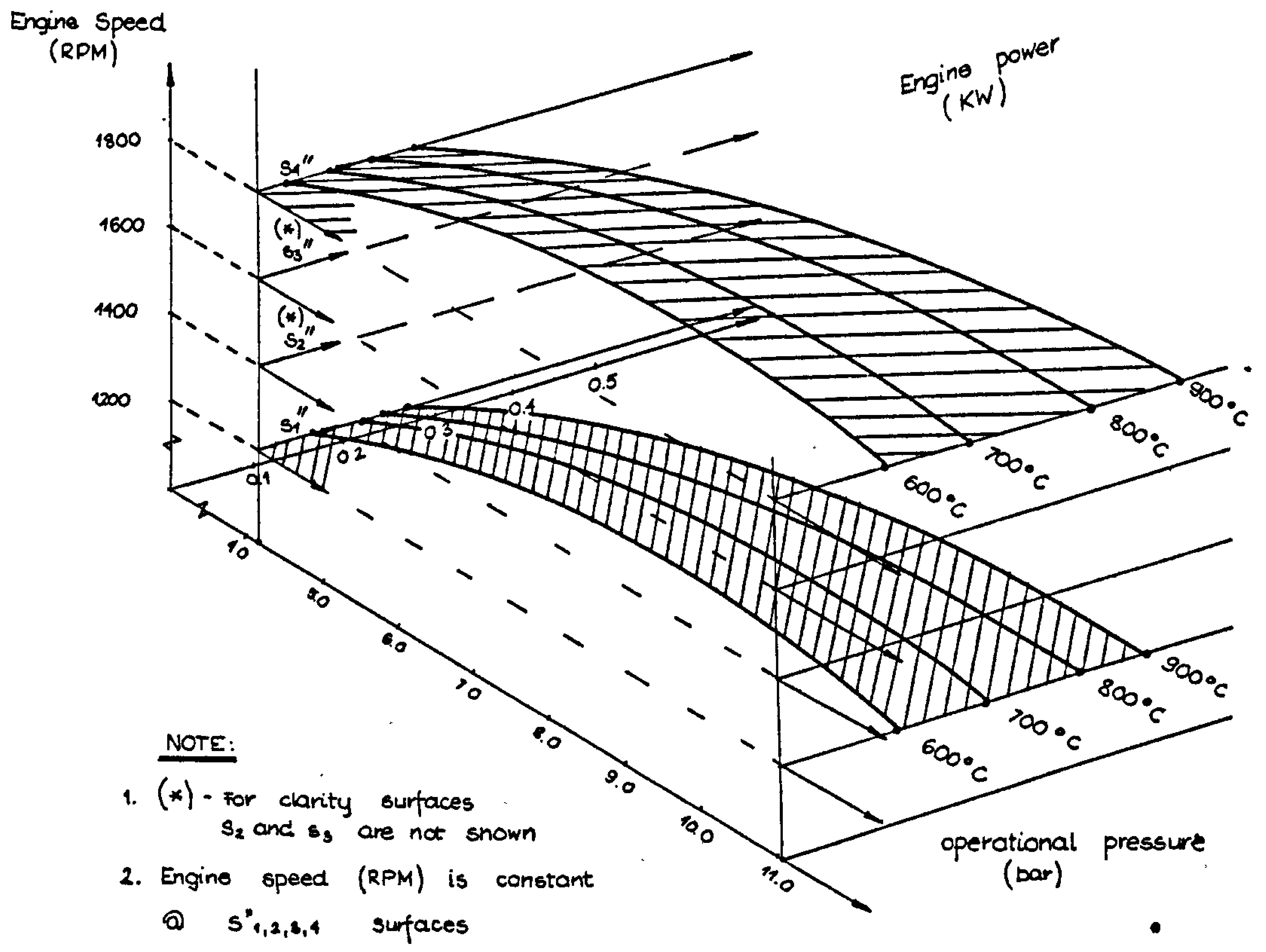
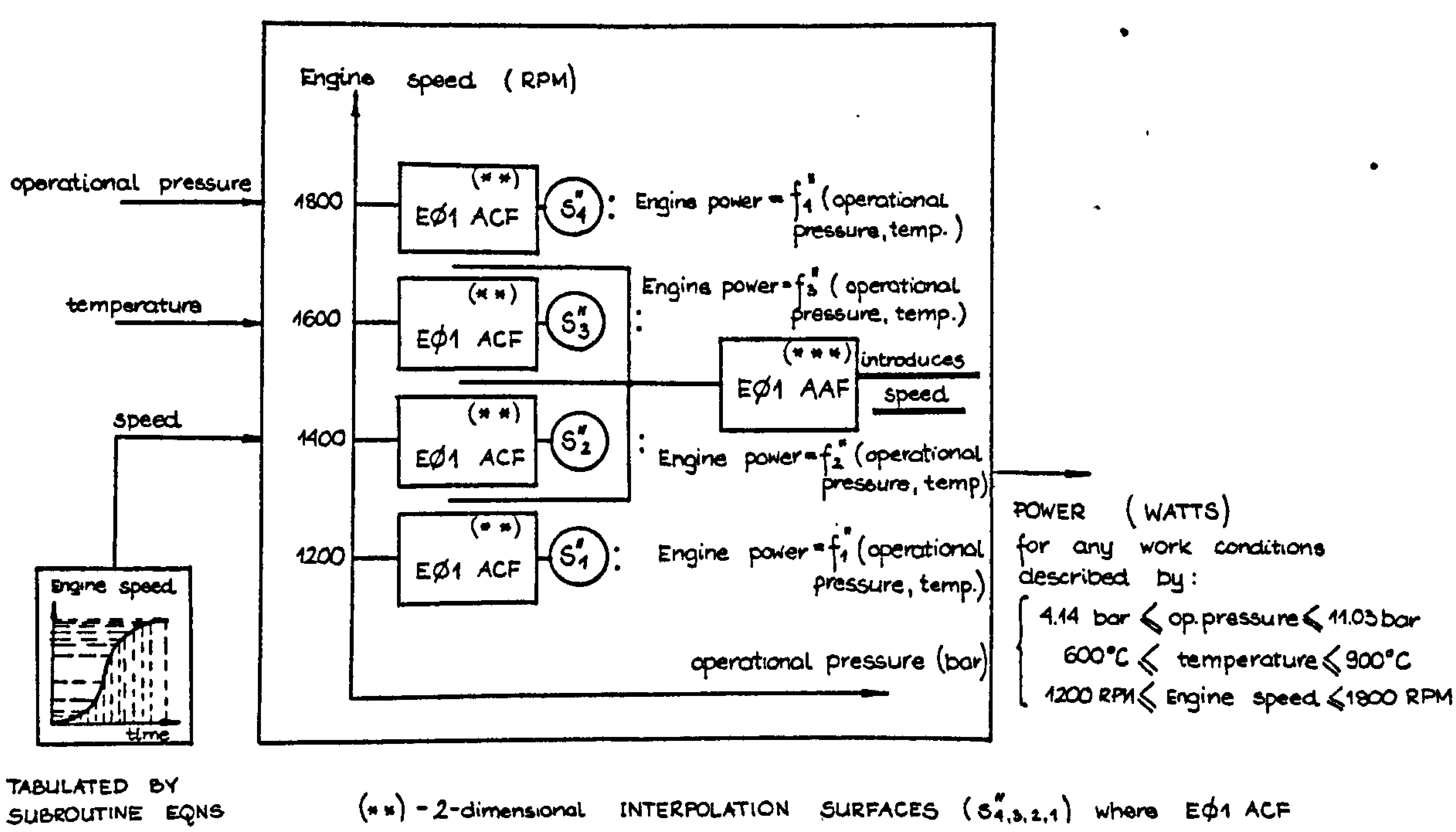



**Fig.2 One of the possible data interpolation techniques as used in POWTOR subroutine.**




(**) - 2-dimensional INTERPOLATION SURFACES ($S''_{4,3,2,1}$) where EØ1 ACF interpolates at a given point on a surface by fitting bi-cubic spline functions

(***) - 3-dimensional interpolation at a given point in form at a table of values, from function values evaluated at non-equidistant or equidistant points on a line, using AITKEN's technique of SUCCESSIVE linear interpolation.

**Fig.3 Subroutine POWTOR-structure of interpolation ie. 4xEØ1ACF and 1xEØ1AAF.**

BIBLIOGRAPHY

1. Performance Characteristics of Small Philips Stirling Cycle Air Engines.

Paper prepared for presentation at the Conference: "Stirling Engines for Automotive and Other Applications" Institution of Mechanical Engineers, Reading, UK. March 1977. By G.L.Ward, J. Slowley, G. Walker

2. Subroutine EØ1 ACF, Document No. 596. Nottingham Algorithms Group (NAG). Published 8 January 1973 (Replaces Document No. 142)

3. Subroutine EØ1 AAF, Document No: 598. Nottingham Algorithms Group (NAG) Published 8 January 1973 (Replaces Document No: 98)

## 5.4. SUBROUTINE SLGAIN

The third subroutine of the PCS simulation program is called SLGAIN and provides a set of necessary GAIN values for the specific GAIN block in EQNS. Data for SLGAIN subroutine was found as the slopes of the engine's steady-state characteristics (compare with Fig.2/Section 5.2 Chapter 5), where each of the basic steady-state curves were divided into three operational speed regions:

a) above 1200 RPM and less than 1400 RPM
b) above 1400 RPM and less than 1600 RPM
c) above 1600 RPM and less than 1800 RPM

The slopes of these relevant steady-state lines were found graphically and are shown in Fig,1 (Plots) or in tabulated form in Tab.1. Referring back to Fig.1. Section 5.1./Chapter 5, Subroutine SLGAIN uses the EØ1 ACF package solely and it is called three times only as the interpolation process is relatively simpler than in EQNS or POWTOR. The specific gain values thus are calculated within a range of system stability using Routh's criterions and SLGAIN subprogram also solves a set of "characteristic function" equations derived in section 4.4/Chapter 4.

For program abbreviation details (other than used in EQNS/POWTOR) see Tab.2.

```
      SUBROUTINE SLGAIN (PRSSUR,TEMPER,DELAY,DELAYV,ICON)
CVERSION
CVERSION                    THIS VERSION  17/08/77
CVERSION
      COMMON/DEH/ SLOPE1(4,6),SLOPE2(4,6),SLOPE3(4,6),
     1DTEMP(10),WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4
      COMMON/DEH/ TEMP(10),PRESS(10)
      COMMON/DEHK/ G1,G2,G3
      DATA NT,NP,IG1,IFAIL/4,6,5,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      READ 5001,(TEMP(I),I=1,NT)
      PRINT 6003,(TEMP(I),I=1,NT)
      READ 5001,(PRESS(I),I=1,NP)
      PRINT 6004,(PRESS(I),I=1,NP)
      DO 1 J=1,NT
1     READ 5002,(SLOPE1(J,K),K=1,NP)
      DO 5 J=1,NT
5     READ 5002,(SLOPE2(J,K),K=1,NP)
      DO 6 J=1,NT
6     READ 5002,(SLOPE3(J,K),K=1,NP)
      PRINT 6006,((SLOPE1(J,I),I=1,NP),J=1,NT)
      PRINT 6007,((SLOPE2(J,I),I=1,NP),J=1,NT)
      PRINT 6008,((SLOPE3(J,I),I=1,NP),J=1,NT)
      PRINT 6001,PRSSUR,TEMPER
C
C
C
  100 CONTINUE
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,SLOPE1,F1,F2,
     *           IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,SLOPE2,F3,F4,
     *           IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      CALL E01ACF (TEMPER,PRSSUR,TEMP,PRESS,SLOPE3,F5,F6,
     *           IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NT)
      B1=0.5*(F1+F2)
      B2=0.5*(F3+F4)
      B3=0.5*(F5+F6)
      A=1.0/(DELAYV*60.0)
      D=1.0/(DELAY*60.0)
      C=1.0
      T =((B1*C+A+D)*(A*D+B1*A*C+B1*D*C)-A*D*C*B1)/(A*D*C)
      T2=((B2*C+A+D)*(A*D+B2*A*C+B2*D*C)-A*D*C*B2)/(A*D*C)
      T3=((B3*C+A+D)*(A*D+B3*A*C+B3*D*C)-A*D*C*B3)/(A*D*C)
      G1=T1-1.0
      G2=T2-1.0
      G3=T3-1.0
      IF(ICON.EQ.0) PRINT 6009,F1,F2,B1,G1,F3,F4,B2,G2,F5,F6,B3,G3
      RETURN
5001  FORMAT(16F5.0)
5002  FORMAT(10F6.0)
6001  FORMAT(/32H INITIAL PRESSURE, TEMPERATURE =,F7.3,1H,,F7.1,8X)
6002  FORMAT(35H INTERPOLATION ERROR - PARAMETERS =,3F7.2,5X)
6003  FORMAT(/(13H TEMPERATURES,5X,16F7.1))
```

Progr.1  Subroutine SLGAIN.

```
6004  FORMAT(/(10H PRESSURES,8X,16F7.2))
6005  FORMAT(/(7H SPEEDS,11X,16F7.0))
6006  FORMAT(/108H    SLOPE VALUES AS FUNCTION OF TEMPER    (DOWN THE PAG
     1E) AND PRESSURE (ACROSS) AT SPEED GE 1200 AND LT 1400 /(4(6F10.3/
     2)))
6007  FORMAT(/108H    SLOPE VALUES AS FUNCTION OF TEMPER    (DOWN THE PAG
     1E) AND PRESSURE (ACROSS) AT SPEED GE 1400 AND LT 1600 /(4(6F10.3/
     2)))
6008  FORMAT(/95H    SLOPE VALUES AS FUNCTION OF TEMPER    (DOWN THE PAGE
     1) AND PRESSURE (ACROSS) AT SPEED GE 1600 /(4(6F10.3/)))
6009  FORMAT(1X,19H      ESTIMATES      ,3X,7H SLOPE ,4X,6H GAIN //(4E10.3
     1))
      END
```

```
 LISTING BY SYSTRAN/C PROGRAM HK.4A

 ENQUIRIES:- D.HIRST,EE DEPT,BRUNEL U.

      BEGIN READING CARDS:

 **LIMITS:1000,,120

 **TABULATE:1,2,4,5,7,8

 **PLOTF:1(1150,1810)8(0.1,3.8)7(30,470)2(2.50,7.90)
      WARNING: ONLY FIRST 12 DATA FIELDS USED

 **RUN:0.9,0.01

      SOLUTION STARTED
```

Progr.1 Subroutine SLGAIN -continuation;
SYSTRAN- Control Card Deck.

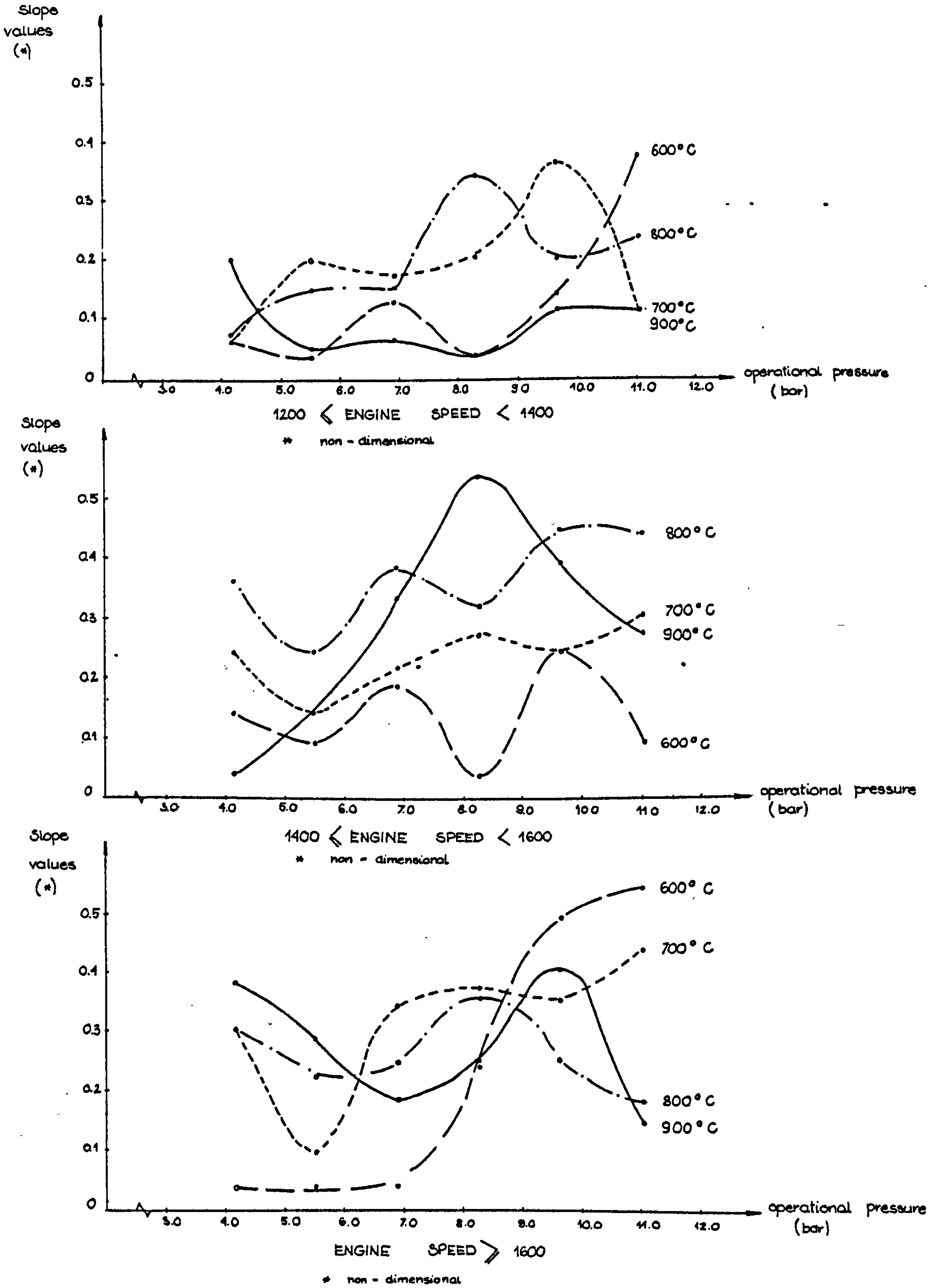



**Fig.1** Three regions slope values VS. operational pressure presented @ constant temperature conditions.

```
TEMPERATURES        600.0   700.0   800.0   900.0

PRESSURES           4.14    5.52    6.90    8.28    9.66    11.03

     SLOPE VALUES AS FUNCTION OF TEMPER   (DOWN THE PAGE) AND

PRESSURE (ACROSS) AT SPEED GE 1200 AND LT 1400
        .076        .153        .153        .346        .201        .240
        .067        .201        .173        .200        .365        .115
        .067        .038        .134        .038        .144        .380
        .201        .048        .067        .038        .115        .115
     SLOPE VALUES AS FUNCTION OF TEMPER   (DOWN THE PAGE) AND

 PRESSURE (ACROSS) AT SPEED GE 1400 AND LT 1600
        .144        .096        .192        .038        .250        .096
        .250        .144        .221        .278        .250        .307
        .365        .250        .384        .423        .451        .442
        .038        .153        .336        .538        .394        .278

     SLOPE VALUES AS FUNCTION OF TEMPER   (DOWN THE PAGE) AND

PRESSURE (ACROSS) AT SPEED GE 1600
        .038        .038        .038        .250        .500        .548
        .307        .096        .346        .375        .355        .442
        .307        .223        .250        .355        .250        .182
        .384        .288        .173        .243        .403        .144

INITIAL PRESSURE, TEMPERATURE =  6.900,  900.0
     ESTIMATES             SLOPE      GAIN

 .670E-01   .670E-01   .670E-01   .293E+02
 .336E+00   .336E+00   .336E+00   .305E+02
 .173E+00   .173E+00   .173E+00   .298E+02
```

Tab.1 Numerical data for Subroutine SLGAIN.

| Abbreviation | Meaning |
|---|---|
| F1, F2, F3<br>F4, F5, F6 | Slope Estimates |
| B1,B2,B3 | Slope Values |
| A,D,C, | System and Auxilary Time lags |
| T1,T2,T3 | ROUTH Critical Stability Gains |
| G1,G2,G3 | Practical Gain Values |

Tab.2.

After listing out the SLGAIN data, the program also prints out the calculated paramters - slope estimates, slope values, Routh critical gain values and practical gain values for all three speed regions of engine's operations considered. As previously mentioned, the SLGAIN subprogram provides, as required by EQNS, a set of gain values for a given pressure, temperature as well as speed conditions, whereas for internal "communication" normal COMMONblock/CALL type arrangements are seperate. SLGAIN uses around 70 lines of program print-out space.

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1. Subroutine EØ1 ACF, Document No. 596, Nottingham Algorithms Group (NAG) Published 8 January 1973 (Replaces Document No.142).

2. Introduction to Control Theory for Engineers. Blackie & Son Ltd. By Allan Sensicle. 1968

3. Digital and Sampled Data Control Systems, McGraw Hill by J.T. Tou. 1959

4. Dynamics of a System of Rigid Bodies. 3rd Edition. Macmillian, London By E.J. Routh. 1977

## 5.5. PROGRAM PLT

Program PLT is basically a fairly simple plotting routine using an external "TEKTRONIX" tele-type plotter facility. As explained in Fig.1/Section 5.1/Chapter 5, Systran's PLOTF statement creates a valuable post-processing file opportunity, used here for special graph-plotting work i.e. engine SPEED/TORQUE characteristics. A program PLT listing is given as Program No.1. Apart from the "TEKTRONIX" output, PLT generates a simple SPEED/TORQUE tabulation of results, which can be useful for additional engine "automotive" analysis. Naturally Program PLT is only one of the many possible applications of the PLOTF backing store data file; other additional post-processing facilities can be easily incorporated and used according to actual modelling requirements. Program PLT SCM TORQUE/SPEED characteristics for specific run i.e.

| | |
|---|---|
| Operational pressure | 6.90 bar |
| Cylinder Head Temperature | $900^{o}C$ |
| Set Speed | 1750 RPM |

is shown in Fig.1.

```
18/12/76 UNIVERSITY OF MINNESOTA 6600 FORTRAN COMPILER SCOPE  3.4.1 VER4.5      18/10/77  16.27.10.
      MNF(T,E=0)
0000000B          PROGRAM PLT (TAPE5,OUTPUT,TAPE6=OUTPUT,PLOT,TAPE27=PLOT)
0051520B          DIMENSION YDOT(500),TORQUE(500)
0051520B          READ(5,5000)YMIN,TMIN,YMAX,TMAX
0071340B     5000 FORMAT(1X/20X,2E18.10/20X,2E18.10)
0071340B          N=0
0071340B          DO 10 I=1,100000
0071370B          READ(5,5001)ICON,Y,T
0071510B     5001 FORMAT(I2,18X,2E18.10)
0071510B          IF(ICON.EQ.0) GO TO 10
0071520B          IF(ICON.EQ.-1) GO TO 12
0071530B          N=N+1
0071540B          YDOT(N)=Y
0071600B          TORQUE(N)=T
0071650B       10 CONTINUE
0071720B       12 CONTINUE
0071740B          DO 11 J=1,N
0072030B          WRITE(6,5002) ICON,YDOT(J),TORQUE(J)
0072250B     5002 FORMAT(1H ,5X,I5,10X,2(E14.7,5X))
0072250B       11 CONTINUE
0072310B          CALL CAM35MM
0072340B          CALL GPSLIDE
0072370B          CALL XAXIS(YMIN,YMAX)
0072420B          CALL YAXIS(TMIN,TMAX)
0072450B          CALL GRAPHIC (YDOT,TORQUE,N)
0072500B          CALL GRFRAME
0072530B          CALL LXVAL
0072560B          CALL LYVAL
0072610B          CALL LXTICK
0072640B          CALL LYTICK
0072670B          CALL GRAFDEF('SCH TORQUE/SPEED CHARACTERISTIC',31,'SPEED',5,'TORQU
         1E',6)
0072720B          CALL ENDFILM
0072750B          STOP
0072760B          END
```

**Progr.1 Program PLT.**

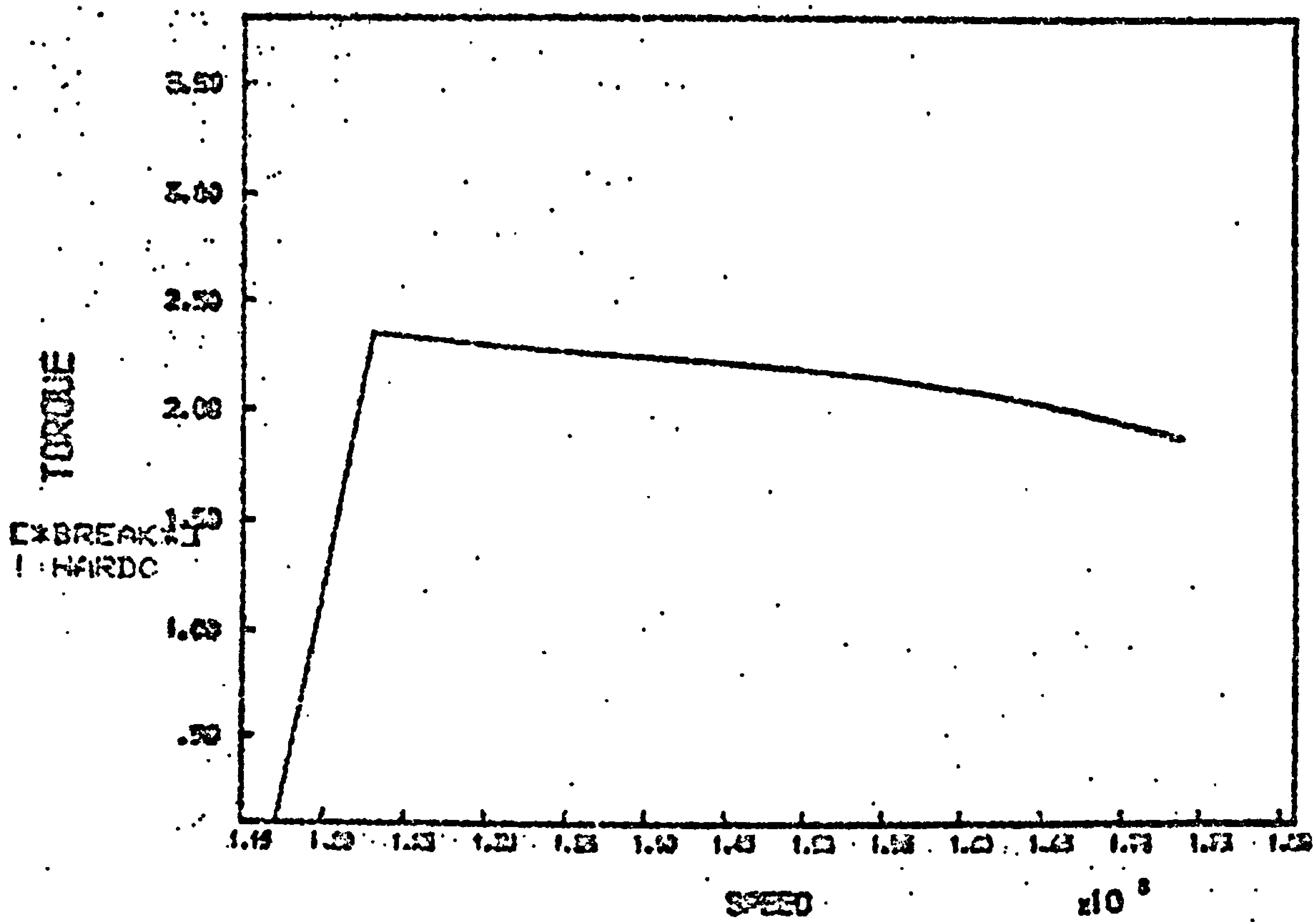


Fig.1 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=6.90bar
YSET=1750 RPM
YDOT=1739 RPM

BIBLIOGRAPHY

1. King's College Computer Unit - Bulletin K.4.8/4
The Job Control for Using DIMFILM
by P.W. Tedder. 1977

2. King's College Computer Unit - Bulletin K4.8/8
Notes on DIMFILM - Description Instructions
by P.W. Tedder. 1973

## 5.6. EXAMPLES OF SELECTED PCS PROGRAM RESULTS

Using the numerical data and initial conditions, the digital program given in 5.1,5.2,5.3,5.4, and 5.5 is easily developed. As an example, the program is run with three different set speed (YSET) values i.e. 1440 RPM, 1620 RPM and 1710 RPM and the corresponding dynamic characteristic of the engine is presented in Fig.1. (YSET = 1440 RMP), Fig.2 (YSET = 1620 RPM), and Fig.3 (YSET = 1710 RPM). Four plotting characters are used i.e. STARS, ZEROS, DOTS and X's, thus four system parameters can be plotted individually against time. These parameters are :engine's output speed (STARS), fuel (ZEROS), power output (X's) and torque(DOTS). The SYSTRAN control card deck used on this particular run is shown in Tab.1. Also a tabulated print-out is given in Tab.2, corresponding to the following initial work conditions :

| | |
|---|---|
| Cylinder Head Temperature | 900$^{o}$C |
| Set Speed | 1620 RPM |
| Operational Pressure | 6.90 Bar |
| Fuel | 5.4 g/min |

and Tab 1.

```
**LIMITS: 1000,,120
**TABULATE:1,2,4,5,7,8
** PLOTF:1(1150,1810)8(0.1,3.8)7(30,470)2(2.50,7.90)
**RUN:0.9,0.01
```

Tab.1. Program PCS' Control Cards (SYSTRAN)


power
torque
fuel
speed
YDOT=1412 RPM
A=28 RPM
Fig. 1 Prog. PCS results Ref YSET=1440 RPM
Temp=900 C
Press.=6.90bar


Fig.2 Progr. PCS results Ref.


Fig. 3 Progr. PCS Results Ref.

INITIAL PRESSURE, TEMPERATURE = 6.900, 900.0 FUEL STEP = 5.400

| TIME | R-K | N-R | WORST ERR | X | X1 | X2 | X3 | X4 | |
|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 6.90000E+00 | 1.62000E+03 | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 6.90000E+00 | 1.49813E+03 | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 6.90000E+00 | 1.35188E+03 | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 6.90000E+00 | 1.20563E+03 | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 6.90000E+00 | 1.05938E+03 | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 6.90000E+00 | 9.13125E+02 | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 6.90000E+00 | 7.66875E+02 | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 6.90000E+00 | 6.20625E+02 | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 6.90000E+00 | 4.74375E+02 | 0. |
| 9.000E-02 | 16 | 0 | 9.967E-04 | 1 | 1.22514E+03 | 5.40513E+00 | 6.90000E+00 | 4.04061E+02 | 3.01714E+02 |
| 1.000E-01 | 18 | 0 | 1.012E-05 | 1 | 1.22650E+03 | 5.41304E+00 | 6.90000E+00 | 3.93776E+02 | 3.01868E+02 |
| 1.100E-01 | 19 | 0 | 3.458E-05 | 1 | 1.22897E+03 | 5.42091E+00 | 6.90000E+00 | 3.91620E+02 | 3.02143E+02 |
| 1.200E-01 | 20 | 0 | 3.333E-05 | 1 | 1.23250E+03 | 5.42872E+00 | 6.90000E+00 | 3.88261E+02 | 3.02571E+02 |
| 1.300E-01 | 21 | 0 | 3.147E-05 | 1 | 1.23701E+03 | 5.43646E+00 | 6.90000E+00 | 3.83902E+02 | 3.03130E+02 |
| 1.400E-01 | 22 | 0 | 2.929E-05 | 1 | 1.24241E+03 | 5.44410E+00 | 6.90000E+00 | 3.78637E+02 | 3.03815E+02 |
| 1.500E-01 | 23 | 0 | 2.732E-05 | 1 | 1.24861E+03 | 5.45162E+00 | 6.90000E+00 | 3.72565E+02 | 3.04623E+02 |
| 1.600E-01 | 24 | 0 | 2.623E-05 | 1 | 1.25554E+03 | 5.45902E+00 | 6.90000E+00 | 3.65753E+02 | 3.05552E+02 |
| 1.700E-01 | 25 | 0 | 2.639E-05 | 1 | 1.26322E+03 | 5.46627E+00 | 6.90000E+00 | 3.58200E+02 | 3.06610E+02 |

Tab.2 Tabulations of results, ref. Temp=900°C, YSET=1620RPM, prssur=6.90bar, fuel=5.4g/min.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.800E-01 | 26 | 0 | 2.898E-05 | 1 | 1.27177E+03 | 5.47336E+00 | 6.90000E+00 | 3.49806E+02 | 3.07821E+02 |
| 1.900E-01 | 27 | 0 | 3.083E-05 | 1 | 1.28133E+03 | 5.48027E+00 | 6.90000E+00 | 3.40423E+02 | 3.09215E+02 |
| 2.000E-01 | 28 | 0 | 3.084E-05 | 1 | 1.29198E+03 | 5.48699E+00 | 6.90000E+00 | 3.29959E+02 | 3.10413E+02 |
| 2.100E-01 | 29 | 0 | 2.701E-05 | 1 | 1.30363E+03 | 5.49348E+00 | 6.90000E+00 | 3.18461E+02 | 3.12611E+02 |
| 2.200E-01 | 30 | 0 | 1.601E-05 | 1 | 1.31586E+03 | 5.49974E+00 | 6.90000E+00 | 3.06263E+02 | 3.14552E+02 |
| 2.300E-01 | 31 | 0 | 2.885E-06 | 1 | 1.32786E+03 | 5.50575E+00 | 6.90000E+00 | 2.94122E+02 | 3.16497E+02 |
| 2.400E-01 | 32 | 0 | 1.807E-05 | 1 | 1.33872E+03 | 5.51153E+00 | 6.90000E+00 | 2.82993E+02 | 3.18278E+02 |
| 2.500E-01 | 33 | 0 | 1.748E-05 | 1 | 1.34822E+03 | 5.51710E+00 | 6.90000E+00 | 2.73263E+02 | 3.19835E+02 |
| 2.600E-01 | 34 | 0 | 9.016E-06 | 1 | 1.35672E+03 | 5.52249E+00 | 6.90000E+00 | 2.64642E+02 | 3.21224E+02 |
| 2.700E-01 | 35 | 0 | 1.579E-06 | 1 | 1.36464E+03 | 5.52771E+00 | 6.90000E+00 | 2.56667E+02 | 3.22521E+02 |
| 2.800E-01 | 36 | 0 | 1.452E-06 | 1 | 1.37222E+03 | 5.53278E+00 | 6.90000E+00 | 2.49050E+02 | 3.23765E+02 |
| 2.900E-01 | 37 | 0 | 1.735E-06 | 1 | 1.37952E+03 | 5.53769E+00 | 6.90000E+00 | 2.41705E+02 | 3.24964E+02 |
| 3.000E-01 | 38 | 0 | 6.422E-07 | 2 | 1.38651E+03 | 5.54247E+00 | 6.90000E+00 | 2.34657E+02 | 3.26109E+02 |
| 3.100E-01 | 39 | 0 | 1.779E-06 | 1 | 1.39309E+03 | 5.54710E+00 | 6.90000E+00 | 2.27994E+02 | 3.27184E+02 |
| 3.200E-01 | 40 | 0 | 3.540E-06 | 1 | 1.39919E+03 | 5.55161E+00 | 6.90000E+00 | 2.21809E+02 | 3.28173E+02 |
| 3.300E-01 | 41 | 0 | 4.596E-06 | 1 | 1.40473E+03 | 5.55614E+00 | 6.90000E+00 | 2.16165E+02 | 3.29066E+02 |
| 3.400E-01 | 42 | 0 | 4.695E-06 | 1 | 1.40972E+03 | 5.56060E+00 | 6.90000E+00 | 2.11085E+02 | 3.29863E+02 |
| 3.500E-01 | 43 | 0 | 4.714E-06 | 1 | 1.41414E+03 | 5.56496E+00 | 6.90000E+00 | 2.06569E+02 | 3.30564E+02 |
| 3.600E-01 | 44 | 0 | 4.152E-06 | 1 | 1.41803E+03 | 5.56923E+00 | 6.90000E+00 | 2.02597E+02 | 3.31176E+02 |

Tab.2- continuation.

Tab.2-continuation

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.700E-01 | 45 | 0 | 3.163E-06 | 1 | 1.42143E+03 | 5.57342E+00 | 6.90000E+00 | 1.99120E+02 | 3.31707E+02 |
| 3.800E-01 | 46 | 0 | 1.968E-06 | 1 | 1.42441E+03 | 5.57755E+00 | 6.90000E+00 | 1.96076E+02 | 3.32170E+02 |
| 3.900E-01 | 47 | 0 | 7.414E-07 | 1 | 1.42704E+03 | 5.58161E+00 | 6.90000E+00 | 1.93396E+02 | 3.32576E+02 |
| 4.000E-01 | 48 | 0 | 4.006E-07 | 1 | 1.42938E+03 | 5.58562E+00 | 6.90000E+00 | 1.91016E+02 | 3.32934E+02 |
| 4.100E-01 | 49 | 0 | 1.396E-06 | 1 | 1.43148E+03 | 5.58959E+00 | 6.90000E+00 | 1.88880E+02 | 3.33255E+02 |
| 4.200E-01 | 50 | 0 | 2.222E-06 | 1 | 1.43339E+03 | 5.59351E+00 | 6.90000E+00 | 1.86939E+02 | 3.33547E+02 |
| 4.300E-01 | 51 | 0 | 2.883E-06 | 1 | 1.43516E+03 | 5.59739E+00 | 6.90000E+00 | 1.85154E+02 | 3.33814E+02 |
| 4.400E-01 | 52 | 0 | 3.397E-06 | 1 | 1.43680E+03 | 5.60124E+00 | 6.90000E+00 | 1.83493E+02 | 3.34062E+02 |
| 4.500E-01 | 53 | 0 | 3.787E-06 | 1 | 1.43835E+03 | 5.60505E+00 | 6.90000E+00 | 1.81933E+02 | 3.34294E+02 |
| 4.600E-01 | 54 | 0 | 4.079E-06 | 1 | 1.43981E+03 | 5.60884E+00 | 6.90000E+00 | 1.80456E+02 | 3.34514E+02 |
| 4.700E-01 | 55 | 0 | 4.295E-06 | 1 | 1.44121E+03 | 5.61259E+00 | 6.90000E+00 | 1.79046E+02 | 3.34723E+02 |
| 4.800E-01 | 56 | 0 | 4.458E-06 | 1 | 1.44256E+03 | 5.61631E+00 | 6.90000E+00 | 1.77694E+02 | 3.34923E+02 |
| 4.900E-01 | 57 | 0 | 4.586E-06 | 1 | 1.44385E+03 | 5.62000E+00 | 6.90000E+00 | 1.76391E+02 | 3.35115E+02 |
| 5.000E-01 | 58 | 0 | 4.684E-06 | 1 | 1.44511E+03 | 5.62367E+00 | 6.90000E+00 | 1.75130E+02 | 3.35301E+02 |
| 5.100E-01 | 59 | 0 | 4.757E-06 | 1 | 1.44633E+03 | 5.62732E+00 | 6.90000E+00 | 1.73905E+02 | 3.35481E+02 |
| 5.200E-01 | 60 | 0 | 4.811E-06 | 1 | 1.44752E+03 | 5.63093E+00 | 6.90000E+00 | 1.72712E+02 | 3.35655E+02 |
| 5.300E-01 | 61 | 0 | 4.849E-06 | 1 | 1.44868E+03 | 5.63452E+00 | 6.90000E+00 | 1.71546E+02 | 3.35825E+02 |
| 5.400E-01 | 62 | 0 | 4.875E-06 | 1 | 1.44981E+03 | 5.63809E+00 | 6.90000E+00 | 1.70405E+02 | 3.35991E+02 |
| 5.500E-01 | 63 | 0 | 4.892E-06 | 1 | 1.45093E+03 | 5.64164E+00 | 6.90000E+00 | 1.69287E+02 | 3.36153E+02 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5.600E-01 | 64 | 0 | 4.901E-06 | 1 | 1.45202E+03 | 5.64516E+00 | 6.90000E+00 | 1.68188E+02 | 3.36312E+02 |
| 5.700E-01 | 65 | 0 | 4.905E-06 | 1 | 1.45310E+03 | 5.64866E+00 | 6.90000E+00 | 1.67108E+02 | 3.36468E+02 |
| 5.800E-01 | 66 | 0 | 4.904E-06 | 1 | 1.45416E+03 | 5.65214E+00 | 6.90000E+00 | 1.66045E+02 | 3.36620E+02 |
| 5.900E-01 | 67 | 0 | 4.899E-06 | 1 | 1.45521E+03 | 5.65559E+00 | 6.90000E+00 | 1.64996E+02 | 3.36770E+02 |
| 6.000E-01 | 68 | 0 | 4.891E-06 | 1 | 1.45624E+03 | 5.65902E+00 | 6.90000E+00 | 1.63962E+02 | 3.36918E+02 |
| 6.100E-01 | 69 | 0 | 4.880E-06 | 1 | 1.45726E+03 | 5.66243E+00 | 6.90000E+00 | 1.62941E+02 | 3.37063E+02 |
| 6.200E-01 | 70 | 0 | 4.868E-06 | 1 | 1.45827E+03 | 5.66582E+00 | 6.90000E+00 | 1.61932E+02 | 3.37206E+02 |
| 6.300E-01 | 71 | 0 | 4.853E-06 | 1 | 1.45926E+03 | 5.66919E+00 | 6.90000E+00 | 1.60933E+02 | 3.37347E+02 |
| 6.400E-01 | 72 | 0 | 4.837E-06 | 1 | 1.46025E+03 | 5.67254E+00 | 6.90000E+00 | 1.59945E+02 | 3.37487E+02 |
| 6.500E-01 | 73 | 0 | 4.820E-06 | 1 | 1.46123E+03 | 5.67587E+00 | 6.90000E+00 | 1.58967E+02 | 3.37624E+02 |
| 6.600E-01 | 74 | 0 | 4.801E-06 | 1 | 1.46219E+03 | 5.67918E+00 | 6.90000E+00 | 1.57998E+02 | 3.37760E+02 |
| 6.700E-01 | 75 | 0 | 4.782E-06 | 1 | 1.46315E+03 | 5.68246E+00 | 6.90000E+00 | 1.57036E+02 | 3.37894E+02 |
| 6.800E-01 | 76 | 0 | 4.761E-06 | 1 | 1.46411E+03 | 5.68573E+00 | 6.90000E+00 | 1.56082E+02 | 3.38026E+02 |
| 6.900E-01 | 77 | 0 | 4.740E-06 | 1 | 1.46505E+03 | 5.68898E+00 | 6.90000E+00 | 1.55136E+02 | 3.38157E+02 |
| 7.000E-01 | 78 | 0 | 4.718E-06 | 1 | 1.46599E+03 | 5.69221E+00 | 6.90000E+00 | 1.54195E+02 | 3.38287E+02 |
| 7.100E-01 | 79 | 0 | 4.695E-06 | 1 | 1.46692E+03 | 5.69542E+00 | 6.90000E+00 | 1.53261E+02 | 3.38416E+02 |
| 7.200E-01 | 80 | 0 | 4.672E-06 | 1 | 1.46785E+03 | 5.69861E+00 | 6.90000E+00 | 1.52333E+02 | 3.38543E+02 |
| 7.300E-01 | 81 | 0 | 4.648E-06 | 1 | 1.46877E+03 | 5.70178E+00 | 6.90000E+00 | 1.51409E+02 | 3.38669E+02 |

Tab.2-continuation.

The tabulation of results is presented in a specific SYSTRAN style with 10 Column notation which includes the following : time, R-K steps, N-R steps, Worst Error, $X_1$ (speed), $X_2$ (fuel), $X_4$ (pressure), $X_5$ (speed error) $X_7$ (power), $X_8$ (torque).

In order to compare various PCS runs or any Systran print outs, it is obvious that control cards such as: LIMITS and RUN should be kept the same for a convenient comparison and analysis. Also the order of tabulation/ Plot can easily be changed, for example, by varying the position of parameters in the control cards such as TABULATE and PLOTF.

The tabulation of speed error is quite a convenient way to observe how the Control Loop (proportional Control system) works, then to analyse its possible drawbacks.

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Brunel University, Dept of Electrical Engineering and Electronics.
By D.E. Hirst. 1971

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BY D.E. Hirst. 1976

## 5.7. DISCUSSION

As shown above the theoretical SYSTRAN's dynamic model conception (Program PCS) incorporates three main program subroutines and one additional plotting program working in conjunction with a post-processing data file.
These are subroutines EQNS, POWTOR, SLGAIN and Program PLT. Program PCS allows for virtually any selection of operating parameters such as : cylinder head temperature operational pressure, fuel input, set-speed etc. - within a reasonably selected operating area and covered by available data material.

The main program solution output incorporates the following : tabulated results, run analysis, graph plot (using line printer) and Auxillaries. The post processing group solution output consists of : tabulated results, graph plot (using "TEKTRONIX" - hard copy tele-type plotter) and usual overall program diagnostics. Section 5.2 describes in detail Subroutine EQNS functional interrelations including : adjustable input quantities group, the output group components, the basic engine simulation circuit, associated subprograms (POWTOR, SLGAIN etc),

SYSTRAN's peripherals (Functions, Program XD etc), and finally data files. The "data continuity line" method set-up is discussed using external software (NAG subroutines) being A01ACF - 2 dimensional interpolation at a given point of a grid by fitting bi-cubic spline functions.

A01AAF 3 dimensional interpolation of a given point in form of a table of values, from function values evaluated at non-equidistant (or equidistant) points on a line, using AITKEN's technique, known also as SUCCESSIVE LINEAR interpolation.

Subroutines POWTOR (see section 5.3) calculate the values of both engine's power/torque and SLGAIN (see section 5.4) which define the system gain according to ROUTH's criterion of stability - both using various combinations of the above mentioned external computer library facilities (EØ1 ACF/ EØ1 AAF) and all mentioned subprograms are carefully explained and relevant program listings are given.

The whole PCS model is proportionally controlled (zero error seeking system). Finally examples of selected PCS program results are presented in two usual (SYSTRAN) forms i.e. Tabulations and plots of dynamic characteristics. The tabulations of results are presented in 10 columns which are interchangeable notation style, and may include the following : time, R-K steps, N-R steps, Worst Error, $X_1$(speed), $X_2$(fuel), $X_4$(pressure), $X_5$(speed error), $X_7$ (power) and finally $X_8$ (torque). The tabulation of speed error is included in order to observe how the control loop works and then to analyse any possible drawbacks.

# 6. PRACTICAL MODEL EVALUATION TESTS AND ANALYSIS USING BODE DIAGRAMS

## 6.1. AUTOMATIC PLOTTING OF BODE DIAGRAMS USING "PROGRAM BODE"

This method based on Bode Diagrams is one of the most commonly used in frequency-domain analysis. Similarly as in the Laplace domain, a basic restriction of frequency -domain techniques is, that they can be applied only to linear systems or systems that have been linearised around some steady-state operation level. Therefore in this example, the non-linear steady-state characteristic is divided into three speed sub-regions being then considered locally as linear and named A,B,C, for example:

Region A : above 1200 RPM and less than 1400 RPM
Region B : above 1400 RPM and less than 1600 RPM
Region C : above 1600 RPM and less than 1800 RPM

Program BODE is designed to output numerical Bode data and if desired to actually plot them on the TEKTRONIX teletype plotter. The Bode Diagrams and corresponding tabulation for the system described by a Laplace transfer function will accept practically any transfer function expressed as one polynomial in terms of "jw" divided by another, however, the number of terms in both numerator and denominator polynomials should be limited to ten only. That is :

$$G(jw) = \frac{A_1 + (jw)\,A_2 + (jw)^2\,A_3 + \ldots\ldots\ldots + (jw)^9 A_{10}}{B_1 + (jw)\,B_2 + (jw)^2 B_3 + \ldots\ldots\ldots + (jw)^9 B_{10}}$$

In order to achieve the above mentioned aims, Program BODE was developed from an original program written by D. Bell and A. Griffin from the Division of Electrical Engineering, University College, Swansea (for details see Lit.1.). However, the BODE method remained the same whilst the original program was completely rewritten into a contemporary FORTRAN language version, because the original version written around 1967 was not accepted by the ULCC Compiling system. Additionally, Program BODE was incorporated also with a modern and economic TEKTRONIX plotter routine which gave, combined with the main BODE program (Program No.1) maximum flexibility for a wide range of problems and solution requirements. The program is quite straightforward and it calculates the amplitude of G(jw), the logarithm of the amplitude and phase angle. The phase angle determination has built into a routine for checking when the numerator or denominator passes through $\pm 90^{o}$, $\pm 270^{o}$, or any other odd number of right angles. This is required since the computer can only find the arctan in the range of $\pm 90^{o}$. A log frequency scale is used.

Program BODE listing is given as Program No.1. and uses the following computational order:

1) Imaginery part of numerator
2) Real part of denominator
3) Imaginery part of denominator
4) Real part of numerator
5) Amplitude and Log. Amplitude
6) Phase and Plotter Phase value
7) Log. Frequency determination
8) Print out
9) Increment Frequency
10) Tektronix Plotting Routine (including Hardcopy Part)

```
      PROGRAM BODE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,PLOT,TAPE27=
     1PLOT)
C     BODE PLOT CALCULATIONS FOR NUMERATOR AND
C     DENOMINATOR POLYNOMIALS   MSS 01.11.77
C     TWO TERMS MINIMUM IN NUMERATOR AND
C     DENOMINATOR
      DIMENSION A(10),B(10),LOF(50),LAMP(50),LPH(50),E(100),G(100),H(100
     1)
      CALL CAM35MM
      CALL GPSLIDE
      CALL LINXY
    1 READ(5,10) N,M
   10 FORMAT(2I10)
      IF(N.EQ.12000000) STOP
      DO 2 IA=1,N
      READ(5,11) A(IA)
   11 FORMAT(F20.10)
    2 CONTINUE
      DO 3 IB=1,M
      READ(5,11) B(IB)
    3 CONTINUE
      DO 100 I=1,N
  100 WRITE(6,1000) A(I)
      DO 200 I=1,M
  200 WRITE(6,1000) B(I)
 1000 FORMAT(1X,F20.10)
      READ(5,82) FLOW,FNUL,FHIG,SCA1,SCA2,SCA3,SPHA
   82 FORMAT(7F10.8)
      WRITE(6,50)
   50 FORMAT(1H1,9X,*LOG FREQ.*,10X,*AMPLITUDE*,11X,*LOG AMP*,12X,*PHASE
     1*)
      F=FLOW
      IC=0
      K=1
      AUO=0.0
      AMO=0.0
      ANN=0.0
   40 FNRL=0.0
      FNIM=0.0
      FDRL=0.0
      FDIM=0.0
      W=2.*3.1416*F
      SGN=1.0
      DO 12 JA=1,N,2
      JB=JA-1
      FNRL=FNRL+SGN*W**JB*A(JA)
      SGN=-SGN
   12 CONTINUE
C     IMAGINARY PART OF NUMERATOR
      SGN=1.0
      DO 13 JC=2,N,2
      JD=JC-1
      FNIM=FNIM+SGN*W**JD*A(JC)
      SGN=-SGN
   13 CONTINUE
C     REAL PART OF DENOMINATOR
      SGN=1.0
      DO 14 JE=1,M,2
      JF=JE-1
      FDRL=FDRL+SGN*W**JF*B(JE)
      SGN=-SGN
   14 CONTINUE
```

Progr.1 PROGRAM BODE.

```
C     IMAGINARY PART OF DENOMINATOR
      SGN=1.0
      DO 15 JG=2,M,2
      JH=JG-1
      FDIM=FDIM+SGN*W**JH*R(JG)
      SGN=-SGN
   15 CONTINUE
C     AMPLITUDE AND LOG AMPLITUDE
      AMP=SQRT((FNRL*FNRL+FNIM*FNIM)/(FDRL*FDRL+FDIM*FDIM))
      AMOG=ALOG(AMP)/2.30259
      LAMP(K)=SCA1*AMOG
C     PHASE AND PLOTTER PHASE VALUE
      AMN=ATAN(FNIM/FNRL)
      ANN=ATAN(FDIM/FDRL)
      IF(ANN*ANO)30,31,31
   30 IF(ANN*ANN-.25)31,31,32
   32 IF(ANN)33,33,34
   33 SPHA=SPHA+180.0
      GO TO 31
   34 SPHA=SPHA-180.0
   31 IF(AMN*AMO)35,36,36
   35 IF(AMN*AMN-.25)36,36,37
   37 IF(AMN)38,38,39
   38 SPHA=SPHA-180.0
      GO TO 36
   39 SPHA=SPHA+180.0
   36 PHAS=SPHA+57.3*(ANN-AMN)
      ANO=ANN
      AMO=AMN
      LPH(K)=SCA2*PHAS
C     LOG FREQUENCY DETERMINATION
      FLOG=ALOG(F)/2.30259
      FLG1=SCA3*FLOG
      LOF(K)=FLG1
      IC=IC+1
      E(IC)=FLOG
      G(IC)=AMOG
      H(IC)=PHAS
C     PRINT OUT
      WRITE(6,22)F
      WRITE(6,20)FLOG,AMP,AMOG,PHAS
   20 FORMAT(4F20.10)
   22 FORMAT(1X,*FREQUENCY*,F20.10)
C     INCREMENT FREQENCY
      IF(FHIG-F)60,21,21
   21 K=K+1
      F=F*FMUL
      GO TO 40
   60 CONTINUE
      CALL POLY3(E,G,IC)
      CALL GRFRAME
      CALL GRAFDEF('LOGAMP V LOGFREQ',16,'LOGFREQ',7,'LOGAMP',6,0)
      CALL NEWFRAM
      CALL POLY3(E,H,IC)
      CALL GRAFDEF('PHASE V LOGFREQ',15,'LOGFREQ',7,'PHASE',5,0)
      CALL GRFRAME
      CALL ENDFILM
      GO TO 1
      END
```

Progr.1-continuation.

Program BODE generates tabulated results in five columns containing the following parameters: Frequency, Log Frequency, Amplitude, Log Amplitude and finally Phase angle. Also two graph plots are provided i.e. Phase Angle plotted V/s Log Frequency and LOGAMPLITUDE plotted V/s Logfrequency. Program BODE and its plotting routine occupies about 106 program line space including the necessary informative comments.

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Jan 1967 By W.D.T. Davies

5. PDP - 8 FORTRAN MANUAL, Digital Equipment Co. Ltd
Reading.

## 6.2. REVIEW OF SELECTED RESULTS, RELATED TO PCS PROGRAM CONDITIONS

Program BODE data organisation is based on a simple input deck, describing the relevant transfer function and consisting of two polynomials, for the denominator and numerator part of this transfer function. Each of these polynomials can contain up to a maximum of 10 terms, sufficient for most practical situations. However, should a larger number of polynomial terms be required, this can then be incorporated by a small program adjustment with a minimum of programming. Data for the BODE plotting program includes the following parameters :

1. Order of numerator
2. Order of denominator
3. Numerator terms
4. Denominator terms
5. Frequency Start point
6. Frequency Stop Point
7. Scale factors
8. Initial phase value.

Applying Program BODE to the PCS program conditions, (Ref. Temperature 900°C and operation pressure of 6.90bars) the relevant transfer function for regions A,B,C, become :

Region A: $$G_1(s)\Big|_{s=jw} = \frac{200}{S^3+30.067s^2+202.01s+13.4}$$

Region B: $$G_2(s)\Big|_{s=jw} = \frac{200}{S^3+30.336s^2+210.08s+67.2}$$

Region C: $$=G_3(s)\Big|_{s=jw} = \frac{200}{S^3+30.173s^2+205.19s+34.6}$$

Program BODE printouts for regions A,B,C, are presented in Tab.1,2,3, respectively, with the Phase v/s Logfrequency and Logamplitude v/s Logfrequency plots shown in Fig.1,2 (region A), Fig.3,4(region B) and Fig.5,6(region C). These graphs can be easily calibrated for both Phase as well as gain margins and the gain margin is expressed specifically as modulus of transfer function, when the phase is equal to $180^{o}$, minus unity. On the gain graphs, the gain margin, in decibels, is the value of gain at the frequency where phase equals $180^{o}$. A positive gain margin, as in this case, indicates a stable system. The phase margin is the amount by which the phase falls short of $180^{o}$ when the modulus of the transfer function is unity.
Unity of $|G(jw)|$ occurs at the frequency at which the gain graph crosses the log w axis, as ($20 \, Log_{10} 1 = 0 dB$). And again, a positive gain margins indicates a stable system.

| | LOG FREQ. | AMPLITUDE | LOG AMP | PHASE |
|---|---|---|---|---|
| FREQUENCY .0100000000 | -1.9999957378 | 10.8867652230 | 1.0368900476 | -43.7044786574 |
| FREQUENCY .0140000000 | -1.8538680136 | 9.0432072967 | .9563204483 | -53.4645498400 |
| FREQUENCY .0196000000 | -1.7077402893 | 7.1321621716 | .8532193913 | -62.5146192116 |
| FREQUENCY .0274400000 | -1.5616125650 | 5.4052261934 | .7328123110 | -70.2503888515 |
| FREQUENCY .0384160000 | -1.4154848408 | 3.9905362657 | .6010299811 | -76.5664334479 |
| FREQUENCY .0537824000 | -1.2693571165 | 2.9006577588 | .4624955049 | -81.6949357984 |
| FREQUENCY .0752953600 | -1.1232293922 | 2.0899306698 | .3201311971 | -86.0093763763 |
| FREQUENCY .1054135040 | -.9771016680 | 1.4980406519 | .1755232248 | -89.9164942594 |
| FREQUENCY .1475789055 | -.8309739437 | 1.0698809806 | .0293386520 | -93.8131405155 |
| FREQUENCY .2066104678 | -.6848462194 | .7612848780 | -.1184525444 | -98.1499207435 |
| FREQUENCY .2892546550 | -.5387184952 | .5387674808 | -.2685980532 | -103.3762005957 |
| FREQUENCY .4049565170 | -.3925907709 | .3777041970 | -.4228472882 | -110.0123049686 |
| FREQUENCY .5669391238 | -.2464630466 | .2603053841 | -.5845156033 | -118.6238514060 |
| FREQUENCY .7937147733 | -.1003353224 | .1740950598 | -.7592119345 | -129.7337090670 |
| FREQUENCY 1.1112006826 | .0457924019 | .1108694710 | -.9551859889 | -143.6136869061 |
| FREQUENCY 1.5556809555 | .1919201262 | .0657287259 | -1.1822426666 | -159.9991792827 |
| FREQUENCY 2.1779533378 | .3380478504 | .0355824465 | -1.4487611076 | -177.9426942869 |
| FREQUENCY 3.0491346729 | .4841755747 | .0174414586 | -1.7584134500 | -196.0064248518 |
| FREQUENCY 4.2687885421 | .6303032990 | .0077857779 | -2.1086934932 | -212.7015142831 |

200.0000000000
0
13.4000000000
202.0100000000
30.0670000000
1.0000000000

Tab.1-Progr.BODE results
Region A

Tab.1-continuation.

```
FREQUENCY        5.9763030589
    .7764310232         .0032216705        -2.4919135686        -226.9483233288
FREQUENCY        8.3668255425
    .9225587475         .0012639611        -2.8982601031        -238.3321733324
FREQUENCY       11.7135557595
   1.0686864718         .0004797090        -3.3190150492        -247.0125784622
FREQUENCY       16.3989780634
   1.2148141969         .0001786365        -3.7480217359        -253.4401802158
FREQUENCY       22.9585692887
   1.3609419203         .0000658381        -4.1815137684        -258.1201448215
FREQUENCY       32.1419970042
   1.5070696446         .0000241334        -4.6173724358        -261.4965633983
FREQUENCY       44.9987953053
   1.6531973689         .0000088212        -5.0544598843        -263.9207685402
FREQUENCY       62.9983141282
   1.7993250931         .0000032196        -5.4921799448        -265.6569361449
FREQUENCY       88.1976397795
   1.9454528174         .0000011743        -5.9302242641        -266.8987380426
FREQUENCY      123.4766956912
   2.0915805417         .0000004231        -6.3684344138        -267.7863540335
FREQUENCY      172.3673739677
   2.2377082659         .0000001560        -6.8067242735        -268.4205897458
FREQUENCY      242.0143235548
   2.3838359902         .0000000569        -7.2450673791        -268.8736970484
FREQUENCY      338.8200529765
   2.5299637145         .0000000207        -7.6834275560        -269.1973749403
FREQUENCY      474.3480741675
   2.6760914387         .0000000076        -8.1217989956        -269.4285843033
FREQUENCY      664.0873038345
   2.8222191630         .0000000028        -8.5601751819        -269.5937378095
FREQUENCY      929.7222253582
   2.9683468873         .0000000010        -8.9985563003        -269.7117060433
FREQUENCY     1301.6111155155
   3.1144746116         .0000000004        -9.4369379147        -269.7959695937
 5185 CHARACTERS OUTPUT
```

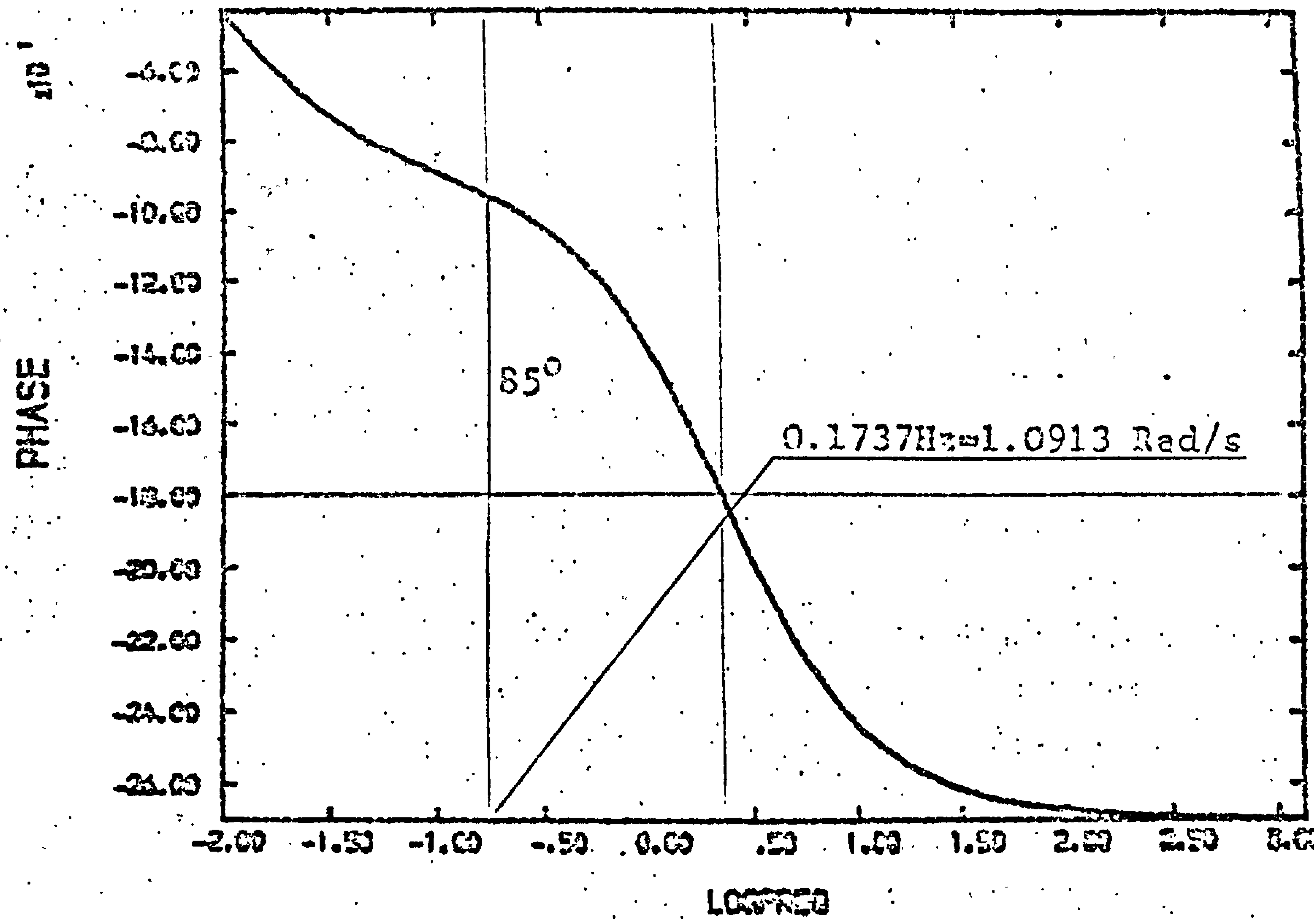



Fig.1 PHASE V LOGFREQ /Reg.A/

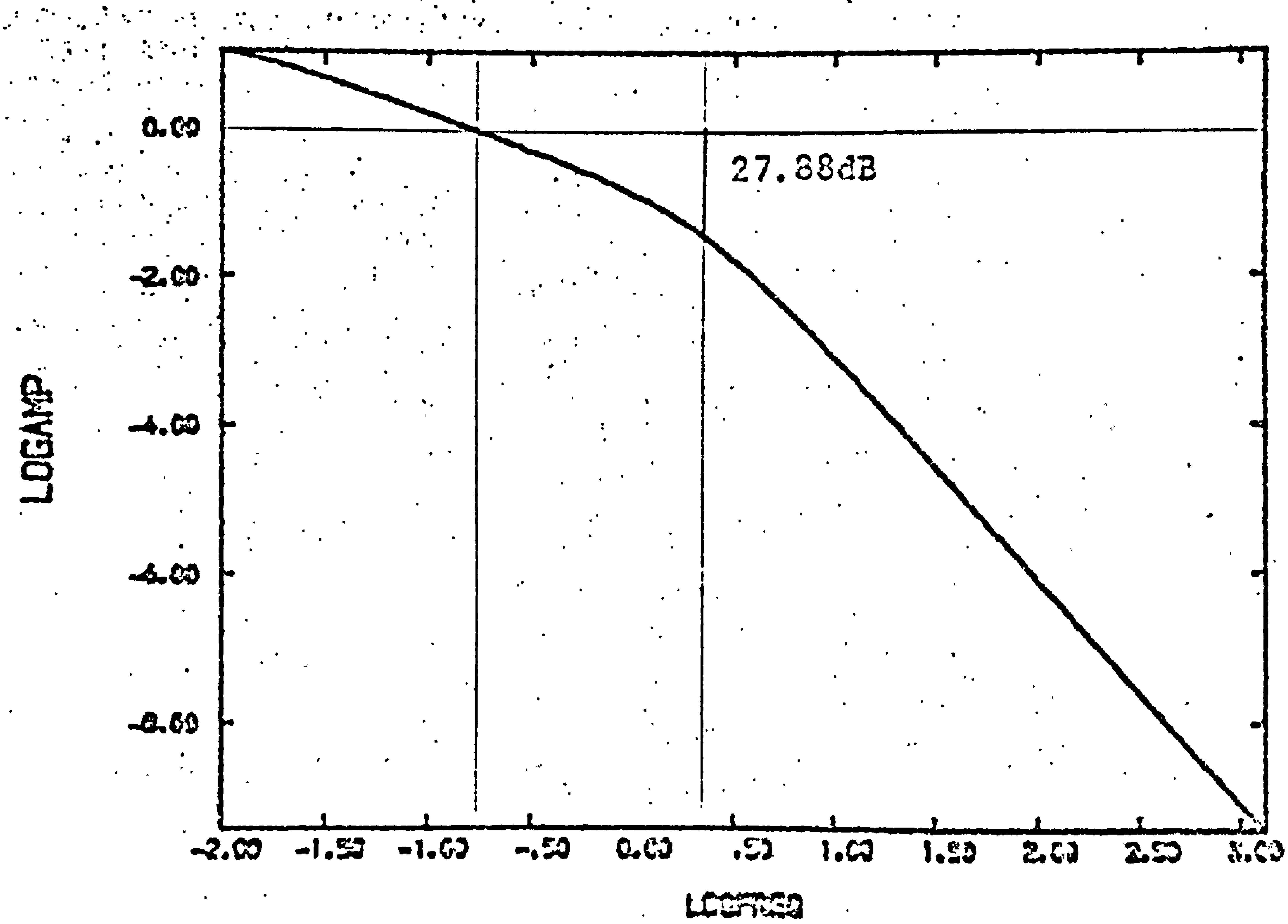



Fig.2 LOGAMP V LOGFREQ /Reg.A/

200.0000000000
0
67.2000000000
210.0800000000
30.3360000000
1.0000000000

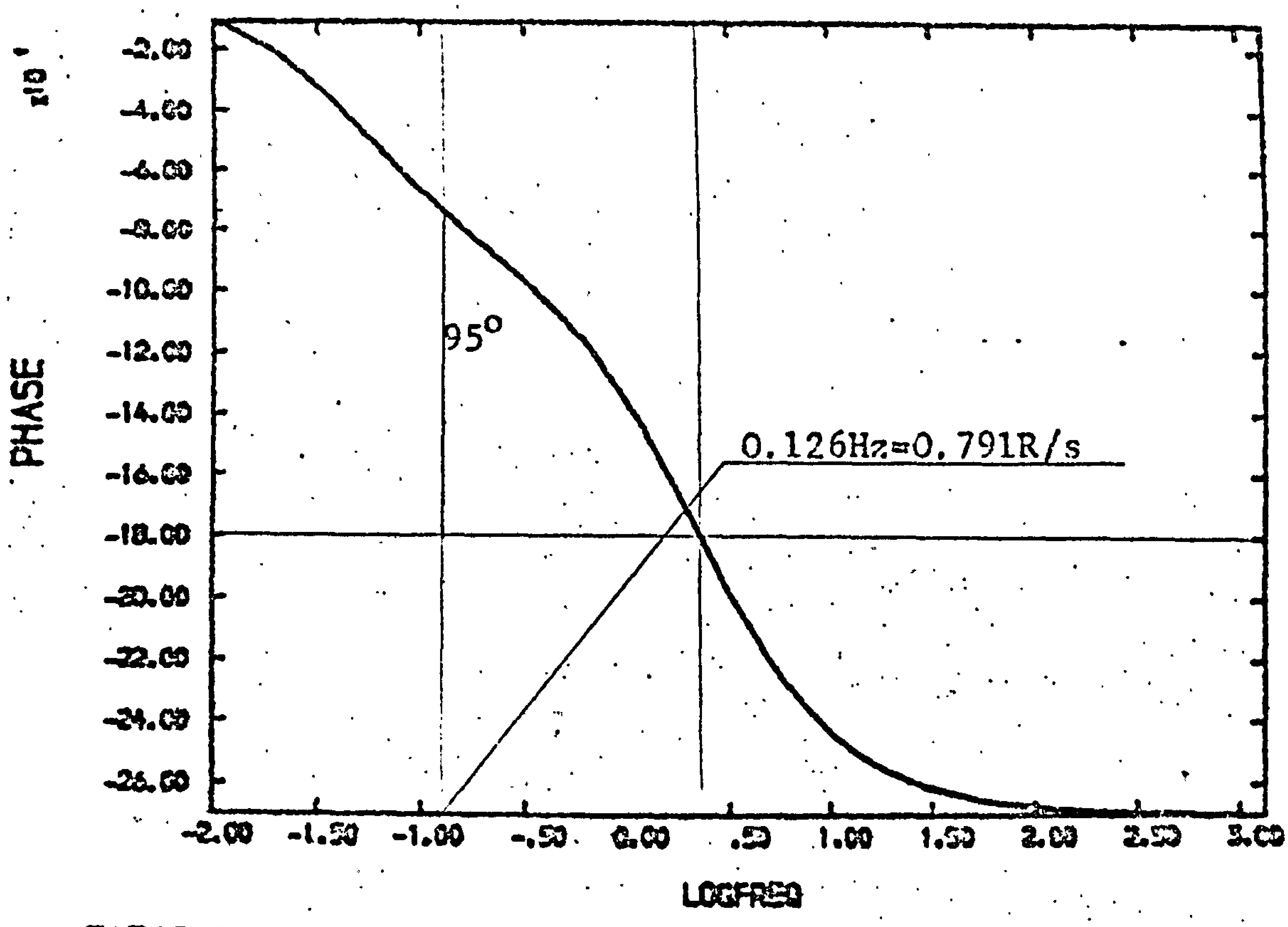
Tab.2-Progr.BODE results

Region B

| FREQUENCY | LOG FREQ. | AMPLITUDE | LOG AMP | PHASE |
|---|---|---|---|---|
| .0100000000 | -1.9999957378 | 2.9254072683 | .4661853426 | -11.1327937591 |
| .0140000000 | -1.8538680136 | 2.8790186789 | .4592435039 | -15.4278993800 |
| .0196000000 | -1.7077402893 | 2.7941423468 | .4462475764 | -21.1888186231 |
| .0274400000 | -1.5616125650 | 2.6474437426 | .4228258392 | -28.6473926454 |
| .0384160000 | -1.4154848408 | 2.4162573125 | .3831423649 | -37.7696292129 |
| .0537824000 | -1.2693571165 | 2.0969664230 | .3215907912 | -48.0706765605 |
| .0752953600 | -1.1232293922 | 1.7209228801 | .2357609063 | -58.6850470170 |
| .1054135040 | -.9771016680 | 1.3427842260 | .1280059580 | -68.7926458057 |
| .1475789056 | -.8309739437 | 1.0085103448 | .0036803495 | -78.0397466939 |
| .2066104678 | -.6848462194 | .7379800198 | -.1319551150 | -86.6056613308 |
| .2892546550 | -.5387184952 | .5301496502 | -.2756009335 | -95.0125446402 |
| .4049565170 | -.3925907709 | .3745832199 | -.4264507727 | -103.9976471140 |
| .5669391238 | -.2464630466 | .2592009206 | -.5863622107 | -114.3126412073 |
| .7937147733 | -.1003353224 | .1737169412 | -.7601562063 | -126.6487548119 |
| 1.1112006825 | .0457924019 | .1107464076 | -.9556683159 | -141.4081290971 |
| 1.5556809555 | .1919201262 | .0656914705 | -1.1824854967 | -158.4230435491 |
| 2.1779533378 | .3380478504 | .0355721520 | -1.4488867730 | -176.8166140757 |
| 3.0491346729 | .4841755747 | .0174380836 | -1.7584775749 | -195.2019837698 |
| 4.2687885421 | .6303032997 | .0077851914 | -2.1087264125 | -212.1268777593 |

Tab.2-continuation.

```
FREQUENCY        5.9763039589
     .7764310232          .0032215467        -2.4919374628      -226.5378556384
FREQUENCY        8.3668255425
     .9225587475          .0012639363        -2.8982686207      -238.0389773760
FREQUENCY       11.7135557595
    1.0686864718          .0004797042        -3.3190193950      -246.8031510483
FREQUENCY       16.3989780634
    1.2148141960          .0001786356        -3.7480239532      -253.2905885750
FREQUENCY       22.9585692887
    1.3609419203          .0000658379        -4.1815148997      -258.0132934196
FREQUENCY       32.1419970042
    1.5070696446          .0000241333        -4.6173730129      -261.4202408847
FREQUENCY       44.9937958050
    1.6531973689          .0000088212        -5.0544601788      -263.8662524285
FREQUENCY       62.9983141482
    1.7993250931          .0000032196        -5.4921800950      -265.6179960540
FREQUENCY       88.1976397795
    1.9454528174          .0000011743        -5.9302243407      -266.8709236879
FREQUENCY      123.4766956912
    2.0915805417          .0000004281        -6.3584344529      -267.7664866358
FREQUENCY      172.8673739677
    2.2377082659          .0000001560        -6.8067292934      -268.4063987469
FREQUENCY      242.0143235546
    2.3838359902          .0000000569        -7.2450673893      -268.8635606204
FREQUENCY      339.8200529763
    2.5299637145          .0000000207        -7.6834275612      -269.1901346345
FREQUENCY      474.3480741675
    2.6760914387          .0000000076        -8.1217989982      -269.4234126562
FREQUENCY      664.0873038345
    2.8222191630          .0000000028        -8.5601761832      -269.5900437759
FREQUENCY      929.7222253682
    2.9683468873          .0000000010        -8.9985563010      -269.7090674479
FREQUENCY     1301.6111155155
    3.1144746115          .0000000004        -9.4369379151      -269.7940848826
 5702 CHARACTERS OUTPUT
```




[*EOF*]

Fig.3 PHASE V LOGFREQ /Reg.B/

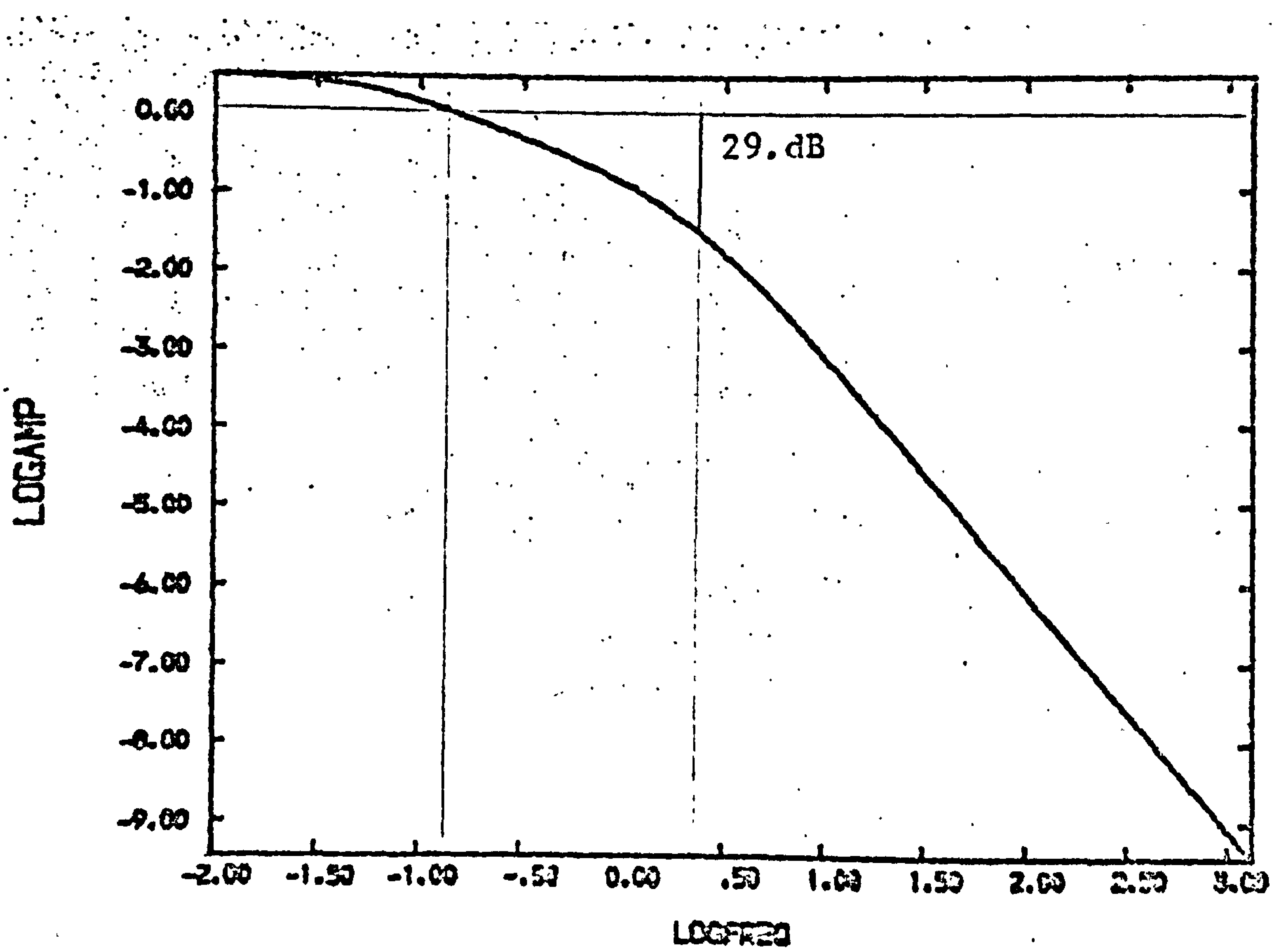



Fig.4 LOGAMP V LOGFREQ /Reg.B/

| FREQUENCY | LOG FREQ. | AMPLITUDE | LOG AMP | PHASE |
|---|---|---|---|---|
| .0100000000 | -1.9999957378 | 5.4329750201 | .7350361418 | -20.5020594339 |
| .0140000000 | -1.8538680136 | 5.1522816979 | .7119980825 | -27.7098645269 |
| .0196000000 | -1.7077402893 | 4.7086256091 | .6728927263 | -36.5063610213 |
| .0274400000 | -1.5616125650 | 4.0935256615 | .6120962128 | -46.3873698783 |
| .0384160000 | -1.4154848408 | 3.3661196027 | .5271284196 | -56.4481764910 |
| .0537824000 | -1.2693571165 | 2.6322310205 | .4203231072 | -65.7982317587 |
| .0752953600 | -1.1232293922 | 1.9823995541 | .2971905580 | -73.9828594091 |
| .1054135040 | -.9771016683 | 1.4563107778 | .1634027976 | -81.0536215741 |
| .1475789056 | -.8309739437 | 1.0544828423 | .0230394685 | -87.3905160776 |
| .2066104678 | -.6848462194 | .7556180211 | -.1216974336 | -93.5133045674 |
| .2892546550 | -.5387184952 | .5367074214 | -.2702610234 | -100.0496844518 |
| .4049565170 | -.3925907709 | .3769647916 | -.4236933079 | -107.6308399893 |
| .5669391238 | -.2464630466 | .2600449308 | -.5849503612 | -116.9208353381 |
| .7937147733 | -.1003353224 | .1740061045 | -.7594338969 | -128.5165496668 |
| 1.1112006825 | .0457924019 | .1108405548 | -.9552992734 | -142.7440248915 |
| 1.5556809555 | .1919201262 | .0657199774 | -1.1823000748 | -159.3778964333 |
| 2.1779533378 | .3380478504 | .0355890299 | -1.4487906041 | -177.4988859383 |
| 3.0491346727 | .4841755747 | .0174408542 | -1.7584285000 | -195.6894061736 |
| 4.2687885421 | .6303032990 | .0077856403 | -2.1087011719 | -212.4750677360 |

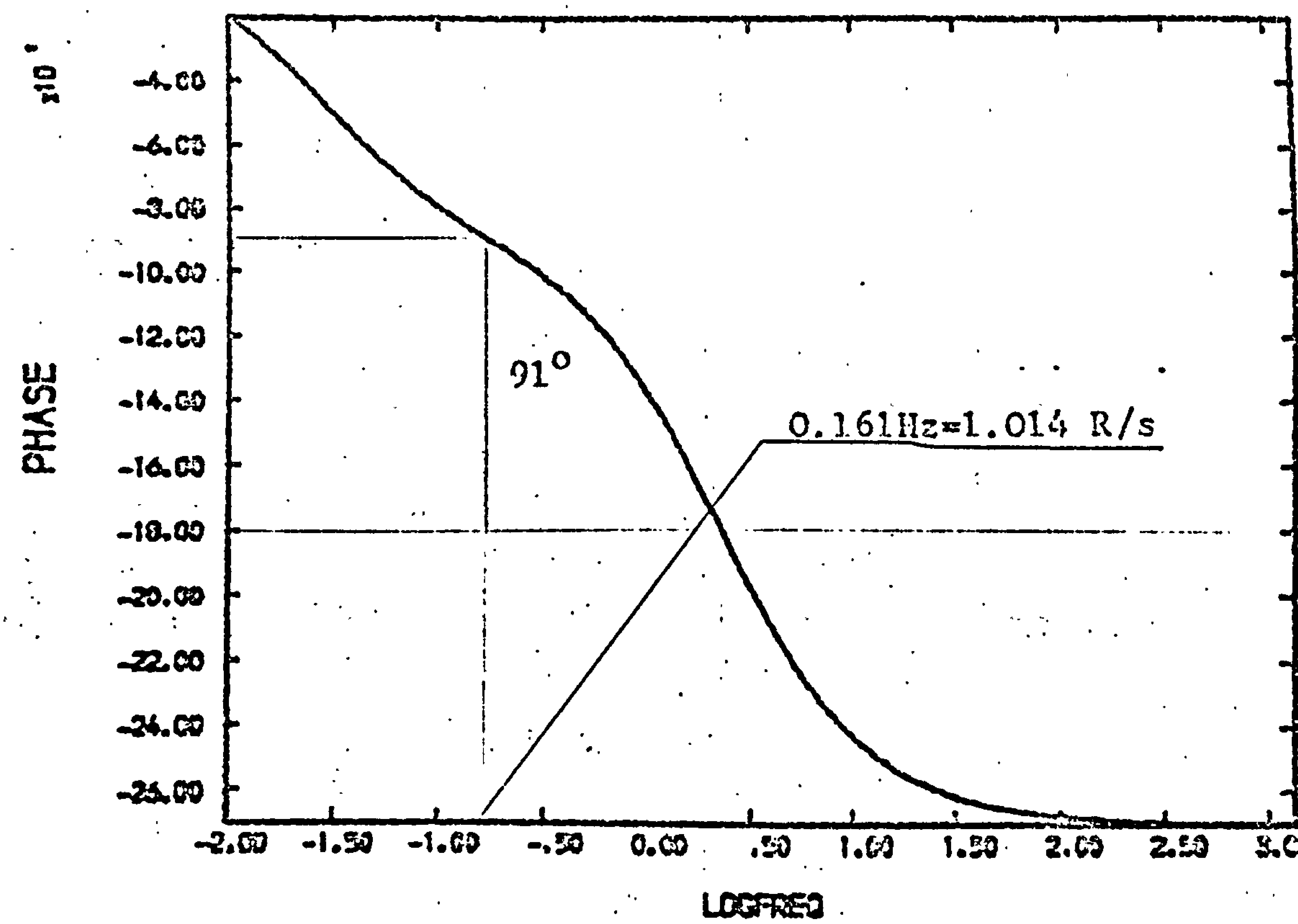
200.0000000000
0
34.6000000000
205.1900000000
30.1730000000
1.0000000000

Tab. 3-Progr. BODE results
Region C

Tab. 3-continuation.

| | | | | |
|---|---|---|---|---|
| FREQUENCY | 5.9763039587 | | | |
| | .7764310232 | .0032216414 | -2.4919174864 | -226.7865741063 |
| FREQUENCY | 8.3668255425 | | | |
| | .9225587475 | .0012639553 | -2.8982661019 | -238.2166375580 |
| FREQUENCY | 11.7135557595 | | | |
| | 1.0686364718 | .0004797079 | -3.3190160690 | -246.9300526848 |
| FREQUENCY | 16.3939780534 | | | |
| | 1.2149141960 | .0001736363 | -3.7480224562 | -253.3812331501 |
| FREQUENCY | 22.9585692887 | | | |
| | 1.3609419203 | .0000658381 | -4.1815140339 | -258.0780397448 |
| FREQUENCY | 32.1419970042 | | | |
| | 1.5070696446 | .0000241333 | -4.6173725712 | -261.4664883327 |
| FREQUENCY | 44.9987958059 | | | |
| | 1.6531973689 | .0000088212 | -5.0544599534 | -263.8992863465 |
| FREQUENCY | 62.9983141282 | | | |
| | 1.7993250931 | .0000032196 | -5.4921799800 | -265.6415917194 |
| FREQUENCY | 88.1976397795 | | | |
| | 1.9454528174 | .0000011743 | -5.9302242820 | -266.8877777382 |
| FREQUENCY | 123.4766956912 | | | |
| | 2.0915305417 | .0000004281 | -6.3684344230 | -267.7785252444 |
| FREQUENCY | 172.8673739677 | | | |
| | 2.2377082659 | .0000001560 | -6.8067292781 | -268.4149977535 |
| FREQUENCY | 242.0143235548 | | | |
| | 2.3838359902 | .0000000569 | -7.2450673815 | -268.8697027682 |
| FREQUENCY | 338.8200529763 | | | |
| | 2.5299637145 | .0000000207 | -7.6834275572 | -269.1945218830 |
| FREQUENCY | 474.3480741675 | | | |
| | 2.6760914387 | .0000000076 | -8.1217989962 | -269.4265464052 |
| FREQUENCY | 664.0873038345 | | | |
| | 2.8222191633 | .0000000028 | -8.5501761822 | -269.5922821680 |
| FREQUENCY | 929.7222253682 | | | |
| | 2.9683468873 | .0000000010 | -8.9985563005 | -269.7106662994 |
| FREQUENCY | 1301.6111155155 | | | |
| | 3.1144746115 | .0000000004 | -9.4369379148 | -269.7952269194 |

5258 CHARACTERS OUTPUT




[*EOF*]

Fig.5 PHASE V LOGFREQ /Reg.C/

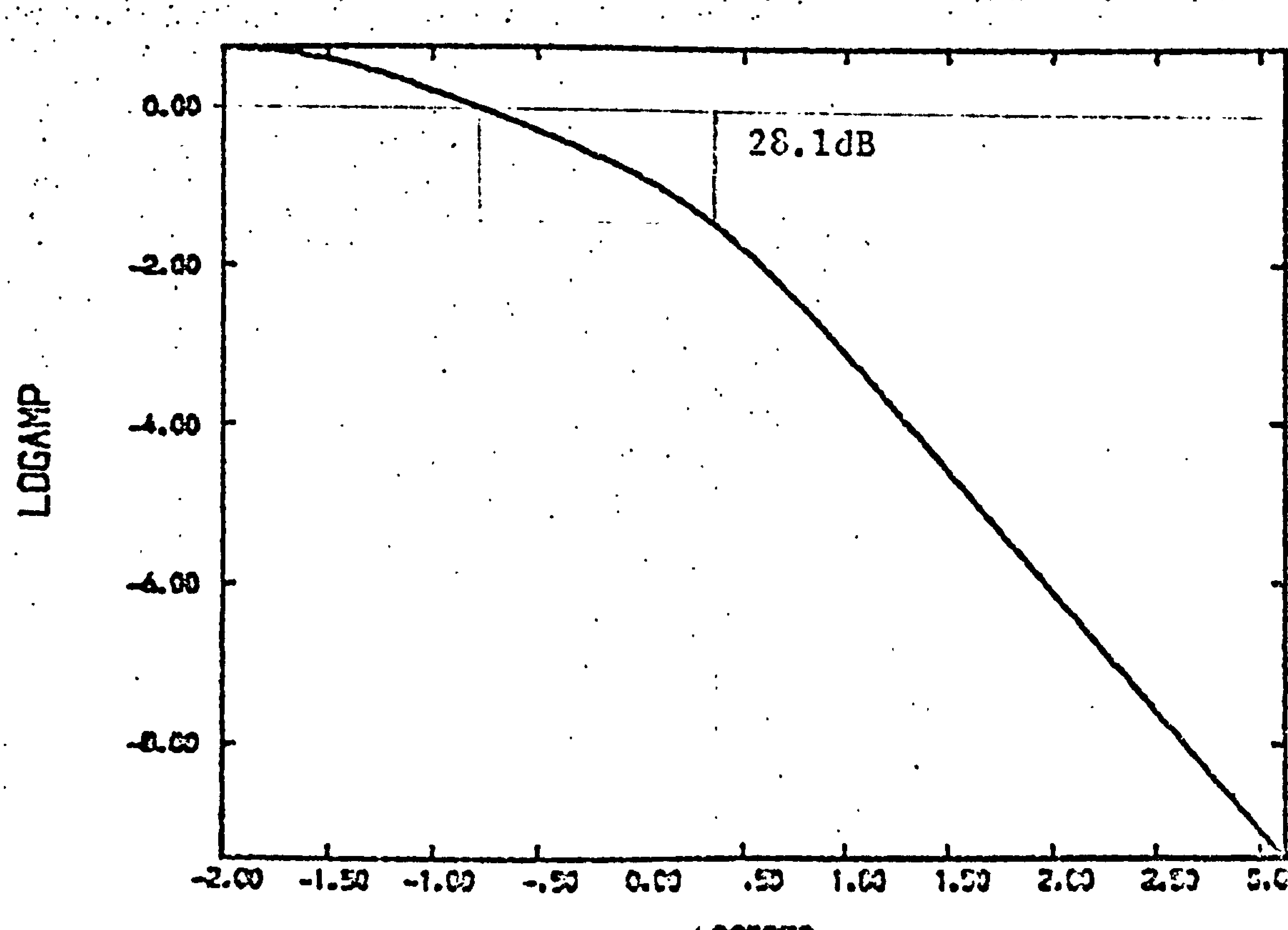



Fig.6 LOGAMP V LOGFREQ /Reg.C/

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2. Introduction to Control Theory for Engineers, Blackie & Son Ltd. By Allen Sensicle . 1968

## 6.3. DISCUSSION

From theories presented, and results obtained in this Chapter, it can be concluded that the PCS model is conditionally stable according to BODE diagram analysis. For the purpose of this analysis " Program Bode" is used because of the non-linear character of the model's steady state characteristics, the three Laplace transfer functions represent linearised portions of the engine's working area. Program Bode is designed to output numerical Bode data for practically any type of polynomial form transfer functions (up to ten terms in T.F. numerator/ denominator) and plot them in the form of two basic Bode diagrams on the TEKTRONIX teletype equipment. The results, in the form of basic Bode Diagrams i.e. Phase v/s Logfrequency and Logamplitude v/s Logfrequency are calibrated for both phase and gain margins and can be deciphered from these diagrams. The phase margin is not less than $85^{o}$ and in the second of the considered regions reaches $95^{o}$. It is also seen that the logarithmic gain margin alters from 27.88dB to approximately 29 dB due to slight differences in considered area "characteristic equations". It is worth noting that both phase and gain margins discussed here, defined for the PCS model (using BODE method) represent uncompensated system and as such will be analysed in detail in the next chapter.

# 7. DERIVATION OF CONTROL SYSTEM REQUIREMENTS AND IMPLEMENTATION OF COMPENSATION THEORY

## 7.1. PROGRAM PCS RESULTS IN LIGHT OF COMPENSATION THEORY

Selected examples of PCS program results presented in section 5.6. Chapter 5 in the form of an engine's dynamic characteristics show the generally unsatisfactory speed transient response of the output engine's shaft. The feed-forward error derivative compensation seems unlikely to offer the solution required. Therefore we shall consider now the compensation of unity feedback control system (compare with Fig.1./Section 5.2/Chapter 5) with the compensator which will modify the system error, and which would be inserted in the forward path. Fig.1. shows a fragment of a simplified block diagram of this improved control system.

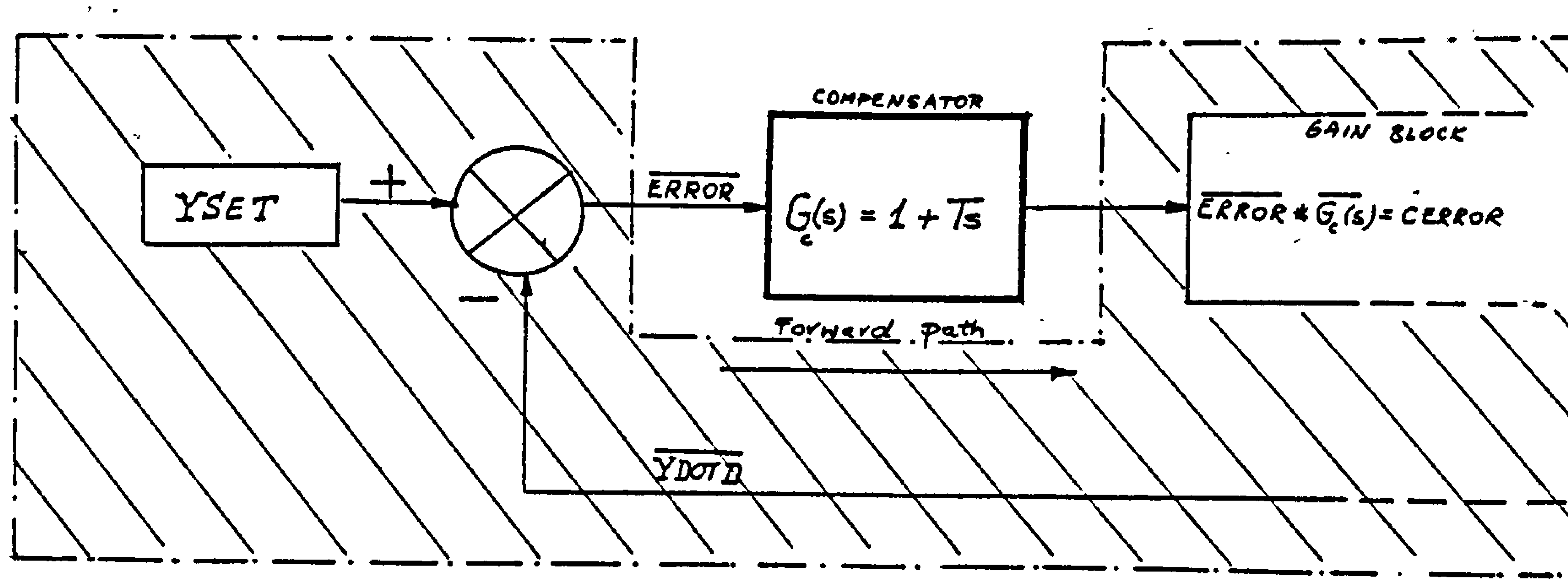



Fig.1. Fragment of Block Diagram representing The Original PCS Model

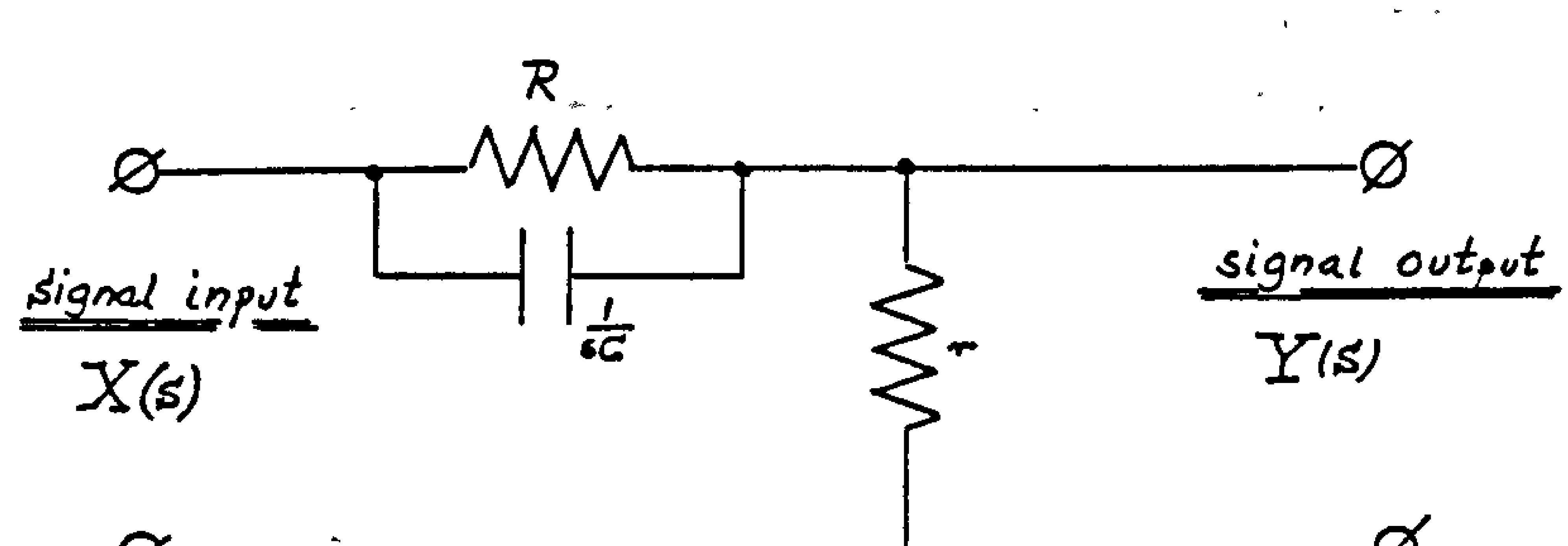
This feed-forward compensation is obtained by modifying the error in the compensation block which itself has a transfer function $\overline{G_c}(s)$. In order to improve the system's transient response, the effect of the ERROR signal through the system must be made substantially faster, therefore, we should add to the original error an estimation of its rate of change, this will then anticipate changes in the system resulting in the faster response. The new error signal will then be the original signal ERROR plus a signal proportional to the rate of change of the original error i.e. proportional to $\frac{d\,\text{ERROR}}{dt} = S.\overline{\text{ERROR}}$

Modified error $= \overline{\text{ERROR}} + T\ s*\overline{\text{ERROR}} = \overline{\text{ERROR}}\ (1+Ts)$

where : T - constant

$(1+Ts) = G_c(s)$ - the compensator transfer function.

As this type of compensation method employs the derivative of the original error, it is called error derivative compensation. Also in electronic analog technique this can be usefully approximated by a simple passive electronic network as shown in Fig.2.




where: s=jw

Fig. 2 Passive Electronic Network (DEC - Analog)

The transfer function is :

$$G_c(s) = \frac{Y(s)}{X(s)} = \frac{r}{R+r}\left[\frac{1+sRC}{1+sRC(\frac{r}{R}+r)}\right] = \frac{r}{\frac{\frac{R}{sC}}{R+\frac{1}{sC}}+r}$$

therefore if CR =T and $\frac{r}{R+r} = A$, the above tranfer function can be expressed in a new form as shown below:

$$G_c(s) = A\left(\frac{1+Ts}{1+ATs}\right)$$

Assuming that A is relatively small, let us say less than 0.1 the numerator of $G_c(s)$ is more effective than the denominator and the compensator transfer function can be approximated as :

$G_c(s) = A_1(1+Ts)$ and the amplifier of the overal system is then readjusted to compensate for the network attenuation. Finally effective compensation is approximated to (1+Ts) as will be used in the following DEC (Derivative Error Compensation) described in the next section.

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2. Three Term Controller Parameter Selection, Using Suboptimal Regulator Theory. I.E.E.E. Trans. Auto Control. Vol. AC-16, No.1. February 1971.
By D. Williams and J.B. Moore.

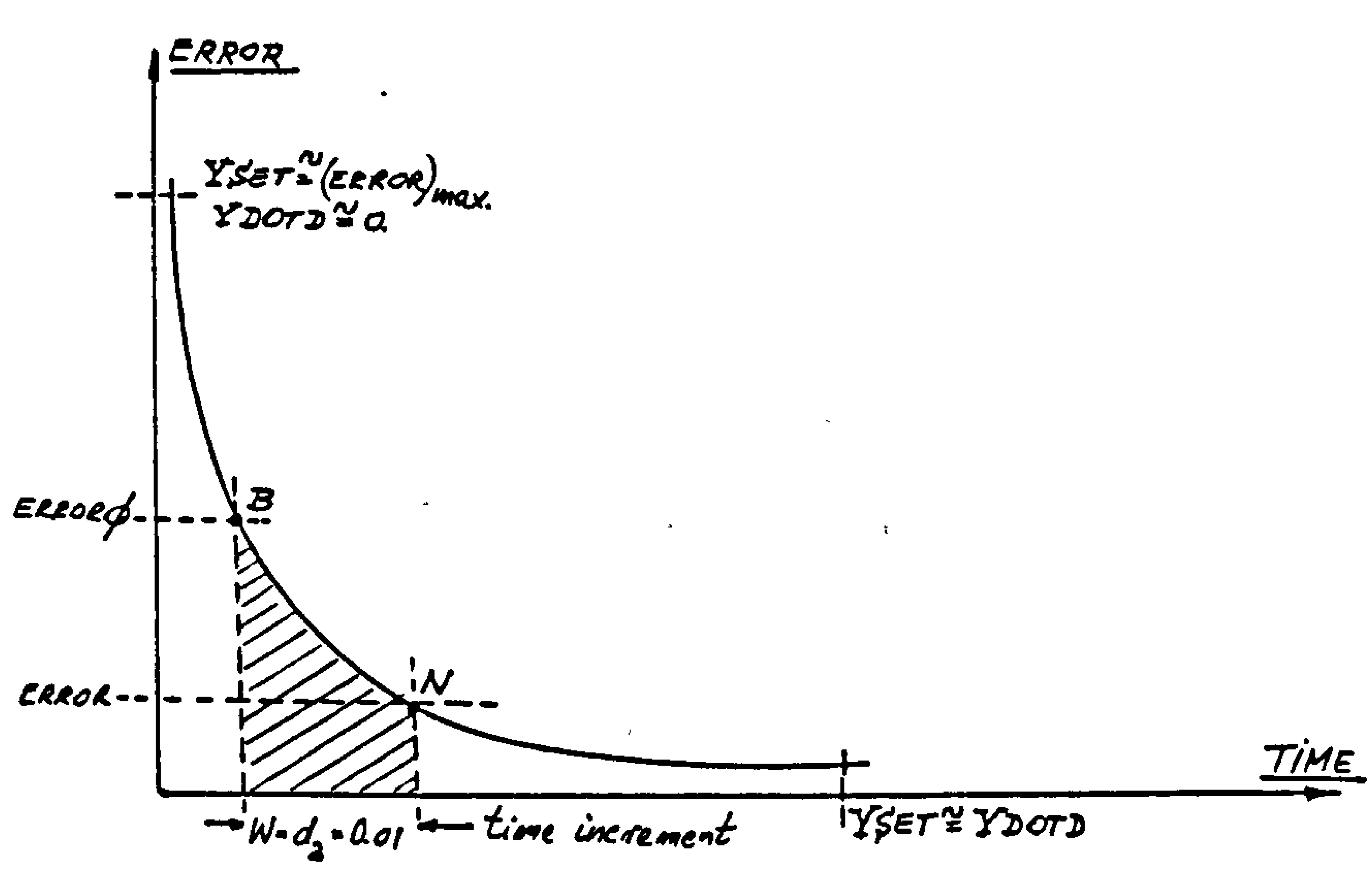
3. Teoria Automatycznego Sterowania. A.G.H. Skrypt No.196.
By W. Zapałowicz. 1970

4. Principles of Control Systems Engineering.
McGraw Hill Book Co. Inc.
By G.J. Thaler and R.G. Brown · 1973

## 7.2. DERIVATIVE ERROR COMPENSATION - PROGRAM DEC

Program DEC introduces Error Derivative Compensation in order to improve the PCS performance whose output responds more slowly to an input than desired i.e. the system transient response is not good enough. The compensation network is included in SUBROUTINE EQNS of the PCS Program and its modified version listing is presented as Program No.1. In order to calculate the new error signal CERROR (compensated error), which consists of the ERROR (original error) signal and the TD*DERROR (signal proportional to the rate of change of error), use is made for a thrid time of the SYSTRAN XD transport delay subprogram, which calculates value of ERRORØ in respect of the last value of original ERROR, allocated as X type variable (Integer type) in COMMON/VBLES/Block, as shown in Fig.1b. The signal proportional to the rate of change of error is calculated as

$$\text{DERROR} = \underset{\text{Value}}{\text{Absolute}} \left[ (\text{ERROR} - \text{ERRORØ})/\text{W} \right]$$




Where: $d_2$- print interval in RUN($d_1$,$d_2$) control cards ( e.g. $d_2$=W=0.01)

Fig.1a ERROR v/s Time, and Graphical Interpretation of Modified error part i.e. an estimation of its rate of Change.

```
      SUBROUTINE EQNS
CVERSION
CVERSION                    THIS VERSION   28/02/77
CVERSION
      COMMON /VALES/ YDOT,FUEL,ERROR,TEMPER,PRSSUR,YDOTD,POWER,TORQUE,
     1FUELD,DERROR,ERROR0,DUM(23),E,DFUEL
      COMMON /CONTRL/TP,TE,TF,ICON
      COMMON/DFHK/ G1,G2,G3,TD1,TD2,TD3,X1,X2,X3
      DIMENSION PRESS(10),FUELS(10),C(25),TEMP(10),SPEEDS(25),CURR(10)
     *  FEEDB1(21,6),FEEDB2(21,6),FEEDB3(21,6),FEEDB4(21,6),DTEMP(10),
     *  WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4)
      DATA N,NT,N2,NP,NS,IG1,IFAIL/3,4,6,6,21,21,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      TD1=1.1
      TD2=0.8
      TD3=0.2
      W=0.01
      Y=100.
      V=50.
      Z=14625.0
      DELAY=Y/60000.
      DELAYV=V/60000.
      TD=0.0
      YDOT=0.
      TEMPER=900.
      YSET=1710.
      PRSSUR=6.90
      CALL SLGAIN (PRSSUR,TEMPER,DELAY,DELAYV,ICON)
      FUEL=5.4
      DFUEL=0.0
C
C
      READ 5001,(TEMP(I),I=1,NT)
      PRINT 6003,(TEMP(I),I=1,NT)
      READ 5001,(PRESS(I),I=1,NP)
      PRINT 6004,(PRESS(I),I=1,NP)
      READ 5001,(SPEEDS(I),I=1,NS)
      PRINT 6005,(SPEEDS(I),I=1,NS)
      DO 1 J=1,NP
1     READ 5001,(FEEDB1(K,J),K=1,NS)
      DO 5 J=1,NP
5     READ 5001,(FEEDB2(K,J),K=1,NS)
      DO 6 J=1,NP
6     READ 5001,(FEEDB3(K,J),K=1,NS)
      DO 7 J=1,NP
7     READ 5001,(FEEDB4(K,J),K=1,NS)
      PRINT 6006,((FEEDB1(I,J),I=1,NS),J=1,NP)
      PRINT 6007,((FEEDB2(I,J),I=1,NS),J=1,NP)
      PRINT 6008,((FEEDB3(I,J),I=1,NS),J=1,NP)
      PRINT 6009,((FEEDB4(I,J),I=1,NS),J=1,NP)
      PRINT 6001,PRSSUR,TEMPER,FUEL
C
C
100   CONTINUE
      YDOTD=XD(1,DELAY)
      IF(YDOTD.LT.1200..OR.YDOTD.GT.1800.) GO TO 4
```

**Progr.1 PROGRAM DEC.**

```
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB1,VAL1(1),VAL2(1),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB2,VAL1(2),VAL2(2),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB3,VAL1(3),VAL2(3),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB4,VAL1(4),VAL2(4),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      DO 2 K=1,NT
      DTEMP(K)=TEMP(K)
      V1=VAL1(K)
      V2=VAL2(K)
      IF(ABS(V1-V2).GT.0.001*ABS(V1)) PRINT 6002,YDOTD,PRSSUR,TEMP(K),
     *   V1,V2
2     CURR(K)=0.5*(V1+V2)
      CALL E01AAF (DTEMP,CURR,C,NT,N2,N,TEMPER)
      FBFUEL=C(N2)
C
C
C
4     CONTINUE
      CALL POWTOR (PRSSUR,TEMPER,YDOT,ICON,POWER,TORQUE)
      FUELD=XD(2,DELAYV)
      ERROR=YSET-YDOTD
      IF(YDOTD.LT.1200.0) DFUEL=0
      IF(YDOTD.GE.1600.0)  G=G3
      IF(YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0) G=G2
      IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)  G=G1
      IF(YDOTD.GE.1600.0)  TD=TD3
      IF(YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0)  TD=TD2
      IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)  TD=TD1
      ERROR0=XD(3,W)
      DERROR=ABS((ERROR-ERROR0)/W)
      CERROR=ERROR+TD*DERROR
      IF(YDOTD.GE.1200.0) DFUEL=G*CERROR/Z
      IF(YDOTD.GT.1800.) FBFUEL=FUEL+1.
      IF(YDOTD.LT.1200.) FBFUEL=FUEL-1.
      FBFUEL=AMAX1(FBFUEL,0.0)
      E=(FUELD-FBFUEL)*Z
C     PRINT 555,TP,TE,YDOT,YDOTD,FBFUEL,FUEL
      RETURN
5001  FORMAT(16F5.0)
6001  FORMAT(/32H INITIAL PRESSURE, TEMPERATURE =,F7.3,1H,,F7.1,8X,
     *   11HFUEL STEP =,F7.3//4X,4HTIME,6X,3HR-K,2X,3HN-R,2X,13HWORST E
     *RR - X,12X,2HX1,12X,2HX2,12X,2HX3,12X,2HX4/)
6002  FORMAT(35H INTERPOLATION ERROR - PARAMETERS =,3F7.2,5X,
     *   20HINTERPOLATED FUELS =,2F7.3)
6003  FORMAT(/(13H TEMPERATURES,5X,16F7.1))
6004  FORMAT(/(10H PRESSURES,8X,16F7.2))
6005  FORMAT(/(7H SPEEDS,11X,16F7.0))
6006  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
     *) AND SPEED (ACROSS) AT T = 600  /(21F6.2))
6007  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
     *) AND SPEED (ACROSS) AT T = 700  /(21F6.2))
6008  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
     *) AND SPEED (ACROSS) AT T = 800  /(21F6.2))
6009  FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
     *) AND SPEED (ACROSS) AT T = 900  /(21F6.2))
555   FORMAT(1P6G20.6)
      END
```

Progr.1-continuation.

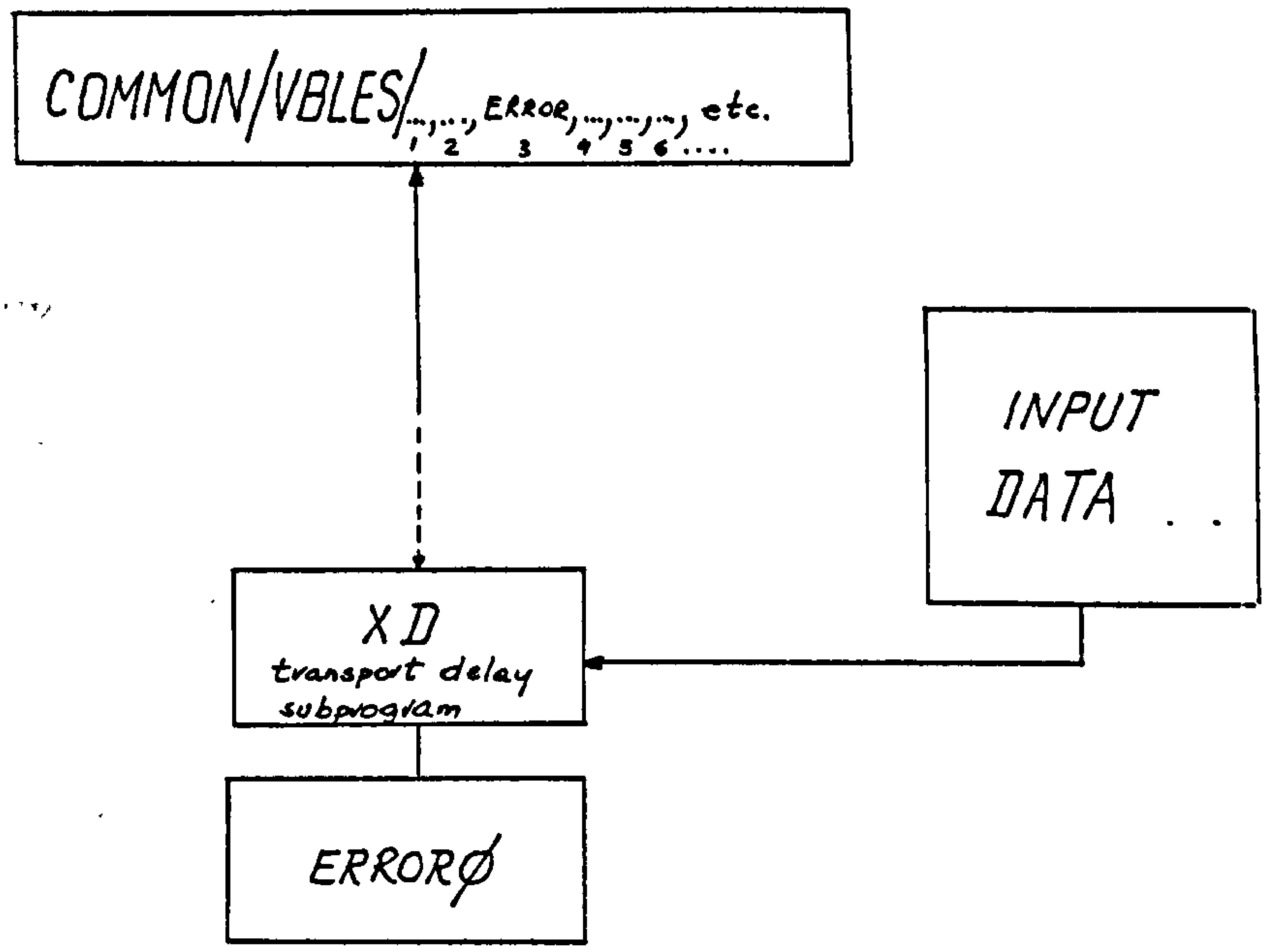



Fig.1b Calculation of ERRORØ value by Modified EQNS Subprogram.

Where W is the time increment equal to print interval as shown in Fig.1a. The value of TD is usually used as an error derivative compensation constant, and because our engine's operational area is split into three subregions, TD is represented by the appropriate $TD_1$, $TD_2$ and $TD_3$ values. Optimisation of these parameters is discussed in the next section. The $TD_1$, $TD_2$, $TD_3$ constants as well as the time increment W values are included in the input data deck, as it is necessary to make sure that the value of time increment W is equal to print interval in Systran's RUN($d_1$=W,$d_2$) control card. As one can see later the newly introduced Error Derivative Compensation network by the fact of its compensated Error will anticipate changes in the system and therefore the time response will be substantially faster.

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## 7.3. OPTIMISATION OF COMPENSATION NETWORK PARAMETERS ON AN EXPERIMENTAL BASIS

As mentioned previously in Section 7.2., the Error Derivative Compensation constant TD is represented in the DEC program as three independent values $TD_1$, $TD_2$, $TD_3$ acting seperately on three different speed levels, as indicated below -

```
IF (YDOTD.GE.1600.0)                          TD=TD3
IF (YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0)      TD=TD2
IF (YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)      TD=TD1
```

To show how these constants $TD_1$, $TD_2$, $TD_3$ are related to the engine's transient response, let us consider three examples of program DEC runs as shown in TAB.1.

| | Region A | Region B | Region C |
|---|---|---|---|
| Run No.1. | $TD_1 = 0.5$ | $TD_2 = 0.5$ | $TD_3 = 0.5$ |
| Run No.2 | $TD_1 = 0.7$ | $TD_2 = 0.7$ | $TD_3 = 0.7$ |
| Run No.3 | $TD_1 = 1.1$ | $TD_2 = 0.8$ | $TD_3 = 0.2$ |

One must assume also that the set speed YSET is equal to 1710 RPM in all three cases. In both RUN No.1. and RUN No.2 situations (see Fig.1, Fig.2), the engine's output response is relatively too slow and the acceleration, especially below 1400 RPM is not satisfactory. During tests, with various $TD_{1,2,3}$ settings, it was found that a relatively large $TD_1$ value produces a fast response in Reg. A's area but this also causes an overshoot problem similar to real overaccelerated engines. It seems reasonable therefore to

SYMBOLS AND SCALES:

| | | | | | | |
|---|---|---|---|---|---|---|
| * | X1 | 1.150E+03 | 1.282E+03 | 1.414E+03 | 1.546E+03 | 1.678E+03 |
| | X8 | 1.000E-01 | 8.400E-01 | 1.580E+00 | 2.320E+00 | 3.060E+00 |
| X | X7 | 3.000E+01 | 1.180E+02 | 2.060E+02 | 2.940E+02 | 3.820E+02 |
| 0 | X2 | 2.500E+00 | 3.580E+00 | 4.660E+00 | 5.740E+00 | 6.820E+00 |

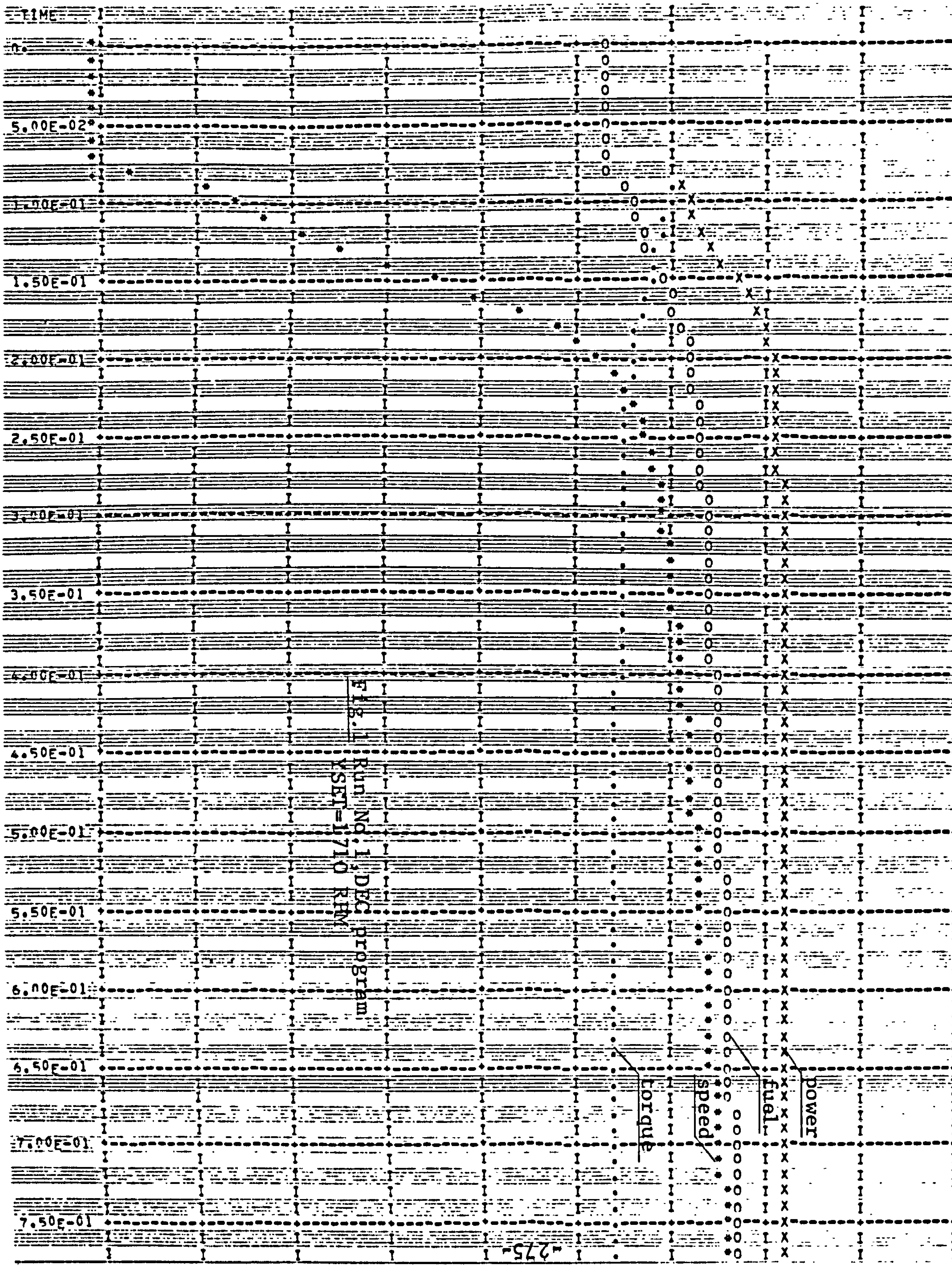


Fig. 1 Run No. 1, DEC program
YSET=1710 RPM


Fig. 2 Run No.2 DEC program
YSET = 1710 RPM


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Fig. 3 Run No3, DEC program YSET=1710 RPM adjust TD values as shown in Fig.3 where the main acceleration is achieved in Region A with Region B and C TD-constants decaying to the nominal value of 0.2. After trying various settings of $TD_1$, $TD_2$ and$TD_3$, the optimum performance was found at the TD values of :

$$TD_1 = 1.1$$
$$TD_2 = 0.8$$
$$TD_3 = 0.2$$

as also shown in Fig.3. The engine's output (Ref. RUN No.3 conditions) responds very fast and the control loop also allows for an accurate setting of the output RPM speed according to the YSET requirements.

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## 7.4. STABILITY EVALUATION OF THE DERIVATIVE ERROR COMPENSATION SYSTEM USING BODE DIAGRAMS

For the evaluation of the stability of the DEC system, use is made of the previously described BODE program which plots the PHASE and the LOGAMPLITUDE against LOGFREQUENCY. Both these graphs allow one to evaluate the system frequency response, the gain and phase margins as explained earlier. Data for program BODE in the form of numerator/denominator polynomials of the transfer function is obtained from the formerly optimised RUN No.3 example in section 7.3. A complete set of results i.e. tabulated and plotted are presented:

| | | | |
|---|---|---|---|
| Region A | Tab.1. | Fig.1. | Fig.2 |
| Region B | Tab.2 | Fig.3 | Fig.4 |
| Region C | Tab.3 | Fig.5 | Fig.6 |

For a direct comparison of both sets of results of this program, uncompensated and compensated systems,see Tab.4a/4b in a tabular form. A fast transient response was obtained after the Error Derivative Compensation had been introduced and this was achieved by a substantial increase of the specific phase margin frequency i.e. in the first region from 1.0913 (RADIAN/SEC) to 4.88 (RADIAN/SEC),in the second region from 0.791 (RADIAN/SEC) to 2.474(RADIAN/SEC), and finally from 1.014(RADIAN/SEC) to 3.625(RADIAN/SEC). Referring to the BODE diagrams, the lead network improves the transient response by virtue of its phase lead characteristic, as the required phase margin frequency is increased by a change in the phase curve. The overall "cost"of these improvements are paid for by PHASE and GAIN margins parameters which are decreased after compensation, but still remaining safely within stability margins. As far as the PHASE margin is concerned it is reduced to within the lower speed regions (below 1400 RPM) with any level above this, the PHASE margin remains similar, for example, in the second region, the phase margin was $95^{o}$ (uncompensated) and $88^{o}$ (compensated) in the third region - $91^{o}$ (uncompensated) and $90^{o}$ (compensated). Whereas the gain margin is then reduced about 4.5 times to a lower speed area (below 1400 RPM) and then steadily increases to 20dB (compensated system) as compared with just 28.1dB for uncompensated.

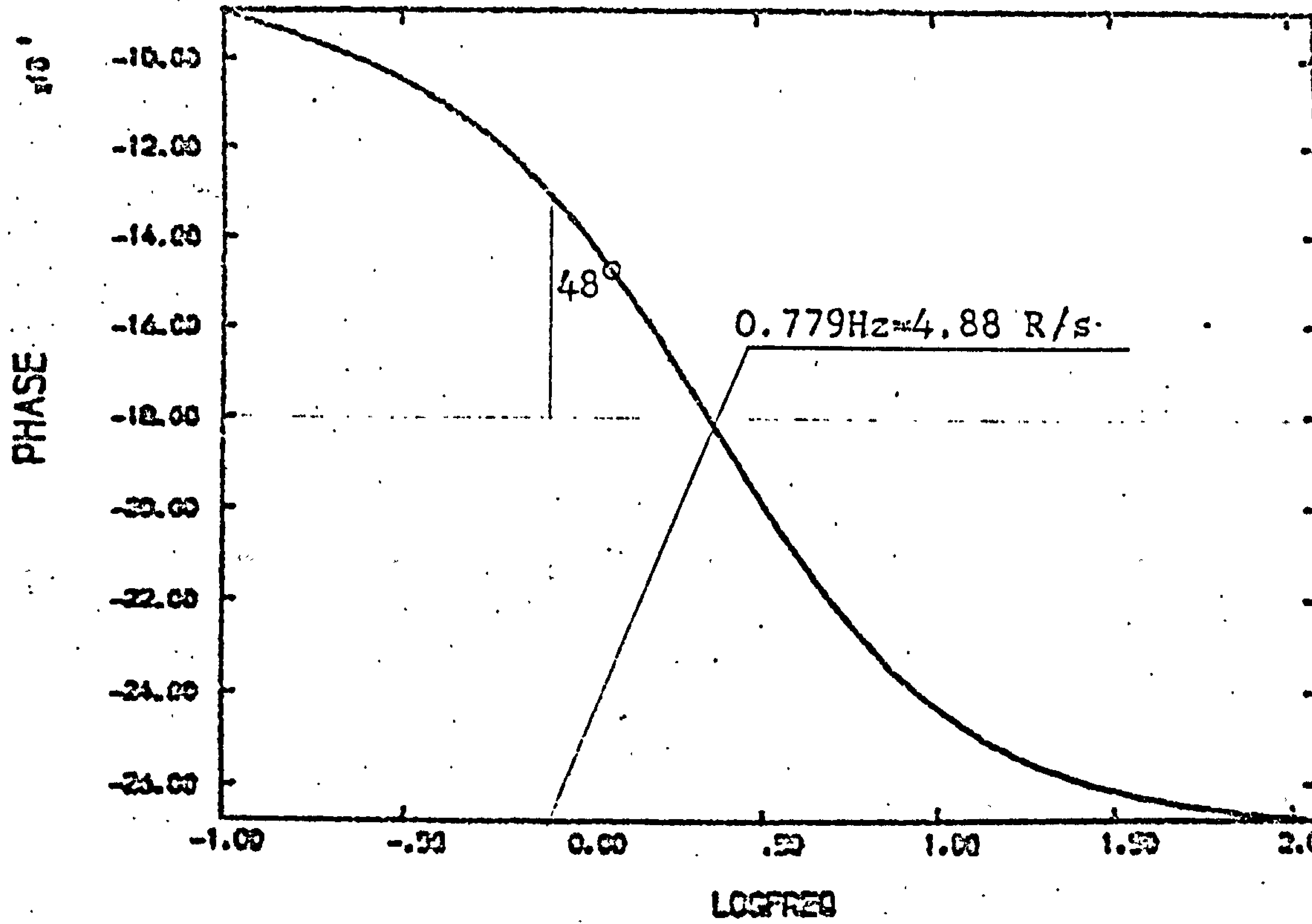
Calculations and plots by Program BODE for the phase margin frequency are initially given as logarithmic values expressed in (Hz) and because (RADIAN/SEC) notation is interchangeable here, Tab. 4a and 4 b includes the phase margin frequency calculated both in Hz and Radian per second.

| | LOG FREQ. | AMPLITUDE | LOG AMR | PHASE |
|---|---|---|---|---|
| FREQUENCY .1000000000 | -.9999978689 | 1.9190485642 | .2830853620 | -89.3146203418 |
| FREQUENCY .1400000000 | -.8538701447 | 1.5697301541 | .1958245887 | -93.1834325205 |
| FREQUENCY .1960000000 | -.7077424204 | 1.3524306054 | .1311147108 | -97.4240442783 |
| FREQUENCY .2744000000 | -.5616146961 | 1.2200181307 | .0863661007 | -102.4781426990 |
| FREQUENCY .3841600000 | -.4154869718 | 1.1339482961 | .0545931364 | -108.8566121182 |
| FREQUENCY .5378240000 | -.2693592476 | 1.0638671987 | .0268873616 | -117.1212255117 |
| FREQUENCY .7529536000 | -.1232315233 | .9852384252 | -.0064586449 | -127.8119891297 |
| FREQUENCY 1.0541350400 | .0228962010 | .8787046922 | -.0561569348 | -141.2586149639 |
| FREQUENCY 1.4757890560 | .1690239252 | .7352619787 | -.1335576067 | -157.2971480485 |
| FREQUENCY 2.0661046784 | .3151516495 | .5644312808 | -.2483883963 | -175.0835275478 |
| FREQUENCY 2.8925465498 | .4612793738 | .3929321399 | -.4056815821 | -193.2312790029 |
| FREQUENCY 4.0495651697 | .6074070980 | .2488060965 | -.6041376948 | -210.2272003156 |
| FREQUENCY 5.6693912375 | .7535348223 | .1455982578 | -.8368420382 | -224.9026059330 |
| FREQUENCY 7.9371477325 | .8996625466 | .0805117448 | -1.0941384299 | -236.7360825205 |
| FREQUENCY 11.1120068256 | 1.0457902708 | .0429514703 | -1.3670190516 | -245.8143324336 |
| FREQUENCY 15.5568095558 | 1.1919179951 | .0224425261 | -1.6489247469 | -252.5610011234 |
| FREQUENCY 21.7795333781 | 1.3380457194 | .0115938426 | -1.9357684742 | -257.4832355670 |
| FREQUENCY 30.4913467293 | 1.4841734436 | .0059534418 | -2.2252271649 | -261.0382894171 |
| FREQUENCY 42.6878854211 | 1.6303011679 | .0030475332 | -2.5160461972 | -263.5921965875 |

Tab.1-Progr.BODE results
Region A.

```
FREQUENCY        59.7630395895
      1.7764288922        .0015574989        -2.8075662786        -265.4217892597
FREQUENCY        83.6682554253
      1.9225566164        .0007953303        -3.0994458847        -266.7806104380
FREQUENCY       117.1355575954
      2.0686843407        .0004059601        -3.3915094088        -267.6662024478
 4240 CHARACTERS OUTPUT
```

Tab.1-continuation.




[*EOF*]

Fig.1 PHASE V LOGFREQ /Reg.A/

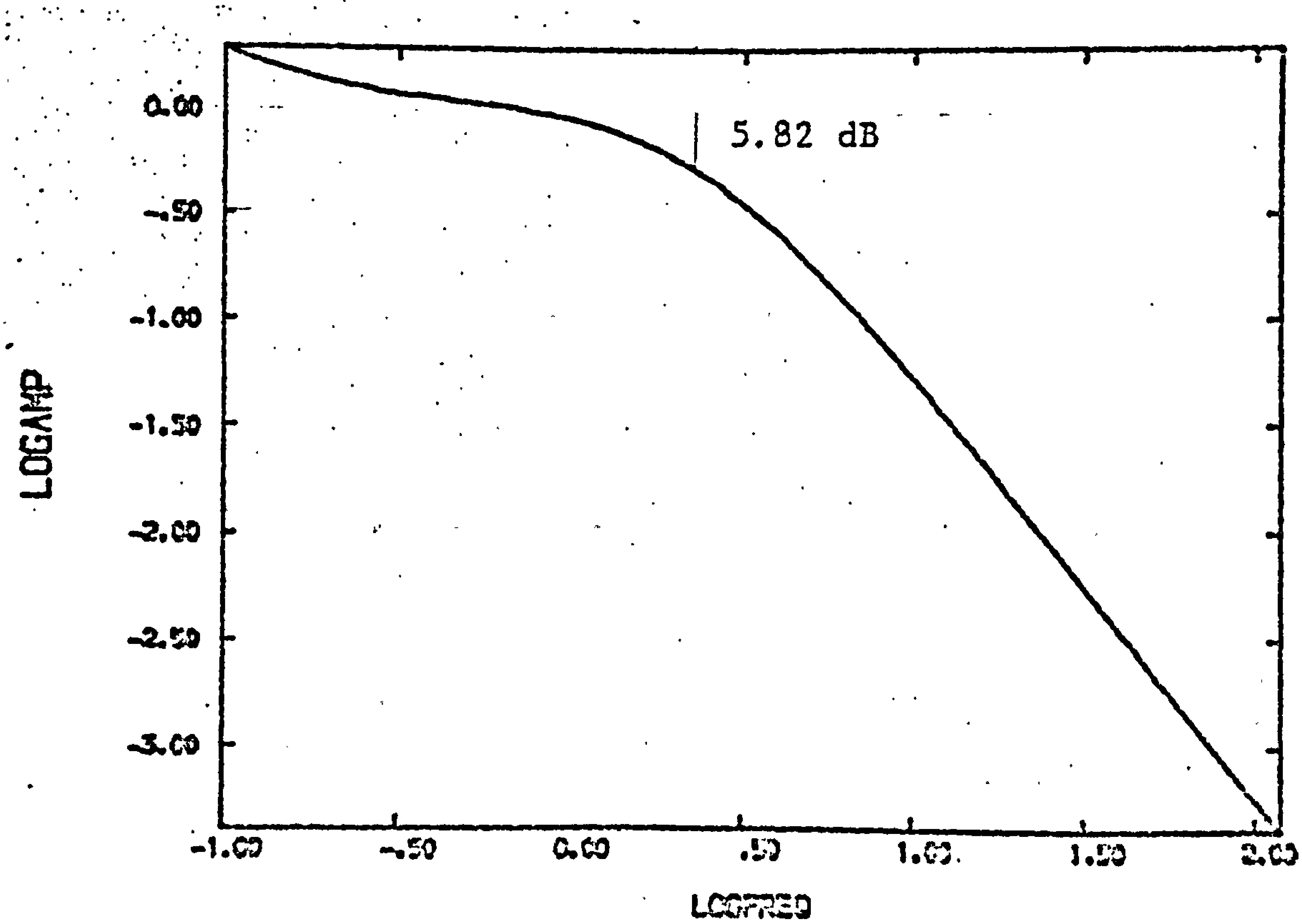


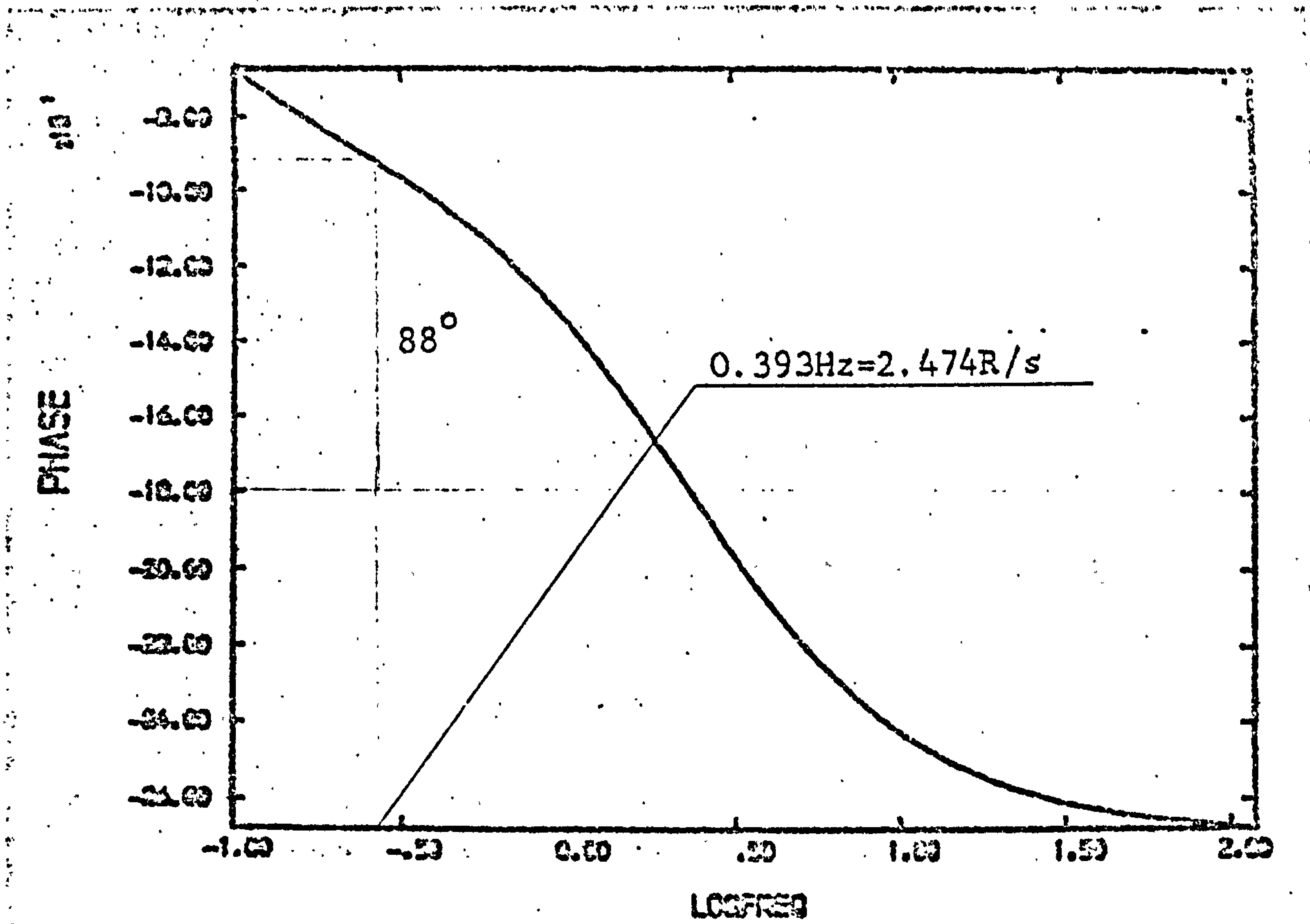

Fig.2 LOGAMP V LOGFREQ /Reg.A/

| | | LOG FREQ. | AMPLITUDE | LOG AMP | PHASE |
|---|---|---|---|---|---|
| FREQUENCY | .1000000000 | -.9999978689 | 1.5669366970 | .1950510360 | -67.2635883188 |
| FREQUENCY | .1400000000 | -.8538701447 | 1.2923389482 | .1113761958 | -76.6456951093 |
| FREQUENCY | .1960000000 | -.7077424204 | 1.0893922781 | .0371842133 | -85.2895698791 |
| FREQUENCY | .2744000000 | -.5616146961 | .9522553950 | -.0212465129 | -93.6751654783 |
| FREQUENCY | .3841600000 | -.4154869718 | .8611466136 | -.0649227686 | -102.5213536915 |
| FREQUENCY | .5378240000 | -.2693592476 | .7928820222 | -.1007912144 | -112.5784708187 |
| FREQUENCY | .7529536000 | -.1232315233 | .7260549704 | -.1390302008 | -124.5607061369 |
| FREQUENCY | 1.0541350400 | .0228962010 | .6435040467 | -.1914483007 | -138.9339060233 |
| FREQUENCY | 1.4757890560 | .1690239252 | .5366600262 | -.2703001764 | -155.6357782231 |
| FREQUENCY | 2.0661046784 | .3151516495 | .4112543274 | -.3858886969 | -173.8965197923 |
| FREQUENCY | 2.8925465498 | .4612793738 | .2860393230 | -.5435731001 | -192.3833014479 |
| FREQUENCY | 4.0495651697 | .6074070980 | .1810374347 | -.7422800311 | -209.6214601864 |
| FREQUENCY | 5.6693912375 | .7535348223 | .1059158047 | -.9750371519 | -224.4699191489 |
| FREQUENCY | 7.9371477325 | .8996625466 | .0585613810 | -1.2323860643 | -236.4270149691 |
| FREQUENCY | 11.1120068256 | 1.0457902708 | .0312394435 | -1.5052935040 | -245.5935678695 |
| FREQUENCY | 15.5568095558 | 1.1919179951 | .0163223732 | -1.7872128876 | -252.4033114102 |
| FREQUENCY | 21.7795333781 | 1.3380457194 | .0084320268 | -2.0740636002 | -257.3705997883 |
| FREQUENCY | 30.4913467293 | 1.4841734436 | .0043298127 | -2.3635258582 | -260.9578351914 |
| FREQUENCY | 42.6878854211 | 1.6303011679 | .0022163974 | -2.6543467061 | -263.5347292476 |

Tab. 2-Program BODE results
Region B.

```
FREQUENCY        59.7630395895
      1.7764288922          .0011327290        -2.9458677154      -265.3807411468
FREQUENCY        83.6682554253
      1.9225566164          .0005784227        -3.2377477950      -266.7012903526
FREQUENCY       117.1355575954
      2.0686843407          .0002952439        -3.5298115577      -267.6452595279
 4386 CHARACTERS OUTPUT
```

Tab.2-continuation.




[*EOF*] Fig.3 PHASE V LOGFREQ /Reg.B/

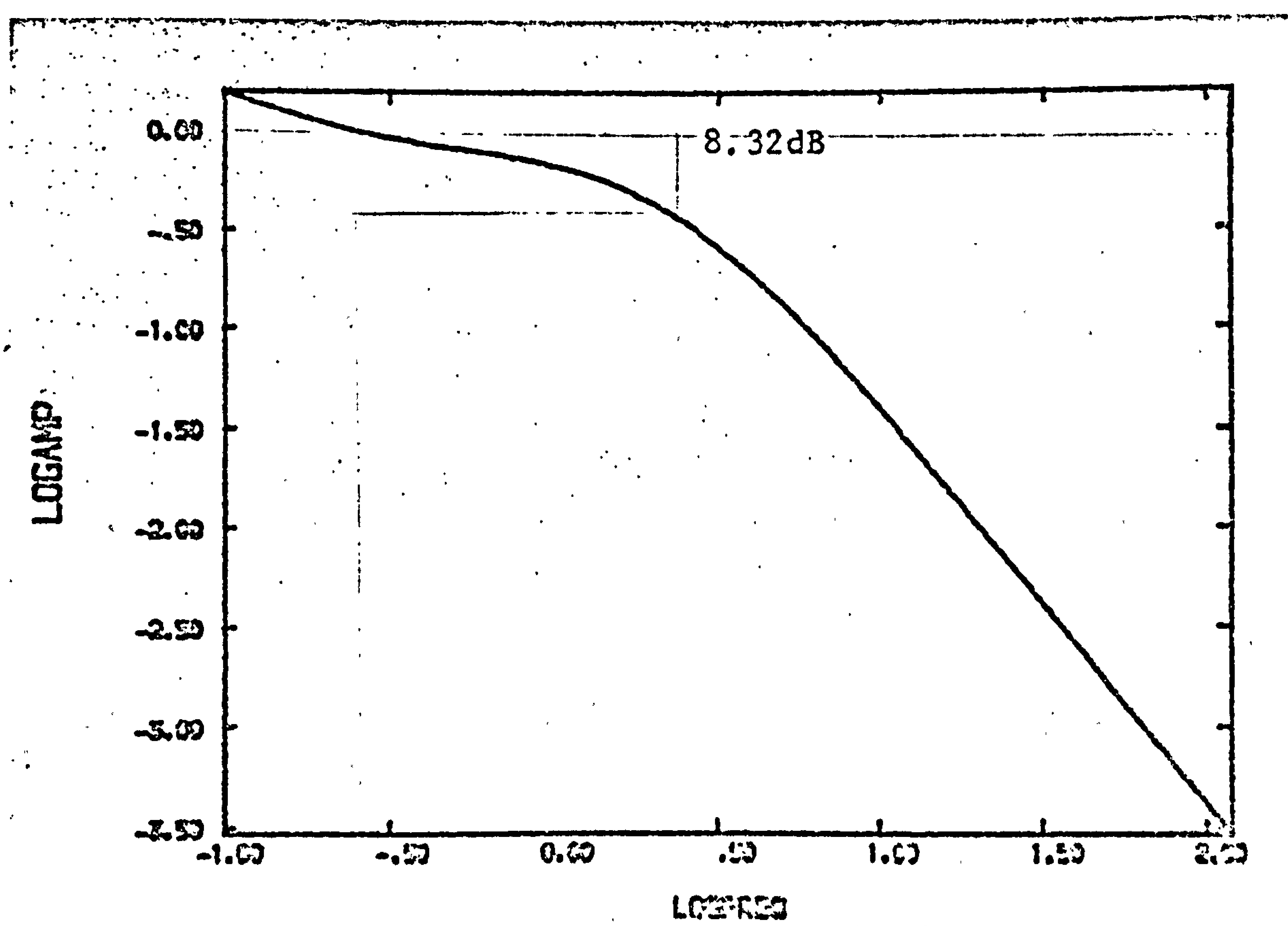



Fig.4 LOGAMP V LOGFREQ /Reg.B/

| | LOG FREQ. | AMPLITUDE | LOG AMB | PHASE |
|---|---|---|---|---|
| FREQUENCY | .1000000000 | | | |
| | -.9999978689 | 1.5427077919 | .1882832718 | -80.0063649520 |
| FREQUENCY | .1400000000 | | | |
| | -.8538701447 | 1.1271352950 | .0519759387 | -86.4254853208 |
| FREQUENCY | .1960000000 | | | |
| | -.7077424204 | .8203860079 | -.0859815721 | -92.5412998678 |
| FREQUENCY | .2744000000 | | | |
| | -.5616146961 | .5993592216 | -.2223123346 | -98.9733425963 |
| FREQUENCY | .3841600000 | | | |
| | -.4154869718 | .4428409129 | -.3537515088 | -106.3468852274 |
| FREQUENCY | .5378240000 | | | |
| | -.2693592476 | .3332069480 | -.4772849341 | -115.3262574082 |
| FREQUENCY | .7529536000 | | | |
| | -.1232315233 | .2558090220 | -.5920828809 | -126.5290266495 |
| FREQUENCY | 1.0541350400 | | | |
| | .0228962010 | .1982600565 | -.7027632767 | -140.3419058175 |
| FREQUENCY | 1.4757890560 | | | |
| | .1690239252 | .1511332996 | -.8206380871 | -156.6422437111 |
| FREQUENCY | 2.0661046784 | | | |
| | .3151516495 | .1096956896 | -.9598083918 | -174.6156978765 |
| FREQUENCY | 2.8925465498 | | | |
| | .4612793738 | .0740011277 | -1.1307592522 | -192.8971000584 |
| FREQUENCY | 4.0495651697 | | | |
| | .6074070980 | .0460723294 | -1.3365569815 | -209.9884956415 |
| FREQUENCY | 5.6693912375 | | | |
| | .7535348223 | .0267229465 | -1.5731123057 | -224.7321006161 |
| FREQUENCY | 7.9371477325 | | | |
| | .8996625466 | .0147093687 | -1.8324020615 | -236.6142922875 |
| FREQUENCY | 11.1120068256 | | | |
| | 1.0457902708 | .0078286739 | -2.1063073005 | -245.7273391472 |
| FREQUENCY | 15.5568095558 | | | |
| | 1.1919179951 | .0040856121 | -2.3887377750 | -252.4988629659 |
| FREQUENCY | 21.7795333781 | | | |
| | 1.3380457194 | .0021093300 | -2.6758497664 | -257.4388511339 |
| FREQUENCY | 30.4913467293 | | | |
| | 1.4841734436 | .0010827999 | -2.9654454683 | -261.0065862379 |
| FREQUENCY | 42.6878854211 | | | |
| | 1.6303011679 | .0005541899 | -3.2563344324 | -263.5695514548 |

Tab. 3-Program BODE results
Region C

```
FREQUENCY        59.7630395895
       1.7764288922          .0002832059         -3.5478802062      -265.4056141633
FREQUENCY        83.6682554253
       1.9225566164          .0001446118         -3.8397880235      -266.7190567971
FREQUENCY       117.1355575954
       2.0686843407          .0000738126         -4.1318608373      -267.6579498469
 4482 CHARACTERS OUTPUT
```

Tab. 3-continuation.

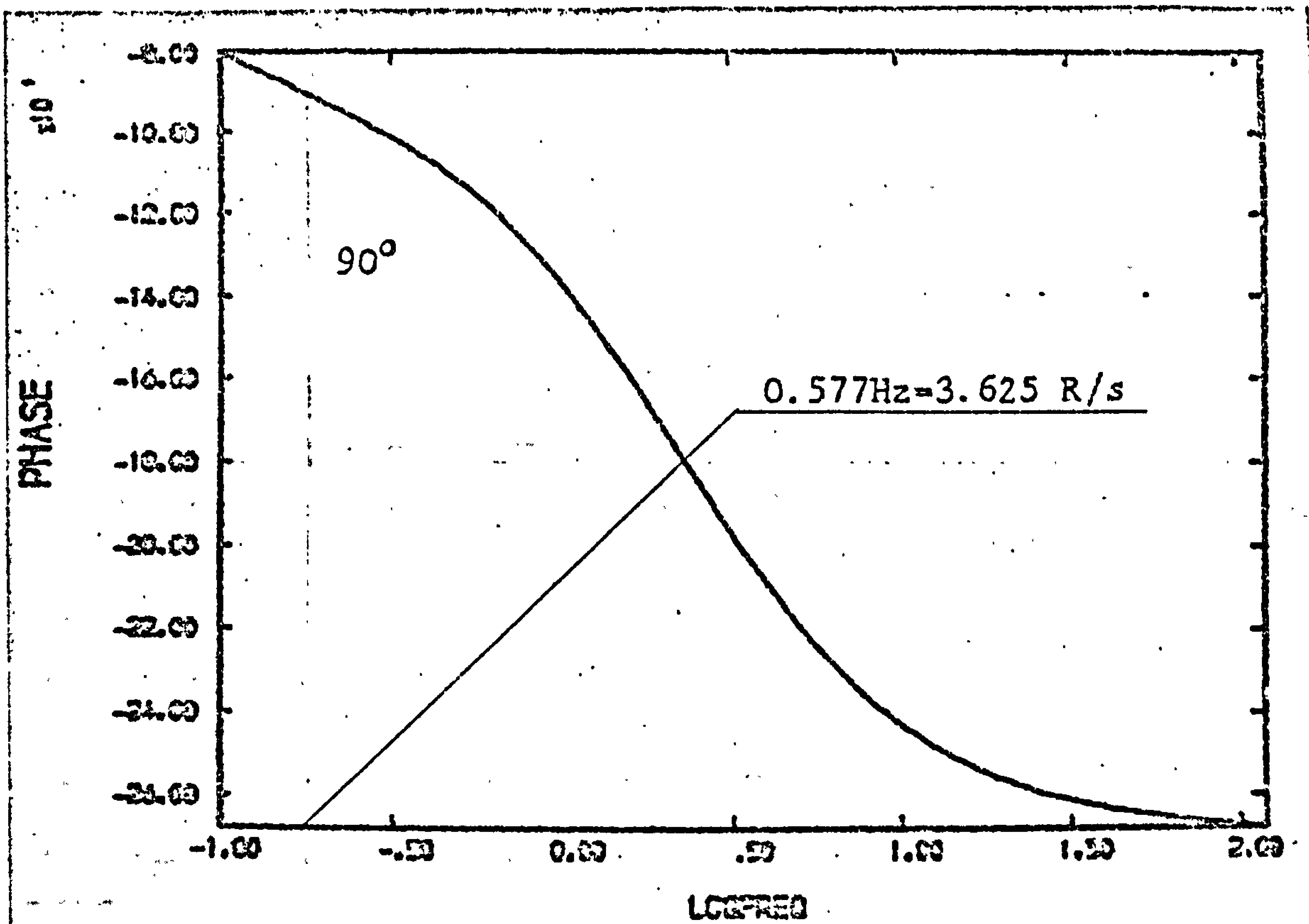



Fig.5 PHASE V LOGFREQ /Reg.C/

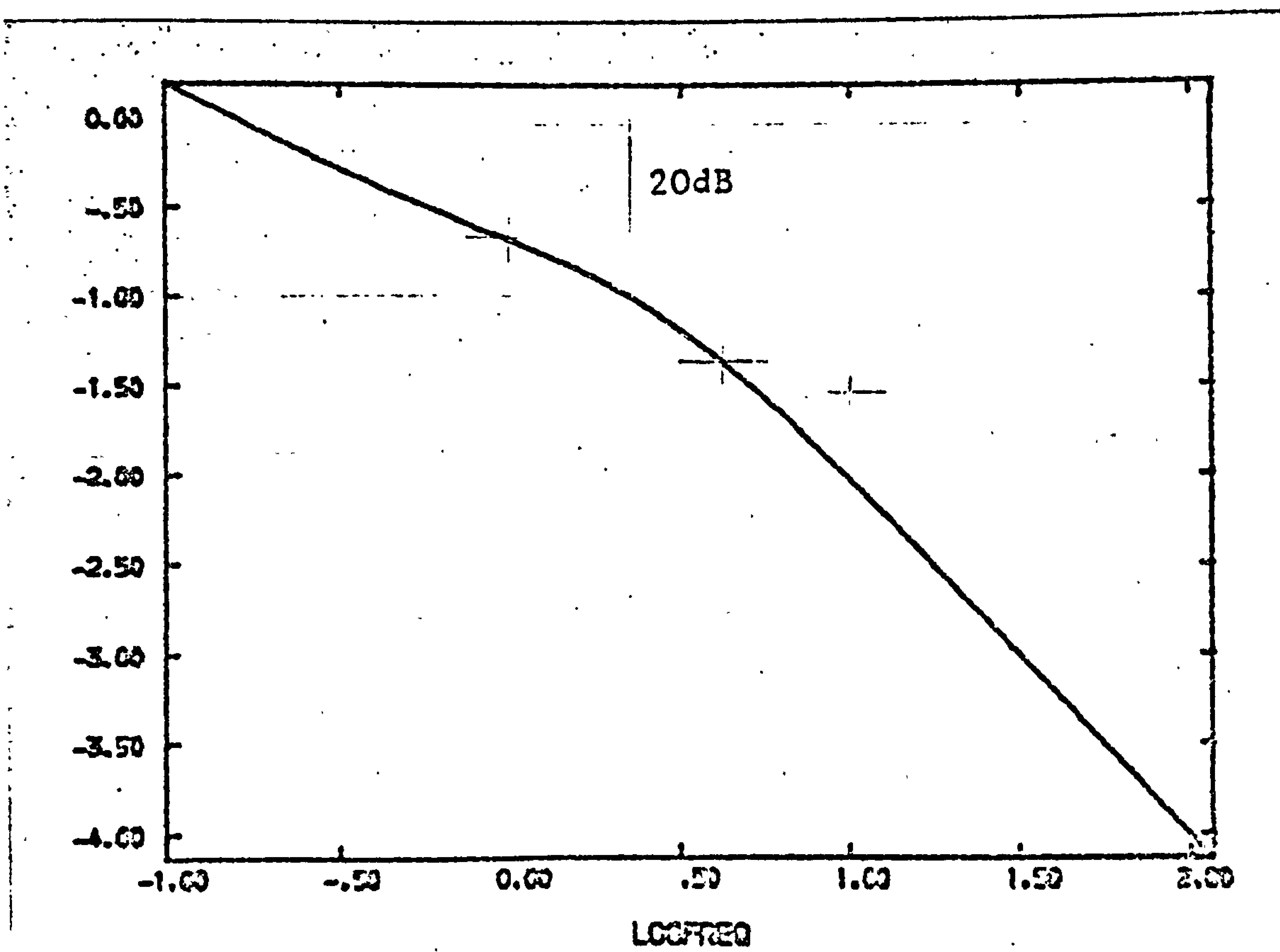



Fig.6 LOGAMP V LOGFREQ /Reg.C/

| UNCOMPENSATED SYSTEM | | | |
|---|---|---|---|
| | Reg 1 $(B_1)$ $B_1 = 0.067$ | Reg 2 $(B_2)$ $B_2 = 0.336$ | Reg 3 $(B_3)$ $B_3 = 0.173$ |
| Transfer Function $G_n(s)$ | $G_1(s) = \frac{200.}{s^3 + 30.067s^2 + 202.01s + 13.4}$ | $G_2(s) = \frac{200.}{s^3 + 30.336s^2 + 210.08s + 67.2}$ | $G_3(s) = \frac{200.}{s^3 + 30.173s^2 + 205.19s + 34.6}$ |
| Phase Margin Frequency $f$ [Hz] $\omega$ $\left[\frac{Radian}{sec}\right]$ | $f = 0.173$ [Hz] $\omega = 1.0913$ $\left[\frac{Radian}{sec}\right]$ | $f = 0.126$ [Hz] $\omega = 0.791$ $\left[\frac{Radian}{sec}\right]$ | $f = 0.161$ [Hz] $\omega = 1.014$ $\left[\frac{Radian}{sec}\right]$ |
| Phase Margin $\Theta_n$ | $\Theta_1 = 85$ [°] | $\Theta_2 = 95$ [°] | $\Theta_3 = 91$ [°] |
| Gain Margin $K_m$ | $K_1 = 27.88$ [dB] | $K_2 = 29.0$ [dB] | $K_3 = 28.1$ [dB] |

**Tab.4a** Summary of uncompensated system.

| COMPENSATED SYSTEM | | | |
|---|---|---|---|
| | Reg 1 $(B_1, T_{d1})$ $B_1 = 0.067$ $T_{d1} = 1.1$ | Reg 2 $(B_2, T_{d2})$ $B_2 = 0.336$ $T_{d2} = 0.8$ | Reg 3 $(B_3, T_{d3})$ $B_3 = 0.173$ $T_{d3} = 0.2$ |
| Transfer Function $G'_n(s)$ | $G'_1(s) = \frac{220s + 200.}{s^3 + 30.067s^2 + 202.01s + 13.4}$ | $G'_2(s) = \frac{160s + 200.}{s^3 + 30.336s^2 + 210.08s + 67.2}$ | $G'_3(s) = \frac{40s + 200.}{s^3 + 30.173s^2 + 205.19s + 34.6}$ |
| Phase Margin Frequency $f'$ [Hz] $\omega'$ $\left[\frac{Radian}{sec}\right]$ | $f' = 0.779$ [Hz] $\omega' = 4.88$ $\left[\frac{Radian}{sec}\right]$ | $f' = 0.393$ [Hz] $\omega' = 2.474$ $\left[\frac{Radian}{sec}\right]$ | $f' = 0.577$ [Hz] $\omega' = 3.625$ $\left[\frac{Radian}{sec}\right]$ |
| Phase Margin $\Theta'_n$ | $\Theta'_1 = 48$ [°] | $\Theta'_2 = 88$ [°] | $\Theta'_3 = 90$ [°] |
| Gain Margin $K'_m$ | $K'_1 = 5.82$ [dB] | $K'_2 = 8.32$ [dB] | $K'_3 = 20.0$ [dB] |

**Tab.4b** Summary of compensated system.

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## 7.5. DISCUSSION

In this chapter derivation of control system requirements for the earlier program PCS are made and implemented to the practical model called DEC (Derivative Error Compensation System). Selected PCS program results are also discussed in the form of an engine's dynamic characteristics, which show the generally unsatisfactory speed transient response. The compensator (error derivative type) is inserted into the feed-forward control path, in order to modify the error signal. Both practical (effective) and theoretical aspects of the phase-lead characteristics in this type of compensation are discussed, and a fast acceleration rate is achieved within a range of system stability. For the evaluation of stability factors, previously introduced Program BODE is used. A fast transient response was obtained in a modified model DEC Program, when compared with the original PCS Program and this was achieved by a substantial increase of the specific phase margin frequency i.e. in the first speed region from 1.0913 (radian/sec) to 4.88 (radian/sec), in the second from 0.791 (radian/sec) to 2.474 (radian/sec) and finally from 1.014 (radian/sec) to about 3.625 (raidan/sec). The overall improvement in transient response leads to some reduction in PHASE/GAIN margins, calculated in the BODE program, but these parameters still remain safely within the stability margin. A suitable design of Error Derivative Compensator has been theoretically considered and shown to be practically effective.

# 8. COMPARISON OF PERFORMANCE AND ANALYSIS OF COMPENSATED AND UNCOMPENSATED SYSTEMS

## 8.1. ROOT-LOCUS PATTERN FOR UNCOMPENSATED SYSTEM

The root-locus method is another useful analytical technique for determining the behaviour of linear control systems in a time domain, and is generally regarded as complementary to BODE diagrams considered earlier in this thesis. This method is particularly helpful in the determination of compensation network structures by trial and error procedure, and also overall system stability. As shown by D. Bell (Lit.1.), this procedure can then be followed by a simple optimal design technique in which values are assigned to the parameters of this compensating network. Being limited in this work to the three linearised region of the engine's steady-state characteristics, the root-locus method was introduced to the first region by a step-by-step plot of the root-loci, and consequently all three regions' root-locus patterns were analysed automatically by a specially provided FORTRAN program RLT. Let us now consider the first of the indicated features applied for an uncompensated system - PCS and represented by an open-loop transfer function:

$$G(s) = G(jw) = \frac{K}{(s+B)(s+10.)(s+20.)} \qquad \text{Let: } K = 200, \quad B = B_1 = 0.067$$

The preceding notes can be expressed as rules for constructing root-locus patterns :

1. Number of Root-Loci - equal to number of poles

As number of poles $m = 3$,

Number of Zeros $n = 0$

There are three poles at: $s = -B = -0.067$

$s = -10.0$

$s = -20.0$

2. Root-Loci Along the Real Axis (See Fig.1.)

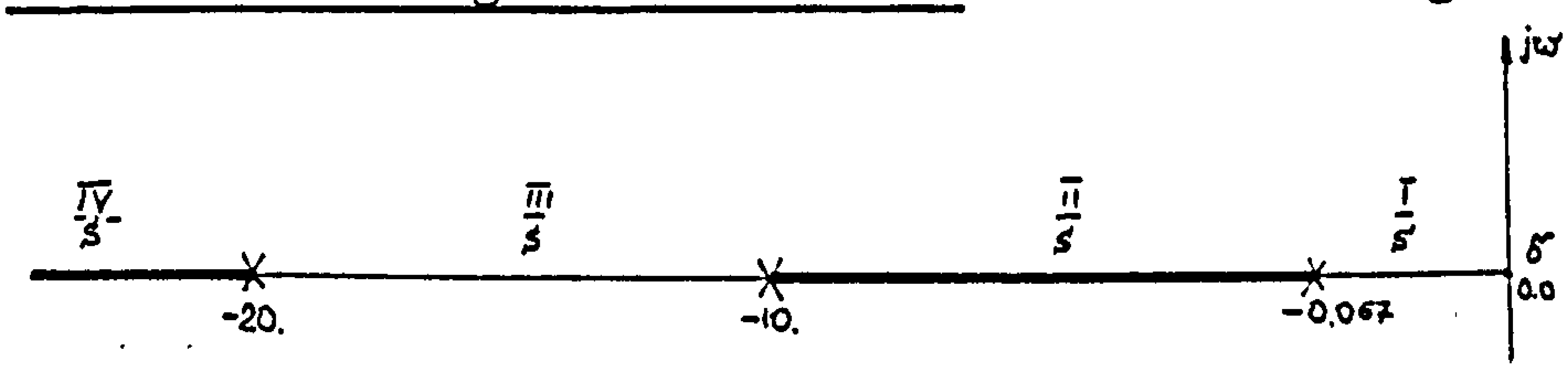


Fig.1. Points s(II) and s(IV) are on root-loci.

3. Break-Away from the Real Axis

There is a breakaway point between the poles $s = -0.067$ and $s = -10.0$ calculated by differentiating the open-loop transfer function with respect to s and equating to zero.

$$\left.\frac{d\,/G(jw)/}{d(jw)}\right|_{jw = s} = 0. \qquad \text{therefore:}$$

$s^3+20s^2+10s^2+200s+0.067s^2+1.34s+0.67s+13.4$ or

$s^3+30.067s^2+202.01s+13.4$ differentiating we have

$3s^2+60.134s+202.01 = 0.$

$\Delta = b^2 - 4ac = 3616.09 - 4(606.03) = 1191.97$

$\sqrt{\Delta} = \pm\ 34.52$

$$s_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-60.134-34.52}{6} = -15.775$$

$$s_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-60.134+34.52}{6} = -4.269$$

$s = s_1$ is not a possible break-away point because it does not lie on a root-locus.
The break-away point is $\underline{s = s_2 = -4.269}$

4. Asymptotes to the Root-loci at Infinity

There are three non-finite zeros and hence three asymptotes. The angles of the asymptotes to the real axis are given by:

$$\Theta = \frac{180^\circ + N.360^\circ}{n - m} \qquad \text{where: } n-m = -3, \quad N = 1,2,3$$

$$\Theta = 60^\circ \ \& \ 180^\circ \ \& \ 300^\circ$$

5. Intersection of the Asymptotes at the Centroid

The centroid is placed on the real axis with the position given by :

$$d = \frac{\sum_{i=1}^{i=n}\text{poles} - \sum_{i=1}^{i=m}\text{zeros}}{m-n}$$

Where : $n-m = -3.$

$m-n = 3.$

$\Sigma$ Poles $\simeq 30.067 \qquad d \simeq 10.0$

$\sim 30.$

6. Intersection of the Root-Locus with the Imaginary Axis

This is found by substituting jw for s in the system transfer function of G(jw) and then equating the imaginery parts to zero.

$$200(-30.067W^2-j(202.01W-W^3)) = 0$$
$$202.01W - W3 = 0$$
$$W(202.01 - W^3) = 0$$
$$W = \pm j\ 14.21$$

A complete root-locus pattern for this control system is shown in Fig.2. The shape of the pattern was sketched in with the available points, however, further points can be checked by reading them off the sketch and using the earlier presented theory in order to verify whether they are accurate and on the root-locus. Summing up, the root-locus pattern is bent into the right half of the S-plane and for a gain constant greater than $\pm$ j 14.2 points the system is unstable. In order to analyse the root-locus pattern, and simplify the effort of classical plotting of the root-loci, a specially developed RLT computer program offers substantial help. Program RLT originates from a Root-Locus Plot program written by J. Galvez (see Lit.2), however, certain small changes were made and a new improved plotting routine is provided. Program RLT contains approximately 94 lines of FORTRAN source program (excluding Comments Lines) and its plotting routine (subroutine PLT) uses an additional 67 lines of print-out space.

This program is carefully described in Lit.2 and it is quite straightforward to use due to the abundance of comment statements throughout. The overall accuracy of this program may easily be verified "manually" or compared with the original discussion of this method by Kuo (See Lit.3). The program listing including subroutine PLT is given as Program No.1. and tabulated print-outs for all three regions are presented in Tab.1, Tab.2, Tab.3, with the corresponding computer plots of root-locus pattern shown in Fig.3, Fig.4, Fig.5. On top of each region's tabulation of results, the transfer function is presented and the tabulations include a maximum of five complex roots of the system where each root is split into two : REAL and IMAGINERY parts, calculated


**Fig. 2** Root-Locus pattern for uncompensated system.

individually for any given value of gain K. Naturally an increment of K and its maximum value are externally adjusted in the input data deck and since the change in the values of the roots of the polynomial are very small, each time K is changed during program execution, the previous roots are used as initial approximations only, in order to obtain new roots. Convergence to the new roots is therefore quite rapid (second order) and usually takes less than four iterations to obtain the desired degree of accuracy.

```
      PROGRAM RLT (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION CI(6),A(6),B(6),AR(6),AI(6),BR(6),BI(6),CR(6),RR(5,51),
     1RI(5,51),IDD(6)
C
C     INITIALIZE THE OUTPUT MATRIX.
C
    1 DO 20 I=1,5
      DO 20 J=1,50
      RR(I,J)=0.0
   20 RI(I,J)=1.0
      WRITE(6,9999)
 9999 FORMAT(1H0)
C
C
C     INPUT ALL PARAMETERS.
C
C
      READ(5,2000)NA
      IF(NA)190,190,30
   30 NDEG=NA
      NA=NA+1
      READ(5,1000) (A(I),I=1,NA)
      READ(5,2000)NB
      NB=NB+1
      READ(5,1000)(B(I),I=1,NB)
      READ(5,1000)XKINC
      READ(5,1000)XHIGH,YHIGH
      WRITE(6,6000)
      NN=NB-1
      DO 35 I=1,NN
   35 IDD(I)=NB-I
      WRITE(6,3100) (B(I),IDD(I),I=1,NB)
      WRITE(6,3200)
      NN=NA-1
      DO 36 I=1,NN
   36 IDD(I)=NA-I
      WRITE(6,3100)(A(I),IDD(I),I=1,NA)
      WRITE(6,4000)
      NDIFF=NA-NB
      NDIF1=NDIFF+1
C
C     COMPUTER ACCURACY REQUIRED,IT WILL BE ONE PART IN ONE THOUSANDTHS
C
C
C
C     COMPUTATIONS START HERE FOR 50 POINTS ON ROOT LOCUS OF EACH ROOT.
C
C
      DELTA=ABS(XHIGH)
      IF(DELTA-ABS(YHIGH))50,50,40
   40 DELTA=ABS(YHIGH)
   50 DELTA=DELTA/100000.
      DO 160 IN=1,50
      XK=(IN-1)*XKINC
C
C     ROOT LOCUS PROGRAM FINDS ROOT LOCUS WITHIN A SPECIFIC REGION.
C
      N=NDEG
      N1=NDEG+1
C
C     COMPUTE THE POLYNOMIAL K*V(S)
C
      DO 60 I=1,NDIFF
      AR(I)=A(I)
   60 AI(I)=0.0
      DO 70 I=NDIF1,NA
      K=I-NDIF1+1
      AR(I)=A(I)+XK*B(K)
   70 AI(I)=0.0
```

Progr.1-PROGRAM RLT.

```
C     USE SYNTHETIC DIVISION TO EVALUATE POLYNOMIAL
C     BY USING THIS WE CAN AUTOMATICALLY GET THE REDUCED POLYNONIAL.
C
```


```
      BR(1)=AR(1)
      BI(1)=AI(1)
      CR(1)=AR(1)
      CI(1)=AI(1)
      DO 150 NROOT=1,NDEG
      C=RR(NROOT,IN)
      D=RI(NROOT,IN)
C
C     WE NOW USE NEWTON'S METHOD TO GET A BETTER APPROXIMATION
C     WE USE IT FOR A MAXIMUM OF 200 TIMES.
      DO 110 KK=1,200
      DO 80 I=2,N1
      BR(I)=AR(I)+BR(I-1)*C-BI(I-1)*D
   80 BI(I)=AI(I)+BR(I-1)*D+BI(I-1)*C
      DO 90 I=2,N
      CR(I)=BR(I)+CR(I-1)*C+CI(I-1)*D
   90 CI(I)=BI(I)+CR(I-1)*D+CI(I-1)*C
C
C     COMPUTE NEW ROOTS AND CHECK IF ACCURACY IS ACHIEVED.
C
      DEN=CR(N)*CR(N)+CI(N)*CI(N)
      X1=-(BR(N1)*CR(N)+BI(N1)*CI(N))/DEN
      Y1=+(BR(N1)*CI(N)-BI(N1)*CR(N))/DEN
      C=C+X1
      D=D+Y1
      IF(ABS(X1)+ABS(Y1)-DELTA)130,130,110
  110 CONTINUE
C
C     IT GETS HERE IF NO CONVERGENCE AFTER 200 TRIES
C
      WRITE(6,120)XK
  120 FORMAT(F11.1,10X,'DOES NOT CONVERGE TRYING DIFFERENT K VALUE')
      GO TO 160
  130 N1=N1-1
C
C     USE THE REDUCED POLYNOMIAL PRODUCED BY SYNTHETIC DIVISION.
C
C
C     SAVE THIS ROOT FOR NEXT APPROXIMATION.
C
      N=N-1
      DO 140 I=2,N1
      AR(I)=BR(I)
  140 AI(I)=BI(I)
      RR(NROOT,IN)=C
      RI(NROOT,IN)=D
      RR(NROOT,IN+1)=C
      RI(NROOT,IN+1)=D
  150 CONTINUE
  160 WRITE(6,5000)XK,(RR(I,IN),RI(I,IN),I=1,NDEG)
      WRITE(6,6000)
 6000 FORMAT('1')
      CALL PLT(NDEG,IN,RR,RI)
      GO TO 1
  190 CALL EXIT
 1000 FORMAT(8F10.0)
 2000 FORMAT(I2)
 3100 FORMAT(11X,5(F7.3,'*S**',I2,' + '))
 3200 FORMAT(' F(S) = ',79('-'))
 4000 FORMAT(8X,'K',14X,5(8X,'ROOT',8X)/,25X,5(' REAL   IMAGINARY  ')/
 5000 FORMAT(F11.1,10X,10F10.3)
      STOP
      END
```

Progr.1-continuation.

```
      SUBROUTINE PLT(N1,N2,X,Y)
      LOGICAL Q1,Q2
      DIMENSION PLOT(103,61),VAL(11),X(5,51),Y(5,51),YMARK(7)
      DATA BLANK,SIGNUM,DOT,EYE,ANEG,STAR/' ','+','.','I','_','*'/
      XMIN=1E90
      XMAX=-1E90
      YMIN=1E90
      YMAX=-1E90
      DO 500 I=1,N1
      DO 500 J=1,N2
  501 CONTINUE
      IF(XMIN.GT.X(I,J)) XMIN=X(I,J)
      IF(XMAX.LT.X(I,J)) XMAX=X(I,J)
      IF(YMIN.GT.Y(I,J)) YMIN=Y(I,J)
      IF(YMAX.LT.Y(I,J)) YMAX=Y(I,J)
  500 CONTINUE
      YS=YMAX-YMIN
      XS=XMAX-XMIN
      WRITE(6,350)
  350 FORMAT(///50X,'ROOT LOCUS PLOT'/)
      DO 10 I=1,11
   10 VAL(I)=XMIN+(I-1)*XS/10.
      WRITE(6,100) (VAL(I),I=1,11)
      WRITE(6,101)
      DO 50 I=2,102
      DO 50 J=1,61
   50 PLOT(I,J)=BLANK
      Q1=YMAX.GT.0.
      Q2=YMIN.LT.0.
      IF(Q1.AND.Q2) IXAXIS=-YMIN/YS*60.+1.5
      IF(.NOT.Q1) IXAXIS=1
      IF(.NOT.Q2) IXAXIS=61
      Q1=XMAX.GT.0.
      Q2=XMIN.LT.0.
      IF(Q1.AND.Q2) IYAXIS=-XMIN/XS*101.+2.5
      IF(.NOT.Q1) IYAXIS=102
      IF(.NOT.Q2) IYAXIS=2
      DO 104 I=2,102
  104 PLOT(I,IXAXIS)=ANEG
      DO 105 I=1,61
  105 PLOT(IYAXIS,I)=EYE
      DO 110 I=1,7
  110 YMARK(I)=FLOAT(I-1)/6.*YS+YMIN
      DO 201 I=1,61
      PLOT(1,I)=STAR
  201 PLOT(103,I)=STAR
      DO 200 I=1,61,10
      PLOT(1,I)=SIGNUM
  200 PLOT(103,I)=SIGNUM
      DO 300 I=1,N1
      DO 300 J=1,N2
      IX=(X(I,J)-XMIN)/XS*101.+2.5
      IY=(Y(I,J)-YMIN)/YS*60.+1.5
  300 PLOT(IX,62-IY)=DOT
      DO 400 I=1,6
      IX=(I-1)*10+1
      WRITE(6,405) YMARK(I),(PLOT(J,IX),J=1,103),YMARK(I)
  400 WRITE(6,410) ((PLOT(J,K),J=1,103),K=IX+1,IX+9)
       WRITE(6,405) YMARK(7),(PLOT(J,61),J=1,103),YMARK(7)
  405 FORMAT(6X,1PE11.4,103A1,1PE11. )
  410 FORMAT(17X,103A1)
      WRITE(6,101)
      WRITE(6,100) (VAL(I),I=1,11)
  100 FORMAT(12X,11(1PE9.2,1X))
  101 FORMAT(17X,'*',10('+*********'),'+*')
      RETURN
      END
```

Progr.1-continuation.

$$F(S) = \frac{1.000 \ast S^{\ast\ast}\ 0\ +}{1.000 \ast S^{\ast\ast}\ 3\ +\ 30.067 \ast S^{\ast\ast}\ 2\ +\ 202.010 \ast S^{\ast\ast}\ 1\ +\ 13.400 \ast S^{\ast\ast}\ X\ +}$$

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.067 | .000 | -10.000 | .000 | -20.000 | -.000 |
| 150.0 | -.935 | .000 | -8.451 | -.000 | -20.681 | .000 |
| 300.0 | -2.246 | .000 | -6.563 | -.000 | -21.258 | -.000 |
| 450.0 | -21.763 | -.000 | -4.152 | -.014 | -4.152 | 2.013 |
| 600.0 | -22.217 | .000 | -3.925 | -3.494 | -3.925 | 3.494 |
| 750.0 | -22.631 | -.000 | -3.718 | -4.462 | -3.718 | 4.462 |
| 900.0 | -23.014 | .000 | -3.527 | -5.220 | -3.527 | 5.220 |
| 1050.0 | -23.370 | -.000 | -3.348 | -5.856 | -3.349 | 5.856 |
| 1200.0 | -23.704 | .000 | -3.181 | -6.408 | -3.181 | 6.408 |
| 1350.0 | -24.020 | -.000 | -3.023 | -6.901 | -3.024 | 6.901 |
| 1500.0 | -24.319 | .000 | -2.874 | -7.347 | -2.874 | 7.347 |
| 1650.0 | -24.604 | -.000 | -2.731 | -7.755 | -2.731 | 7.755 |
| 1800.0 | -24.877 | .000 | -2.595 | -8.134 | -2.595 | 8.134 |
| 1950.0 | -25.138 | -.000 | -2.464 | -8.487 | -2.465 | 8.487 |
| 2100.0 | -25.389 | .000 | -2.339 | -8.819 | -2.339 | 8.819 |
| 2250.0 | -25.631 | -.000 | -2.218 | -9.132 | -2.218 | 9.132 |
| 2400.0 | -25.864 | .000 | -2.101 | -9.428 | -2.101 | 9.428 |
| 2550.0 | -26.090 | -.000 | -1.988 | -9.711 | -1.988 | 9.711 |
| 2700.0 | -26.309 | .000 | -1.879 | -9.980 | -1.879 | 9.980 |
| 2850.0 | -26.521 | -.000 | -1.773 | -10.238 | -1.773 | 10.238 |
| 3000.0 | -26.727 | .000 | -1.670 | -10.486 | -1.670 | 10.486 |
| 3150.0 | -26.928 | -.000 | -1.570 | -10.724 | -1.570 | 10.724 |
| 3300.0 | -27.123 | .000 | -1.472 | -10.954 | -1.472 | 10.954 |
| 3450.0 | -27.313 | -.000 | -1.377 | -11.176 | -1.377 | 11.176 |
| 3600.0 | -27.499 | .000 | -1.284 | -11.391 | -1.284 | 11.391 |
| 3750.0 | -27.681 | -.000 | -1.193 | -11.599 | -1.193 | 11.599 |
| 3900.0 | -27.858 | .000 | -1.104 | -11.801 | -1.104 | 11.801 |
| 4050.0 | -28.032 | 0 | -1.018 | -11.997 | -1.018 | 11.997 |
| 4200.0 | -28.202 | 0 | -.933 | -12.187 | -.933 | 12.187 |
| 4350.0 | -28.368 | 0 | -.849 | -12.373 | -.849 | 12.373 |
| 4500.0 | -28.531 | 0 | -.768 | -12.554 | -.768 | 12.554 |
| 4650.0 | -28.691 | 0 | -.688 | -12.730 | -.688 | 12.730 |
| 4800.0 | -28.848 | 0 | -.609 | -12.903 | -.609 | 12.903 |
| 4950.0 | -29.003 | 0 | -.532 | -13.071 | -.532 | 13.071 |
| 5100.0 | -29.154 | 0 | -.456 | -13.236 | -.456 | 13.236 |
| 5250.0 | -29.303 | 0 | -.382 | -13.397 | -.382 | 13.397 |
| 5400.0 | -29.449 | 0 | -.309 | -13.555 | -.309 | 13.555 |
| 5550.0 | -29.593 | 0 | -.237 | -13.709 | -.237 | 13.709 |
| 5700.0 | -29.735 | 0 | -.166 | -13.861 | -.166 | 13.861 |
| 5850.0 | -29.875 | 0 | -.096 | -14.009 | -.096 | 14.009 |
| 6000.0 | -30.012 | 0 | -.027 | -14.155 | -.027 | 14.155 |
| 6150.0 | -30.148 | 0 | .040 | -14.298 | .040 | 14.298 |
| 6300.0 | -30.281 | 0 | .107 | -14.439 | .107 | 14.439 |
| 6450.0 | -30.413 | 0 | .173 | -14.577 | .173 | 14.577 |
| 6600.0 | -30.542 | 0 | .238 | -14.713 | .238 | 14.713 |
| 6750.0 | -30.670 | 0 | .302 | -14.847 | .302 | 14.847 |
| 6900.0 | -30.797 | 0 | .365 | -14.978 | .365 | 14.978 |
| 7050.0 | -30.921 | 0 | .427 | -15.108 | .427 | 15.108 |
| 7200.0 | -31.045 | 0 | .489 | -15.235 | .489 | 15.235 |
| 7350.0 | -31.166 | 0 | .550 | -15.361 | .550 | 15.361 |

Tab.1 Program RLT/Reg.A/-results.

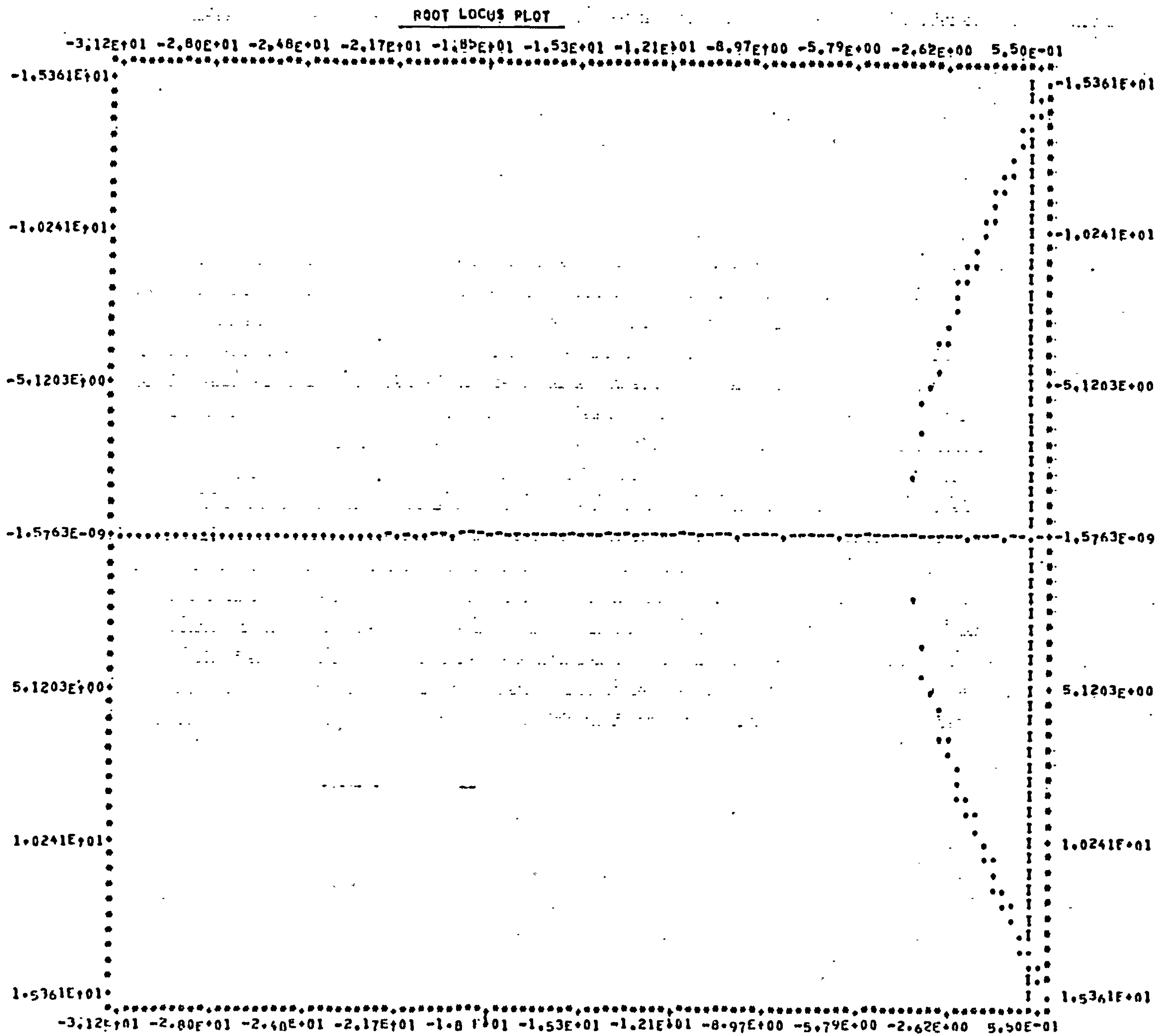


Fig.3 RL Plot/Reg.A/

F(S) = 1.000*S** 0 + / 1.000*S** 3 + 30.336*S** 2 + 210.080*S** 1 + 67.200*S** X +

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.336 | -.000 | -10.000 | -.000 | -20.000 | -.000 |
| 150.0 | -1.250 | -.000 | -8.396 | .000 | -20.689 | -.000 |
| 300.0 | -2.721 | -.000 | -6.343 | -.000 | -21.271 | .000 |
| 450.0 | -21.781 | -.000 | -4.277 | -2.334 | -4.277 | 2.334 |
| 600.0 | -22.238 | .000 | -4.049 | -3.689 | -4.049 | 3.689 |
| 750.0 | -22.655 | -.000 | -3.840 | -4.618 | -3.840 | 4.618 |
| 900.0 | -23.040 | .000 | -3.648 | -5.355 | -3.648 | 5.355 |
| 1050.0 | -23.398 | -.000 | -3.469 | -5.976 | -3.469 | 5.976 |
| 1200.0 | -23.734 | .000 | -3.301 | -6.519 | -3.301 | 6.519 |
| 1350.0 | -24.051 | -.000 | -3.142 | -7.004 | -3.142 | 7.004 |
| 1500.0 | -24.352 | .000 | -2.992 | -7.443 | -2.992 | 7.443 |
| 1650.0 | -24.638 | -.000 | -2.849 | -7.847 | -2.849 | 7.847 |
| 1800.0 | -24.912 | .000 | -2.712 | -8.222 | -2.712 | 8.222 |
| 1950.0 | -25.174 | -.000 | -2.581 | -8.571 | -2.581 | 8.571 |
| 2100.0 | -25.426 | .000 | -2.455 | -8.900 | -2.455 | 8.900 |
| 2250.0 | -25.669 | -.000 | -2.334 | -9.210 | -2.334 | 9.210 |
| 2400.0 | -25.903 | .000 | -2.217 | -9.504 | -2.217 | 9.504 |
| 2550.0 | -26.129 | -.000 | -2.103 | -9.785 | -2.103 | 9.785 |
| 2700.0 | -26.349 | .000 | -1.994 | -10.052 | -1.994 | 10.052 |
| 2850.0 | -26.562 | -.000 | -1.887 | -10.309 | -1.887 | 10.309 |
| 3000.0 | -26.768 | .000 | -1.784 | -10.555 | -1.784 | 10.555 |
| 3150.0 | -26.970 | -.000 | -1.683 | -10.792 | -1.683 | 10.792 |
| 3300.0 | -27.165 | .000 | -1.585 | -11.020 | -1.585 | 11.020 |
| 3450.0 | -27.356 | -.000 | -1.490 | -11.241 | -1.490 | 11.241 |
| 3600.0 | -27.543 | .000 | -1.397 | -11.454 | -1.397 | 11.454 |
| 3750.0 | -27.725 | -.000 | -1.306 | -11.661 | -1.306 | 11.661 |
| 3900.0 | -27.903 | 0 | -1.217 | -11.862 | -1.217 | 11.862 |
| 4050.0 | -28.077 | 0 | -1.130 | -12.057 | -1.130 | 12.057 |
| 4200.0 | -28.247 | 0 | -1.045 | -12.247 | -1.045 | 12.247 |
| 4350.0 | -28.414 | 0 | -.961 | -12.431 | -.961 | 12.431 |
| 4500.0 | -28.577 | 0 | -.879 | -12.611 | -.879 | 12.611 |
| 4650.0 | -28.738 | 0 | -.799 | -12.787 | -.799 | 12.787 |
| 4800.0 | -28.895 | 0 | -.720 | -12.959 | -.720 | 12.959 |
| 4950.0 | -29.050 | 0 | -.643 | -13.126 | -.643 | 13.126 |
| 5100.0 | -29.201 | 0 | -.567 | -13.290 | -.567 | 13.290 |
| 5250.0 | -29.351 | 0 | -.493 | -13.451 | -.493 | 13.451 |
| 5400.0 | -29.497 | 0 | -.419 | -13.608 | -.419 | 13.608 |
| 5550.0 | -29.642 | 0 | -.347 | -13.762 | -.347 | 13.762 |
| 5700.0 | -29.784 | 0 | -.276 | -13.913 | -.276 | 13.913 |
| 5850.0 | -29.924 | 0 | -.206 | -14.061 | -.206 | 14.061 |
| 6000.0 | -30.061 | 0 | -.137 | -14.206 | -.137 | 14.206 |
| 6150.0 | -30.197 | 0 | -.069 | -14.349 | -.069 | 14.349 |
| 6300.0 | -30.331 | 0 | -.003 | -14.489 | -.003 | 14.489 |
| 6450.0 | -30.463 | 0 | .063 | -14.627 | .063 | 14.627 |
| 6600.0 | -30.593 | 0 | .128 | -14.762 | .128 | 14.762 |
| 6750.0 | -30.721 | 0 | .192 | -14.895 | .192 | 14.895 |
| 6900.0 | -30.848 | 0 | .256 | -15.026 | .256 | 15.026 |
| 7050.0 | -30.972 | 0 | .318 | -15.156 | .318 | 15.156 |
| 7200.0 | -31.096 | 0 | .380 | -15.283 | .380 | 15.283 |
| 7350.0 | -31.217 | 0 | .441 | -15.408 | .441 | 15.408 |

Tab.2 Program RLT/Reg.B/-results.

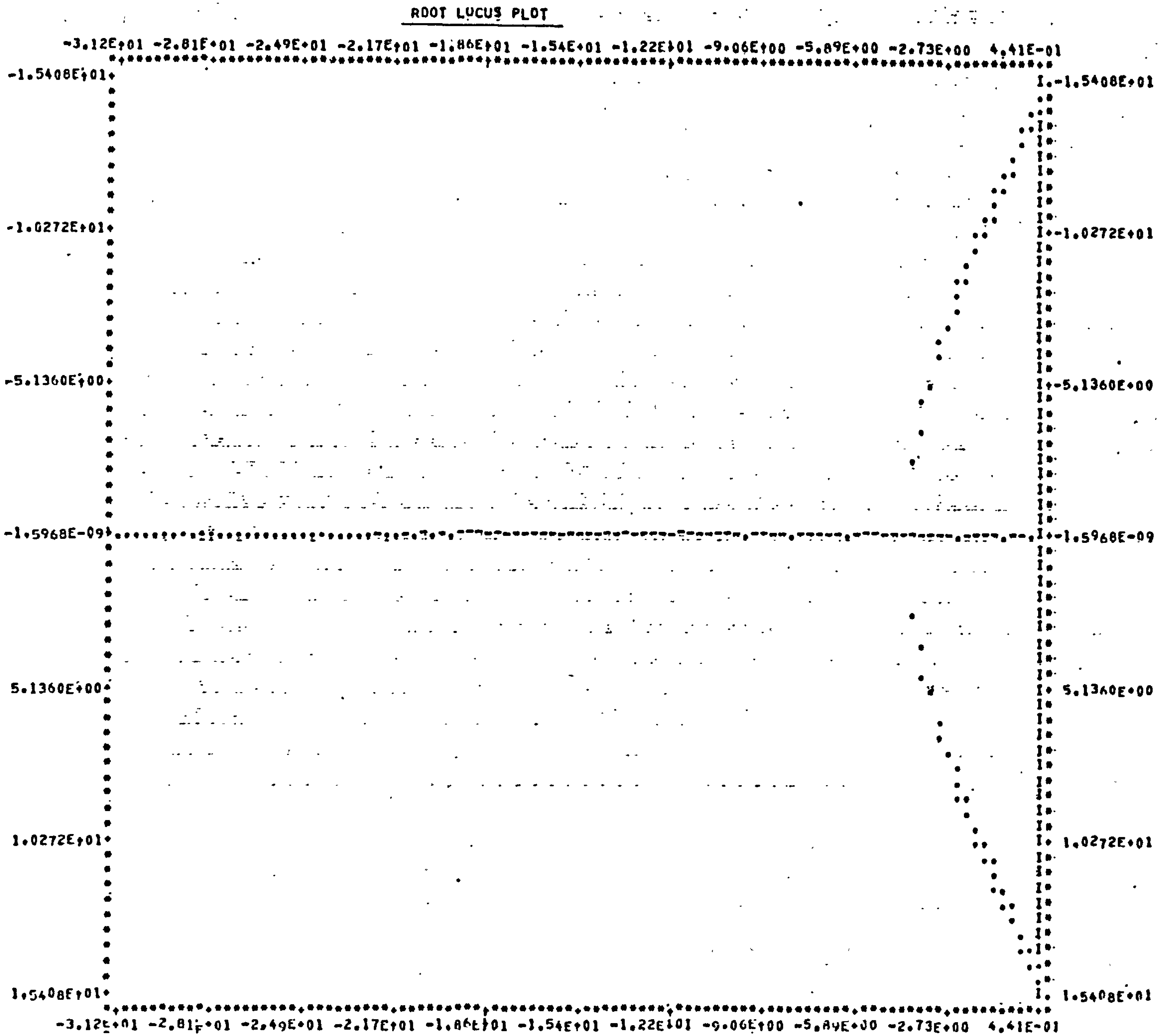


Fig.4. RL Plot/ Reg.B /

$$F(S) = \frac{1.000*S^{**}\ 0\ +}{1.000*S^{**}\ 3\ +\ 30.173*S^{**}\ 2\ +\ 205.190*S^{**}\ 1\ +\ 34.600*S^{**}\ X\ +}$$

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.173 | .000 | -10.000 | .000 | -20.000 | -.000 |
| 150.0 | -1.059 | .000 | -8.430 | -.000 | -20.684 | .000 |
| 300.0 | -2.427 | .000 | -6.483 | -.000 | -21.263 | -.000 |
| 450.0 | -21.770 | -.000 | -4.201 | 2.147 | -4.201 | -2.147 |
| 600.0 | -22.225 | .000 | -3.974 | 3.573 | -3.974 | -3.573 |
| 750.0 | -22.641 | -.000 | -3.766 | 4.525 | -3.766 | -4.525 |
| 900.0 | -23.024 | .000 | -3.574 | 5.274 | -3.574 | -5.274 |
| 1050.0 | -23.381 | -.000 | -3.396 | 5.904 | -3.396 | -5.904 |
| 1200.0 | -23.716 | .000 | -3.228 | 6.453 | -3.229 | -6.453 |
| 1350.0 | -24.032 | -.000 | -3.079 | 6.942 | -3.070 | -6.942 |
| 1500.0 | -24.332 | .000 | -2.920 | 7.385 | -2.920 | -7.385 |
| 1650.0 | -24.618 | -.000 | -2.778 | 7.792 | -2.778 | -7.792 |
| 1800.0 | -24.891 | .000 | -2.641 | 8.169 | -2.641 | -8.169 |
| 1950.0 | -25.152 | -.000 | -2.510 | 8.521 | -2.510 | -8.521 |
| 2100.0 | -25.403 | .000 | -2.385 | 8.851 | -2.385 | -8.851 |
| 2250.0 | -25.646 | -.000 | -2.264 | 9.163 | -2.264 | -9.163 |
| 2400.0 | -25.879 | .000 | -2.147 | 9.459 | -2.147 | -9.459 |
| 2550.0 | -26.106 | -.000 | -2.034 | 9.740 | -2.034 | -9.740 |
| 2700.0 | -26.325 | .000 | -1.924 | 10.009 | -1.924 | -10.009 |
| 2850.0 | -26.537 | -.000 | -1.818 | 10.266 | -1.818 | -10.266 |
| 3000.0 | -26.743 | .000 | -1.715 | 10.513 | -1.715 | -10.513 |
| 3150.0 | -26.944 | -.000 | -1.614 | 10.751 | -1.614 | -10.751 |
| 3300.0 | -27.140 | .000 | -1.517 | 10.980 | -1.517 | -10.980 |
| 3450.0 | -27.330 | -.000 | -1.421 | 11.202 | -1.421 | -11.202 |
| 3600.0 | -27.516 | .000 | -1.328 | 11.416 | -1.328 | -11.416 |
| 3750.0 | -27.698 | -.000 | -1.237 | 11.624 | -1.237 | -11.624 |
| 3900.0 | -27.876 | .000 | -1.149 | 11.825 | -1.149 | -11.825 |
| 4050.0 | -28.047 | 0 | -1.062 | 12.021 | -1.062 | -12.021 |
| 4200.0 | -28.219 | 0 | -.977 | 12.211 | -.977 | -12.211 |
| 4350.0 | -28.386 | 0 | -.894 | 12.396 | -.894 | -12.396 |
| 4500.0 | -28.549 | 0 | -.812 | 12.577 | -.812 | -12.577 |
| 4650.0 | -28.709 | 0 | -.732 | 12.753 | -.732 | -12.753 |
| 4800.0 | -28.867 | 0 | -.653 | 12.925 | -.653 | -12.925 |
| 4950.0 | -29.021 | 0 | -.576 | 13.093 | -.576 | -13.093 |
| 5100.0 | -29.173 | 0 | -.500 | 13.257 | -.500 | -13.257 |
| 5250.0 | -29.322 | 0 | -.426 | 13.4.8 | -.426 | -13.418 |
| 5400.0 | -29.468 | 0 | -.352 | 13.576 | -.352 | -13.576 |
| 5550.0 | -29.612 | 0 | -.280 | 13.730 | -.280 | -13.730 |
| 5700.0 | -29.754 | 0 | -.209 | 13.881 | -.209 | -13.881 |
| 5850.0 | -29.894 | 0 | -.140 | 14.030 | -.140 | -14.030 |
| 6000.0 | -30.032 | 0 | -.071 | 14.175 | -.071 | -14.175 |
| 6150.0 | -30.167 | 0 | -.003 | 14.318 | -.003 | -14.318 |
| 6300.0 | -30.301 | 0 | .064 | 14.459 | .064 | -14.459 |
| 6450.0 | -30.432 | 0 | .130 | 14.597 | .130 | -14.597 |
| 6600.0 | -30.562 | 0 | .195 | 14.733 | .195 | -14.733 |
| 6750.0 | -30.690 | 0 | .259 | 14.866 | .259 | -14.866 |
| 6900.0 | -30.817 | 0 | .322 | 14.997 | .322 | -14.997 |
| 7050.0 | -30.941 | 0 | .384 | 15.127 | .384 | -15.127 |
| 7200.0 | -31.065 | 0 | .446 | 15.254 | .446 | -15.254 |
| 7350.0 | -31.186 | 0 | .507 | 15.380 | .507 | -15.380 |

Tab. 3 Program RLT/Reg.C/-results.

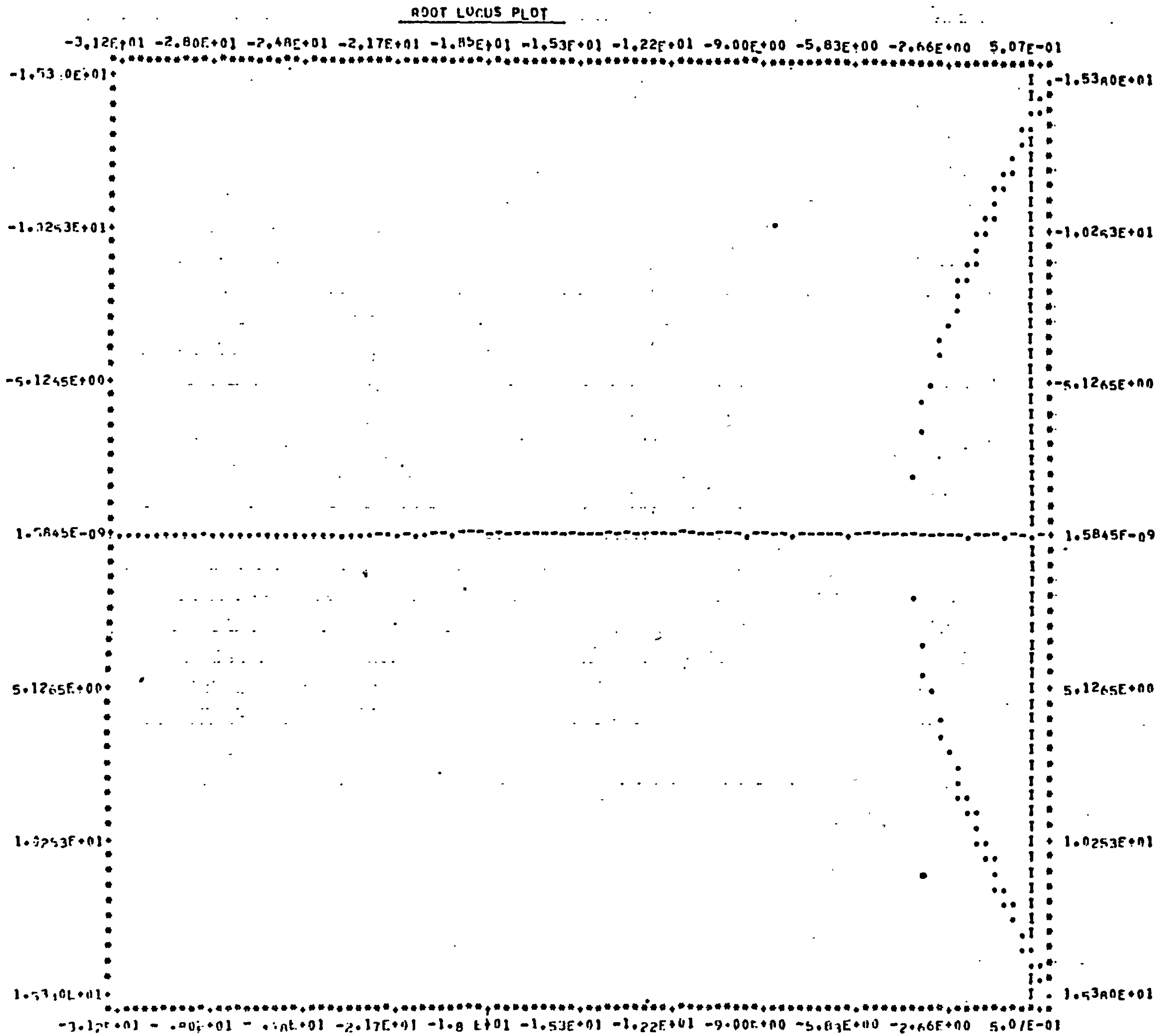


Fig.5. RL Plot /Reg.C/

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## 8.2. ROOT-LOCUS PATTERN FOR COMPENSATED SYSTEMS

The shape of root-locus pattern indicates the system's performance and this performance may be improved by changing this root-locus pattern. The shape can be changed by adding poles and zeros to the open-loop transfer function. In order to make the system stable for all gain constants, the root-loci entering the right half of the S-plane must be reshaped so that they lie in the left half. Let us consider now a pole-zero pattern for an open-loop transfer function of the compensated system as discussed in DEC Program.

$$G\ (s) = G(jw) = \frac{K(s+0.909)}{(s+B)(s+10.)(s+20.)} \qquad \text{Let } K = 220,\quad B = B_1 = 0.067$$

This is a third order system with a number of poles m = 3 and a number of zeros n = 1.
Using similar rules as before, Let us find Root-Loci along the real axis. (See Fig.1.)

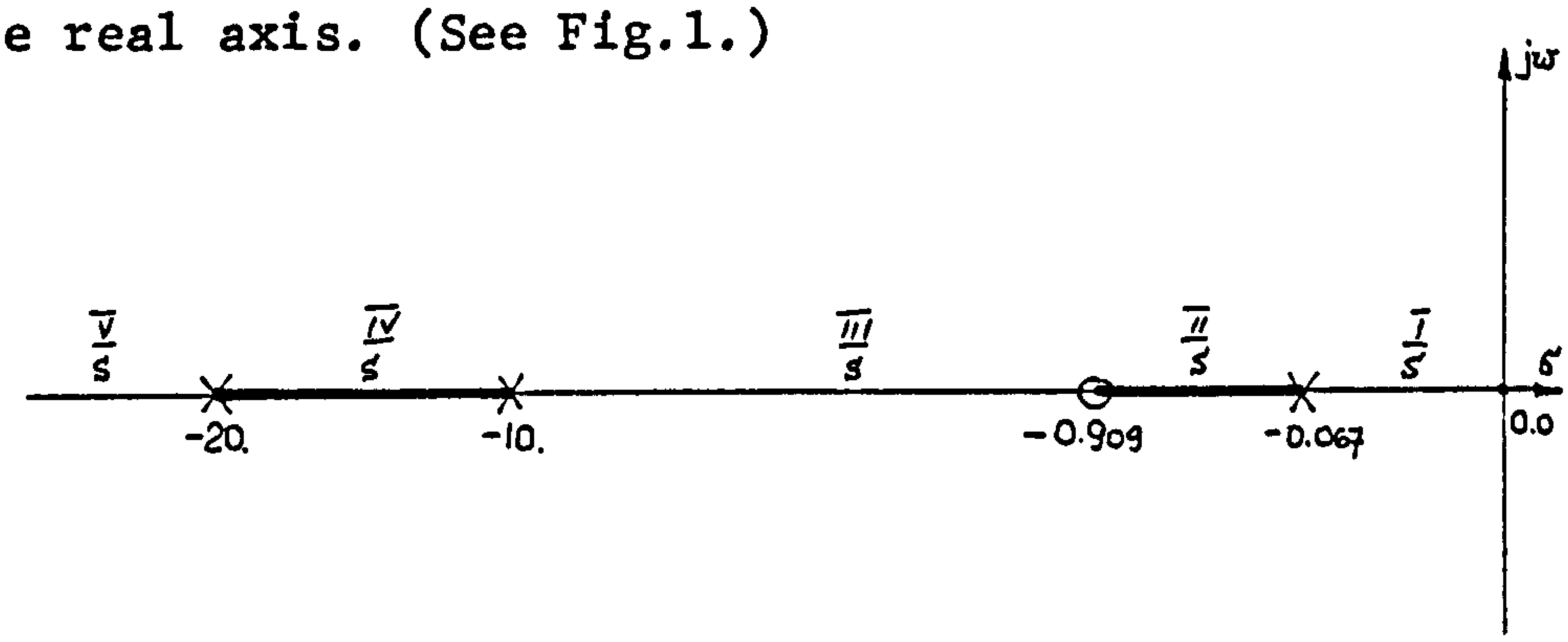



Fig.1.

There are three poles at: $S = -0.067$, $S = -10.0$, $S = -20.0$ and one zero at: $S = -0.909$

Points S($\underline{IV}$) and S ($\underline{II}$) are placed on root-loci.
Intersection of the asymptotes :

$$d = \frac{\Sigma\ \text{poles} - \Sigma\ \text{zeros}}{m-n} = \frac{(-0.067-10.0-20.0)-(-0.909)}{3-1} = -14.579$$

The angles of the asymptotes to the real axis in this case are:

$$\Theta = \frac{180^{\circ} + N.360^{\circ}}{n - m} = \frac{180^{\circ} + N.360^{\circ}}{-2}$$

$$\Theta = 90^{\circ} \quad \& \quad 270^{\circ}$$

BREAK-AWAY point from the real axis :

$$\left. \frac{d\ /G(jw)/}{d(jw)} \right|_{jw = s} = 0.$$

3. $$s^3 + 62.861s^2 + 256.671s + 183.627 + \underbrace{\frac{1}{(s+0.067)(s+10.)(s+20.)}}_{\cong 0.0} = 0.$$

The last EQ can be approximated to zero for the value estimated $\approx$ -15.75 (from the range -10; 20;) when the engine is run at region no.1.

A Complete root-locus pattern for this control system is shown in Fig.2. The root-locus pattern is bent back into the left half of the S-plane and is stable for ALL values of gain constant. Similar to the previous section, tabulated results of the RLT program for all three regions

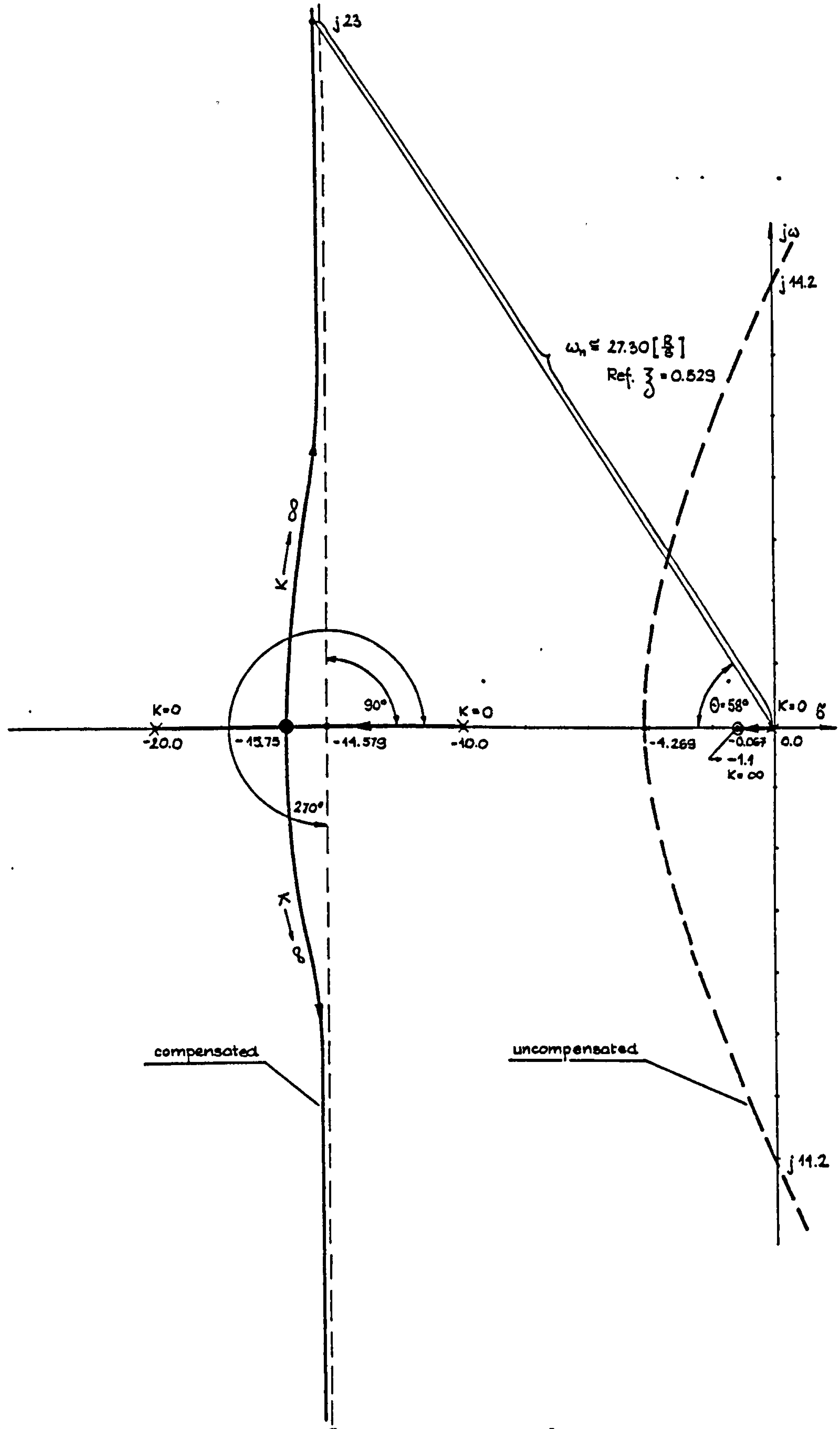



Fig.2 Root-Locus pattern for compensated system.

are presented in Tab.1, Tab.2, Tab.3, with the corresponding computer plots of the root-locus pattern shown in Fig.3, Fig.4, and Fig.5.

```
           1.000*S** 1 +    .909*S** X +
F(S) = ----------------------------------------------------------------
           1.000*S** 3 +  30.067*S** 2 + 202.010*S** 1 +  13.400*S** X +
```

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.067 | .000 | -10.000 | .000 | -20.000 | -.000 |
| 150.0 | -.442 | .000 | -14.813 | 10.932 | -14.813 | -10.932 |
| 300.0 | -.590 | .000 | -14.738 | 16.352 | -14.738 | -16.352 |
| 450.0 | -.668 | .000 | -14.700 | 20.403 | -14.700 | -20.403 |
| 600.0 | -.715 | .000 | -14.676 | 23.783 | -14.676 | -23.783 |
| 750.0 | -.747 | .000 | -14.660 | 26.743 | -14.660 | -26.743 |
| 900.0 | -.770 | .000 | -14.648 | 29.409 | -14.648 | -29.409 |
| 1050.0 | -.788 | .000 | -14.640 | 31.853 | -14.640 | -31.853 |
| 1200.0 | -.801 | .000 | -14.633 | 34.124 | -14.633 | -34.124 |
| 1350.0 | -.812 | .000 | -14.628 | 36.253 | -14.628 | -36.253 |
| 1500.0 | -.821 | .000 | -14.623 | 38.265 | -14.623 | -38.265 |
| 1650.0 | -.828 | .000 | -14.620 | 40.176 | -14.620 | -40.176 |
| 1800.0 | -.834 | .000 | -14.616 | 42.000 | -14.616 | -42.000 |
| 1950.0 | -.839 | .000 | -14.614 | 43.748 | -14.614 | -43.748 |
| 2100.0 | -.844 | .000 | -14.611 | 45.430 | -14.611 | -45.430 |
| 2250.0 | -.848 | .000 | -14.609 | 47.051 | -14.609 | -47.051 |
| 2400.0 | -.852 | .000 | -14.608 | 48.618 | -14.608 | -48.618 |
| 2550.0 | -.855 | .000 | -14.606 | 50.137 | -14.606 | -50.137 |
| 2700.0 | -.858 | .000 | -14.605 | 51.611 | -14.605 | -51.611 |
| 2850.0 | -.860 | .000 | -14.603 | 53.044 | -14.603 | -53.044 |
| 3000.0 | -.863 | .000 | -14.602 | 54.439 | -14.602 | -54.439 |
| 3150.0 | -.865 | .000 | -14.601 | 55.799 | -14.601 | -55.799 |
| 3300.0 | -.867 | .000 | -14.600 | 57.127 | -14.600 | -57.127 |
| 3450.0 | -.868 | .000 | -14.599 | 58.425 | -14.599 | -58.425 |
| 3600.0 | -.870 | .000 | -14.598 | 59.695 | -14.598 | -59.695 |
| 3750.0 | -.872 | 0 | -14.598 | 60.938 | -14.598 | -60.938 |
| 3900.0 | -.873 | 0 | -14.597 | 62.157 | -14.597 | -62.157 |
| 4050.0 | -.874 | 0 | -14.596 | 63.352 | -14.596 | -63.352 |
| 4200.0 | -.875 | 0 | -14.596 | 64.525 | -14.596 | -64.525 |
| 4350.0 | -.877 | 0 | -14.595 | 65.677 | -14.595 | -65.677 |
| 4500.0 | -.878 | 0 | -14.595 | 66.809 | -14.595 | -66.809 |
| 4650.0 | -.879 | 0 | -14.594 | 67.922 | -14.594 | -67.922 |
| 4800.0 | -.879 | 0 | -14.594 | 69.017 | -14.594 | -69.017 |
| 4950.0 | -.880 | 0 | -14.593 | 70.095 | -14.593 | -70.095 |
| 5100.0 | -.881 | 0 | -14.593 | 71.157 | -14.593 | -71.157 |
| 5250.0 | -.882 | 0 | -14.593 | 72.203 | -14.593 | -72.203 |
| 5400.0 | -.883 | 0 | -14.592 | 73.235 | -14.592 | -73.235 |
| 5550.0 | -.883 | 0 | -14.592 | 74.252 | -14.592 | -74.252 |
| 5700.0 | -.884 | 0 | -14.591 | 75.255 | -14.591 | -75.255 |
| 5850.0 | -.885 | 0 | -14.591 | 76.245 | -14.591 | -76.245 |
| 6000.0 | -.885 | 0 | -14.591 | 77.222 | -14.591 | -77.222 |
| 6150.0 | -.886 | 0 | -14.591 | 78.187 | -14.591 | -78.187 |
| 6300.0 | -.886 | 0 | -14.590 | 79.141 | -14.590 | -79.141 |
| 6450.0 | -.887 | 0 | -14.590 | 80.083 | -14.590 | -80.083 |
| 6600.0 | -.887 | 0 | -14.590 | 81.014 | -14.590 | -81.014 |
| 6750.0 | -.888 | 0 | -14.590 | 81.934 | -14.590 | -81.934 |
| 6900.0 | -.888 | 0 | -14.589 | 82.845 | -14.589 | -82.845 |
| 7050.0 | -.889 | 0 | -14.589 | 83.745 | -14.589 | -83.745 |
| 7200.0 | -.889 | 0 | -14.589 | 84.636 | -14.589 | -84.636 |
| 7350.0 | -.890 | 0 | -14.589 | 85.517 | -14.589 | -85.517 |

Tab.1 Program RLT/Reg.A/-results

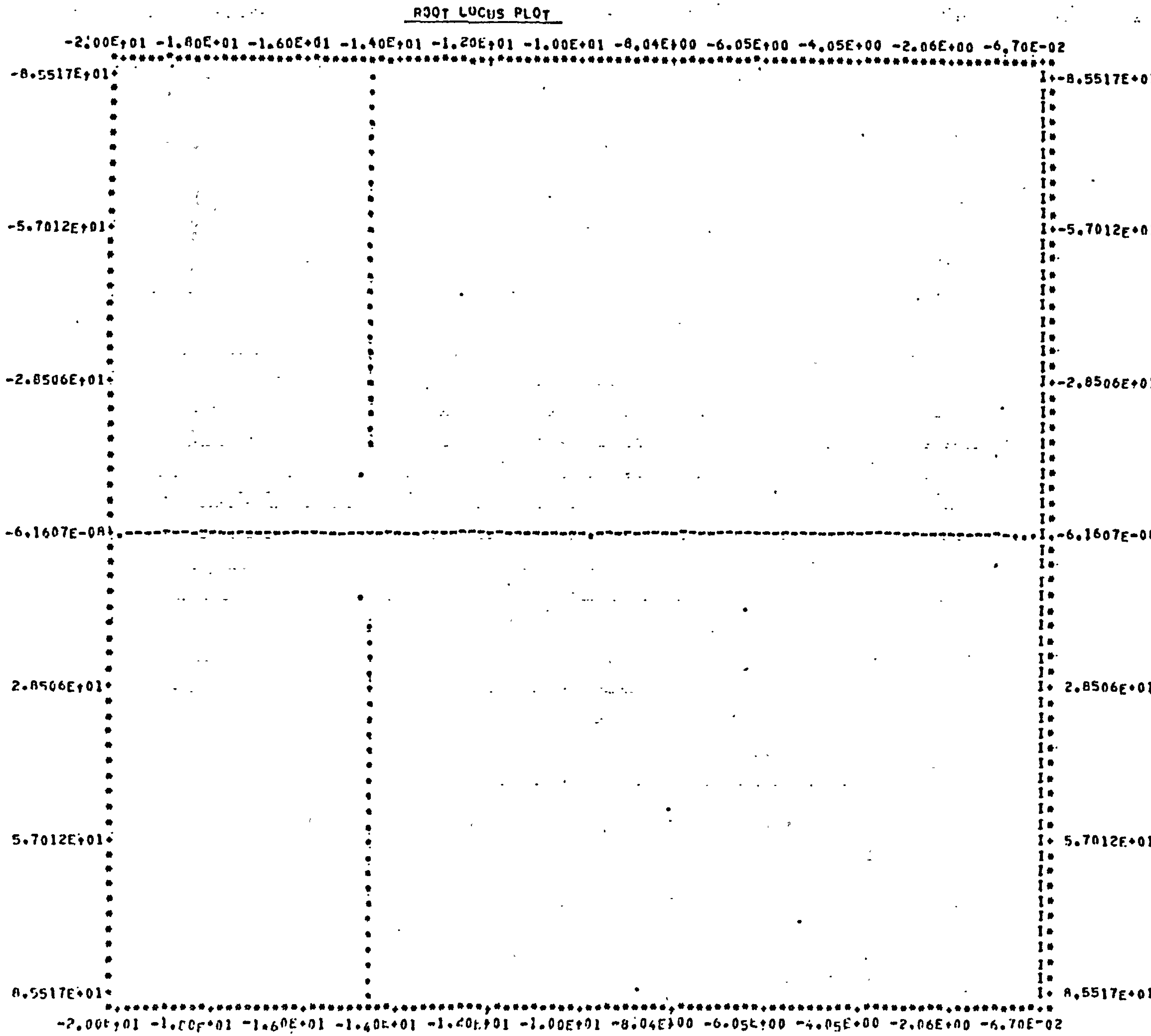
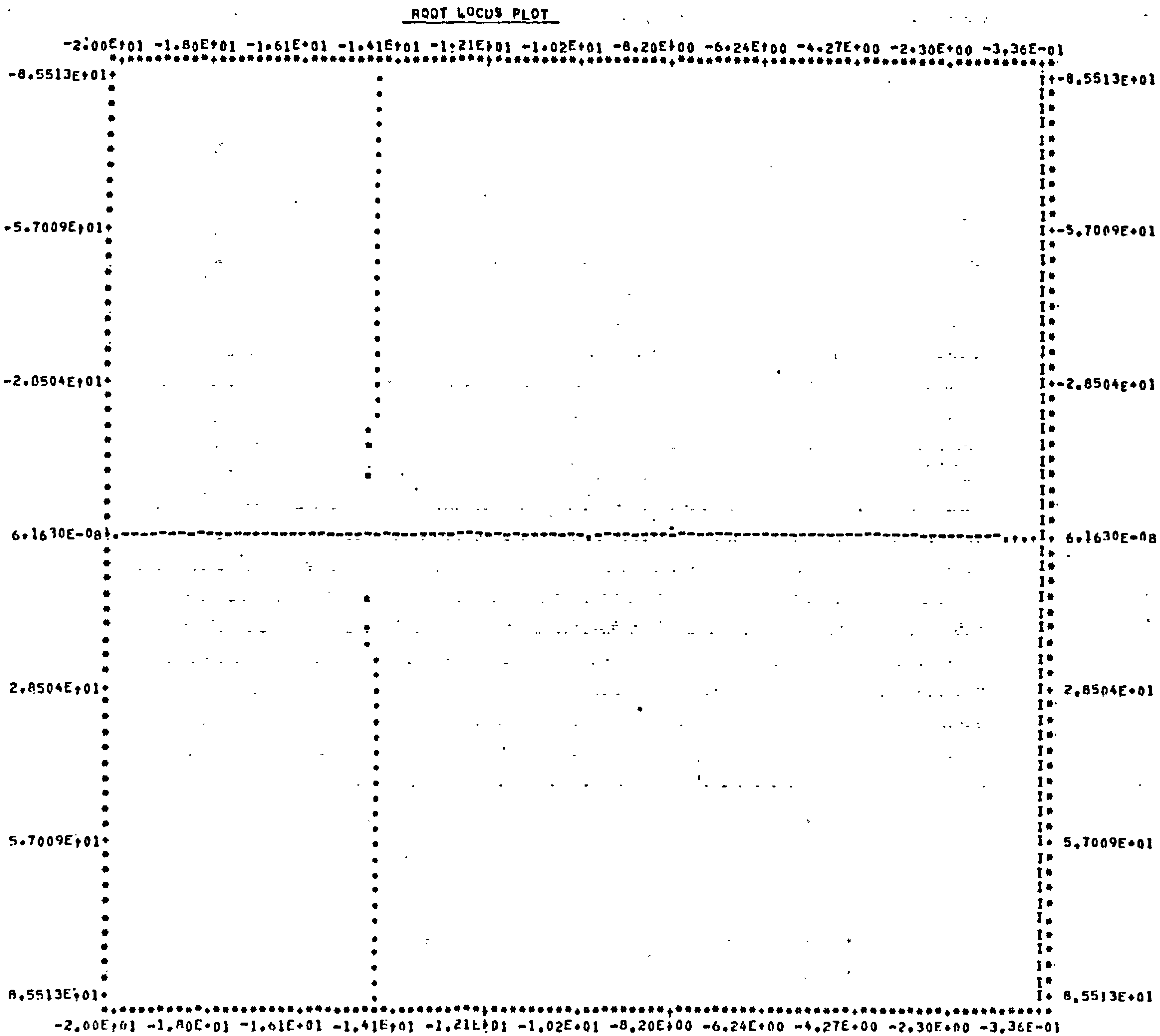



Fig.3. RL Plot /Reg.A./

$$F(S) = \frac{1.000*S**\ 1 + 1.250*S**\ X +}{1.000*S**\ 3 + 30.336*S**\ 2 + 210.080*S**\ 1 + 67.200*S**\ X +}$$

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.336 | -.000 | -10.000 | -.000 | -20.000 | -.000 |
| 150.0 | -.754 | -.000 | -14.791 | -10.909 | -14.791 | 10.909 |
| 300.0 | -.915 | -.000 | -14.710 | -16.333 | -14.710 | 16.333 |
| 450.0 | -.998 | -.000 | -14.669 | -20.387 | -14.669 | 20.387 |
| 600.0 | -1.049 | -.000 | -14.644 | -23.768 | -14.644 | 23.768 |
| 750.0 | -1.082 | -.000 | -14.627 | -26.730 | -14.627 | 26.730 |
| 900.0 | -1.106 | -.000 | -14.615 | -29.396 | -14.615 | 29.396 |
| 1050.0 | -1.124 | -.000 | -14.606 | -31.842 | -14.606 | 31.842 |
| 1200.0 | -1.138 | -.000 | -14.599 | -34.113 | -14.599 | 34.113 |
| 1350.0 | -1.149 | -.000 | -14.593 | -36.243 | -14.593 | 36.243 |
| 1500.0 | -1.159 | -.000 | -14.589 | -38.255 | -14.589 | 38.255 |
| 1650.0 | -1.166 | -.000 | -14.585 | -40.166 | -14.585 | 40.166 |
| 1800.0 | -1.173 | -.000 | -14.582 | -41.991 | -14.582 | 41.991 |
| 1950.0 | -1.178 | -.000 | -14.579 | -43.740 | -14.579 | 43.740 |
| 2100.0 | -1.183 | -.000 | -14.576 | -45.422 | -14.576 | 45.422 |
| 2250.0 | -1.187 | -.000 | -14.574 | -47.043 | -14.574 | 47.043 |
| 2400.0 | -1.191 | -.000 | -14.573 | -48.611 | -14.573 | 48.611 |
| 2550.0 | -1.194 | -.000 | -14.571 | -50.130 | -14.571 | 50.130 |
| 2700.0 | -1.197 | -.000 | -14.569 | -51.604 | -14.569 | 51.604 |
| 2850.0 | -1.200 | -.000 | -14.568 | -53.037 | -14.568 | 53.037 |
| 3000.0 | -1.202 | -.000 | -14.567 | -54.432 | -14.567 | 54.432 |
| 3150.0 | -1.204 | -.000 | -14.566 | -55.793 | -14.566 | 55.793 |
| 3300.0 | -1.206 | -.000 | -14.565 | -57.121 | -14.565 | 57.121 |
| 3450.0 | -1.208 | -.000 | -14.564 | -58.419 | -14.564 | 58.419 |
| 3600.0 | -1.210 | 0 | -14.563 | -59.689 | -14.563 | 59.689 |
| 3750.0 | -1.211 | 0 | -14.562 | -60.932 | -14.562 | 60.932 |
| 3900.0 | -1.213 | 0 | -14.562 | -62.151 | -14.562 | 62.151 |
| 4050.0 | -1.214 | 0 | -14.561 | -63.346 | -14.561 | 63.346 |
| 4200.0 | -1.215 | 0 | -14.560 | -64.519 | -14.560 | 64.519 |
| 4350.0 | -1.217 | 0 | -14.560 | -65.671 | -14.560 | 65.671 |
| 4500.0 | -1.218 | 0 | -14.559 | -66.803 | -14.559 | 66.803 |
| 4650.0 | -1.219 | 0 | -14.559 | -67.916 | -14.559 | 67.916 |
| 4800.0 | -1.220 | 0 | -14.558 | -69.012 | -14.558 | 69.012 |
| 4950.0 | -1.221 | 0 | -14.558 | -70.090 | -14.558 | 70.090 |
| 5100.0 | -1.221 | 0 | -14.557 | -71.152 | -14.557 | 71.152 |
| 5250.0 | -1.222 | 0 | -14.557 | -72.198 | -14.557 | 72.198 |
| 5400.0 | -1.223 | 0 | -14.557 | -73.230 | -14.557 | 73.230 |
| 5550.0 | -1.224 | 0 | -14.556 | -74.247 | -14.556 | 74.247 |
| 5700.0 | -1.224 | 0 | -14.556 | -75.250 | -14.556 | 75.250 |
| 5850.0 | -1.225 | 0 | -14.556 | -76.240 | -14.556 | 76.240 |
| 6000.0 | -1.226 | 0 | -14.555 | -77.218 | -14.555 | 77.218 |
| 6150.0 | -1.226 | 0 | -14.555 | -78.183 | -14.555 | 78.183 |
| 6300.0 | -1.227 | 0 | -14.555 | -79.136 | -14.555 | 79.136 |
| 6450.0 | -1.227 | 0 | -14.554 | -80.078 | -14.554 | 80.078 |
| 6600.0 | -1.228 | 0 | -14.554 | -81.009 | -14.554 | 81.009 |
| 6750.0 | -1.228 | 0 | -14.554 | -81.930 | -14.554 | 81.930 |
| 6900.0 | -1.229 | 0 | -14.554 | -82.840 | -14.554 | 82.840 |
| 7050.0 | -1.229 | 0 | -14.553 | -83.741 | -14.553 | 83.741 |
| 7200.0 | -1.230 | 0 | -14.553 | -84.632 | -14.553 | 84.632 |
| 7350.0 | -1.230 | 0 | -14.553 | -85.513 | -14.553 | 85.513 |

Tab. 2 Program RLT/Reg.B/-results.




<u>Fig.4.</u> RL Plot /Reg.B/

```
        1.000*S** 1 +   5.000*S** X +
F(S) = ---------------------------------------------------------------------------
        1.000*S** 3 +  30.173*S** 2 + 205.190*S** 1 +  34.600*S** X +
```

| K | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY | ROOT REAL | ROOT IMAGINARY |
|---|---|---|---|---|---|---|
| 0 | -.173 | .000 | -10.000 | .000 | -20.000 | -.000 |
| 150.0 | -2.622 | .000 | -13.675 | 9.539 | -13.675 | -9.539 |
| 300.0 | -3.787 | .000 | -13.193 | 15.206 | -13.193 | -15.206 |
| 450.0 | -4.180 | .000 | -12.996 | 19.433 | -12.996 | -19.433 |
| 600.0 | -4.384 | .000 | -12.894 | 22.932 | -12.894 | -22.932 |
| 750.0 | -4.508 | .000 | -12.832 | 25.977 | -12.832 | -25.977 |
| 900.0 | -4.591 | .000 | -12.791 | 28.708 | -12.791 | -28.708 |
| 1050.0 | -4.650 | .000 | -12.762 | 31.203 | -12.762 | -31.204 |
| 1200.0 | -4.694 | .000 | -12.739 | 33.516 | -12.740 | -33.516 |
| 1350.0 | -4.728 | .000 | -12.722 | 35.679 | -12.722 | -35.679 |
| 1500.0 | -4.756 | .000 | -12.709 | 37.720 | -12.709 | -37.720 |
| 1650.0 | -4.778 | .000 | -12.697 | 39.656 | -12.697 | -39.656 |
| 1800.0 | -4.797 | .000 | -12.688 | 41.503 | -12.688 | -41.503 |
| 1950.0 | -4.813 | .000 | -12.680 | 43.271 | -12.680 | -43.271 |
| 2100.0 | -4.826 | .000 | -12.673 | 44.969 | -12.673 | -44.969 |
| 2250.0 | -4.838 | .000 | -12.668 | 46.606 | -12.668 | -46.606 |
| 2400.0 | -4.848 | .000 | -12.663 | 48.188 | -12.663 | -48.188 |
| 2550.0 | -4.857 | .000 | -12.658 | 49.719 | -12.658 | -49.719 |
| 2700.0 | -4.865 | .000 | -12.654 | 51.205 | -12.654 | -51.205 |
| 2850.0 | -4.872 | .000 | -12.650 | 52.649 | -12.650 | -52.649 |
| 3000.0 | -4.879 | .000 | -12.647 | 54.054 | -12.647 | -54.054 |
| 3150.0 | -4.884 | .000 | -12.644 | 55.424 | -12.644 | -55.424 |
| 3300.0 | -4.890 | .000 | -12.642 | 56.760 | -12.642 | -56.760 |
| 3450.0 | -4.894 | .000 | -12.639 | 58.066 | -12.639 | -58.066 |
| 3600.0 | -4.899 | .000 | -12.637 | 59.344 | -12.637 | -59.344 |
| 3750.0 | -4.903 | 0 | -12.635 | 60.594 | -12.635 | -60.594 |
| 3900.0 | -4.907 | 0 | -12.633 | 61.819 | -12.633 | -61.819 |
| 4050.0 | -4.910 | 0 | -12.631 | 63.021 | -12.631 | -63.021 |
| 4200.0 | -4.913 | 0 | -12.630 | 64.199 | -12.630 | -64.199 |
| 4350.0 | -4.916 | 0 | -12.628 | 65.357 | -12.628 | -65.357 |
| 4500.0 | -4.919 | 0 | -12.627 | 66.495 | -12.627 | -66.495 |
| 4650.0 | -4.922 | 0 | -12.626 | 67.613 | -12.626 | -67.613 |
| 4800.0 | -4.924 | 0 | -12.624 | 68.713 | -12.624 | -68.713 |
| 4950.0 | -4.927 | 0 | -12.623 | 69.796 | -12.623 | -69.796 |
| 5100.0 | -4.929 | 0 | -12.622 | 70.862 | -12.622 | -70.862 |
| 5250.0 | -4.931 | 0 | -12.621 | 71.913 | -12.621 | -71.913 |
| 5400.0 | -4.933 | 0 | -12.620 | 72.948 | -12.620 | -72.948 |
| 5550.0 | -4.935 | 0 | -12.619 | 73.969 | -12.619 | -73.969 |
| 5700.0 | -4.936 | 0 | -12.618 | 74.976 | -12.618 | -74.976 |
| 5850.0 | -4.938 | 0 | -12.618 | 75.970 | -12.618 | -75.970 |
| 6000.0 | -4.939 | 0 | -12.617 | 76.950 | -12.617 | -76.950 |
| 6150.0 | -4.941 | 0 | -12.616 | 77.919 | -12.616 | -77.919 |
| 6300.0 | -4.942 | 0 | -12.615 | 78.876 | -12.615 | -78.876 |
| 6450.0 | -4.944 | 0 | -12.615 | 79.821 | -12.615 | -79.821 |
| 6600.0 | -4.945 | 0 | -12.614 | 80.755 | -12.614 | -80.755 |
| 6750.0 | -4.946 | 0 | -12.613 | 81.678 | -12.613 | -81.678 |
| 6900.0 | -4.947 | 0 | -12.613 | 82.591 | -12.613 | -82.591 |
| 7050.0 | -4.948 | 0 | -12.612 | 83.494 | -12.612 | -83.494 |
| 7200.0 | -4.950 | 0 | -12.612 | 84.388 | -12.612 | -84.388 |
| 7350.0 | -4.951 | 0 | -12.611 | 85.272 | -12.611 | -85.272 |

Tab.3 Program RLT/Reg.C/-results.

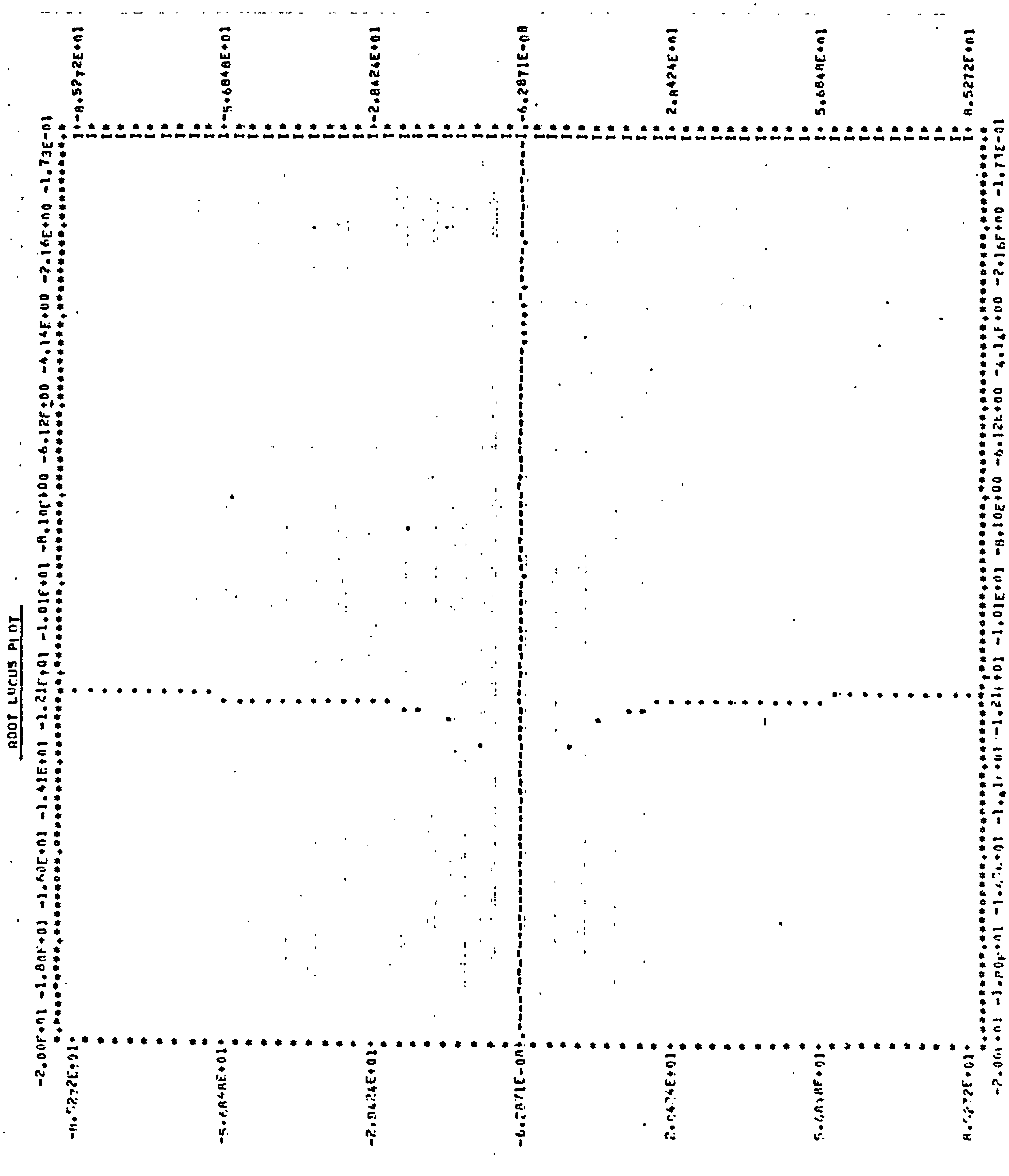



Fig.5. RL Plot/Reg.C/

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## 8.3. EVALUATION OF STEADY-STATE ERROR (CONSTANT VELOCITY) OF SIMULATED ENGINES IN PCS AND DEC PROGRAMS

A system's performance can be partly assessed in terms of its steady-state error, the most common type being the velocity error. The unit ramp function input to a system is an input "velocity" and in steady-state the system output should be a "velocity". If the system has a steady-state error for a unit ramp function it is a velocity error Ev. For our system's purpose, Let us use a RAMP input, - one of the SYSTRAN subprograms, and which acts as an amplitude-limited ramp disturbance generator. The subprogram arguments are:

| | |
|---|---|
| $d_1$ | Initial value of ramp (TYPE REAL) |
| $d_2$ | final value of ramp (TYPE REAL) |
| $t_1$ | value of solution time beyond which ramp is to start (TYPE REAL) |
| $t_2$ | solution time at which ramp is to end (TYPE REAL) where $t_2 > t_1$ |

The form generated by RAMP is shown in Fig.1.

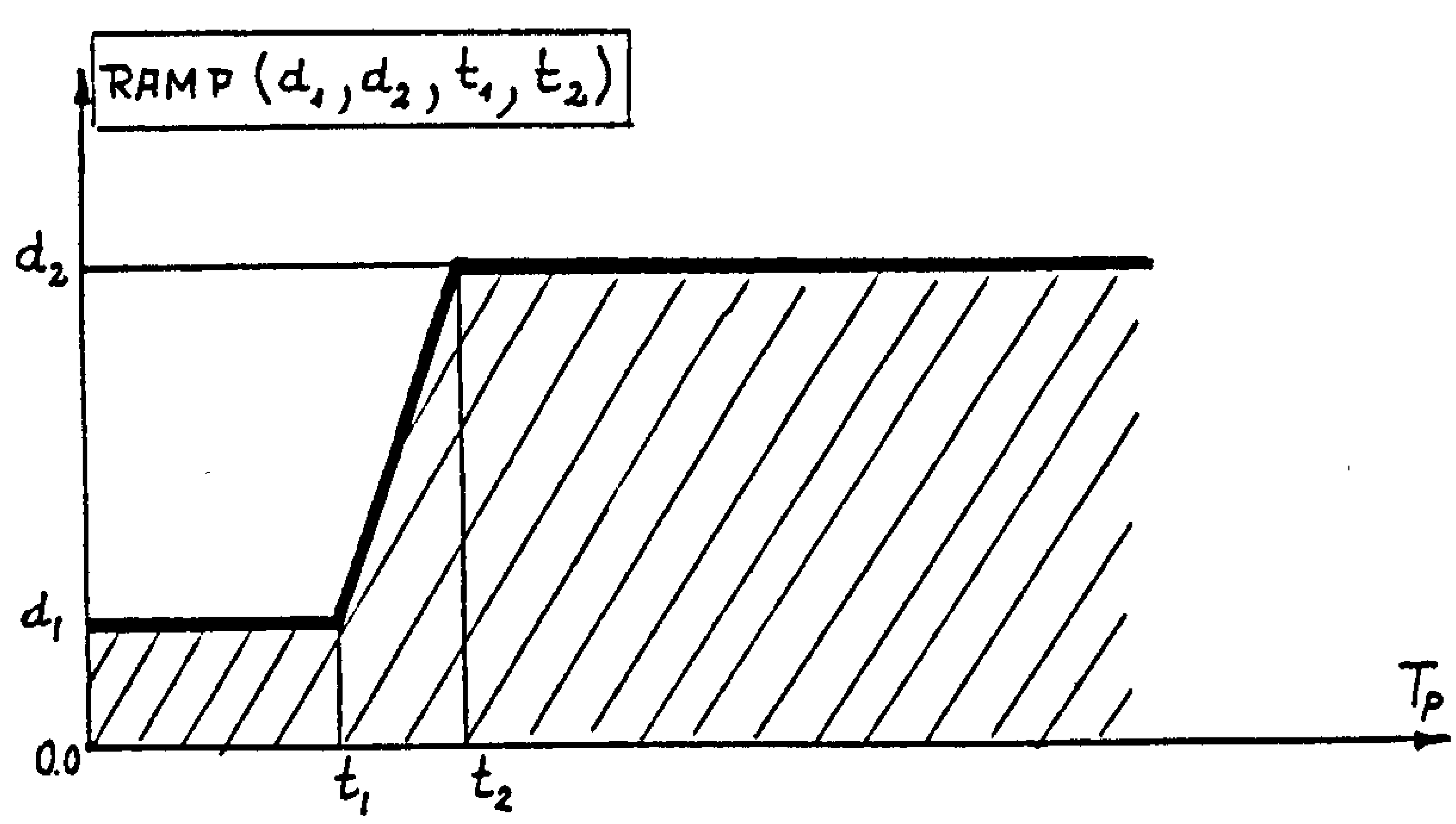


Fig.1.


RAMP input
speed output
Fig.2 PCS o/p Ref.
RAMP:1200/1600/0.083/0.3 1/P
SYMBOLS AND SCALES:
X1 1.150E+03 1.282E+03 1.414E+03 1.546E+03 1.678E+03
X6 1.000E-01 8.400E-01 1.580E+00 2.320E+00 3.060E+00
X7 3.000E+01 1.180E+02 2.060E+02 2.940E+02 3.820E+02
X2 2.500E+00 3.580E+00 4.660E+00 5.740E+00 6.820E+00


RAMP input
speed output
Fig. 3 DEC o/p, Ref.
RAMP: 1200/1600/
0.083/0.3 1/p
0.
5.00E-02
1.00E-01
1.50E-01
2.00E-01
2.50E-01
3.00E-01
3.50E-01
4.00E-01
4.50E-01
5.00E-01
5.50E-01
6.00E-01
6.50E-01
7.00E-01
7.50E-01
TIME
SYMBOLS AND SCALES:
X1 1.150E+03 1.282E+03 1.414E+03 1.546E+03 1.678E+03
X8 1.000E-01 8.400E-01 1.580E+00 2.320E+00 3.060E+00
X7 3.070E+01 1.180E+02 2.060E+02 2.940E+02 3.820E+02
X2 2.500E+00 3.580E+00 4.660E+00 5.740E+00 6.820E+00
1200
1400
1600

In order to compare both PCS and DEC programs the Ramp input used in both cases should be identical, for example:

YSET = RAMP (1200., 1620., 0.083, 0.3)

Both PCS and DEC plots of dynamic characteristics and run in exactly the same conditions are shown in Fig.2 and Fig.3, with the final tabulation of results given in Tab.1. (PCS) and Tab.2. (DEC).

Finally we can evaluate and compare steady-state errors in both PCS and DEC systems:

1. PCS : RAMP: 1200.,1620., 0.083, 0.3

Constant Velocity error: $E_V = \Delta s = 301.76$ (RPM)

Error coefficient : $K_V = \dfrac{1}{E_V} = 0.003313$

Where constant velocity error $E_V$ can be graphically interpolated as shown in Fig.4.

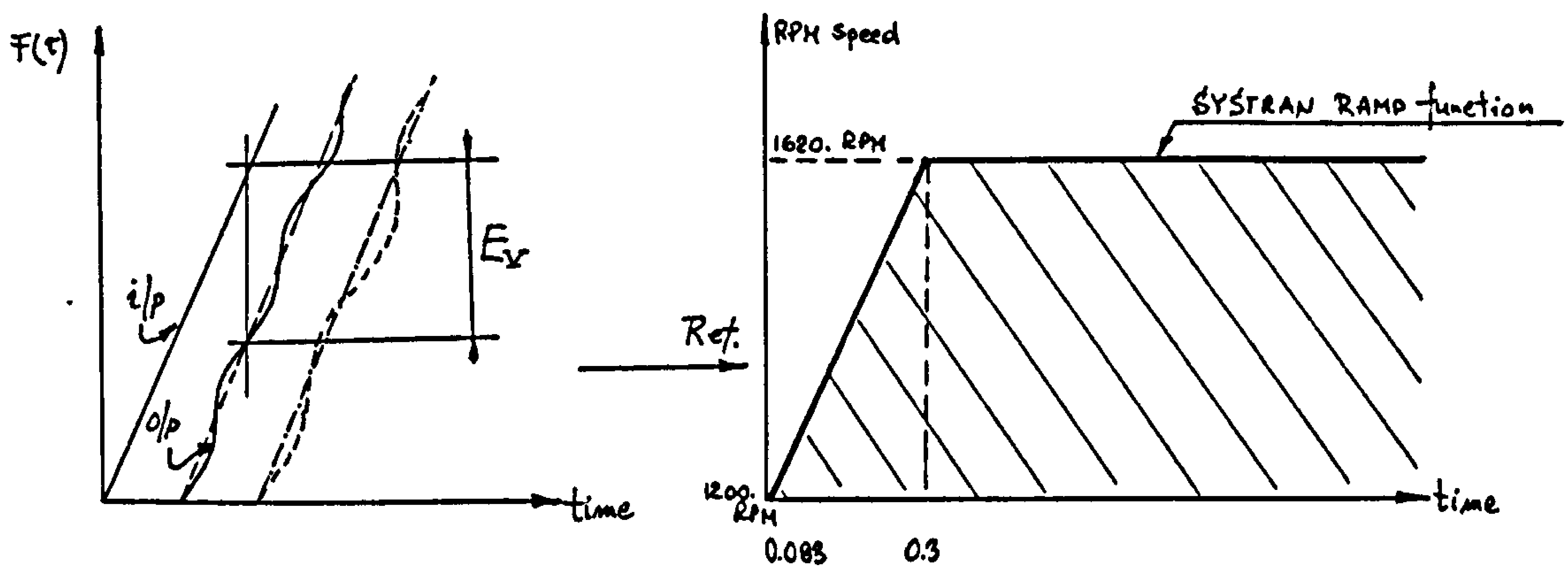


Fig.4.

INITIAL PRESSURE, TEMPERATURE = 6.900, 900.0

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 1.20000E+03 | 6.90000E+00 | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 1.07813E+03 | 6.90000E+00 | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 9.31875E+02 | 6.90000E+00 | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 7.85625E+02 | 6.90000E+00 | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 6.39375E+02 | 6.90000E+00 | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 4.93125E+02 | 6.90000E+00 | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 3.46875E+02 | 6.90000E+00 | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 2.00625E+02 | 6.90000E+00 | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 5.43750E+01 | 6.90000E+00 | 0. |
| 9.000E-02 | 16 | 0 | 9.830E-04 | 1 | 1.23157E+03 | 5.54457E+00 | -7.44578E+00 | 6.90000E+00 | 3.02437E+02 |
| 1.000E-01 | 18 | 0 | 1.018E-04 | 2 | 1.25859E+03 | 5.63188E+00 | -2.08786E+01 | 6.90000E+00 | 3.05941E+02 |
| 1.100E-01 | 20 | 0 | 1.229E-04 | 2 | 1.29264E+03 | 5.66018E+00 | -3.46134E+01 | 6.90000E+00 | 3.10927E+02 |
| 1.200E-01 | 22 | 0 | 1.379E-04 | 2 | 1.33069E+03 | 5.69363E+00 | -5.27425E+01 | 6.90000E+00 | 3.16953E+02 |
| 1.300E-01 | 24 | 0 | 1.356E-04 | 2 | 1.36868E+03 | 5.73308E+00 | -7.14861E+01 | 6.90000E+00 | 3.23204E+02 |
| 1.400E-01 | 26 | 0 | 1.060E-04 | 2 | 1.40630E+03 | 5.77135E+00 | -8.98099E+01 | 6.90000E+00 | 3.29332E+02 |
| 1.500E-01 | 28 | 0 | 9.328E-05 | 2 | 1.44075E+03 | 5.79839E+00 | -1.05689E+02 | 6.90000E+00 | 3.34723E+02 |
| 1.600E-01 | 30 | 0 | 7.304E-05 | 1 | 1.46648E+03 | 5.81711E+00 | -1.13598E+02 | 6.90000E+00 | 3.38432E+02 |
| 1.700E-01 | 32 | 0 | 5.824E-05 | 2 | 1.48289E+03 | 5.82150E+00 | -1.12113E+02 | 6.90000E+00 | 3.40598E+02 |
| 1.800E-01 | 33 | 0 | 1.029E-04 | 2 | 1.49285E+03 | 5.82706E+00 | -1.03951E+02 | 6.90000E+00 | 3.41867E+02 |

Tab.1 Program PCS results,ref.RAMP input.

Tab.1-continuation.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.900E-01 | 34 | 0 | 6.123E-05 | 2 | 1.49996E+03 | 5.84211E+00 | -9.17224E+01 | 6.90000E+00 | 3.42651E+02 |
| 2.000E-01 | 35 | 0 | 4.474E-05 | 2 | 1.50631E+03 | 5.86129E+00 | -7.86177E+01 | 6.90000E+00 | 3.43352E+02 |
| 2.100E-01 | 36 | 0 | 4.069E-05 | 2 | 1.51254E+03 | 5.88138E+00 | -6.54545E+01 | 6.90000E+00 | 3.44035E+02 |
| 2.200E-01 | 37 | 0 | 4.002E-05 | 1 | 1.51876E+03 | 5.90174E+00 | -5.23074E+01 | 6.90000E+00 | 3.44693E+02 |
| 2.300E-01 | 38 | 0 | 3.983E-05 | 1 | 1.52494E+03 | 5.92237E+00 | -3.91410E+01 | 6.90000E+00 | 3.45320E+02 |
| 2.400E-01 | 39 | 0 | 3.947E-05 | 1 | 1.53106E+03 | 5.94333E+00 | -2.59156E+01 | 6.90000E+00 | 3.45909E+02 |
| 2.500E-01 | 40 | 0 | 3.923E-05 | 1 | 1.53707E+03 | 5.96469E+00 | -1.25908E+01 | 6.90000E+00 | 3.46458E+02 |
| 2.600E-01 | 41 | 0 | 3.949E-05 | 1 | 1.54295E+03 | 5.98652E+00 | 8.60376E-01 | 6.90000E+00 | 3.46964E+02 |
| 2.700E-01 | 42 | 0 | 4.018E-05 | 1 | 1.54871E+03 | 6.00883E+00 | 1.44403E+01 | 6.90000E+00 | 3.47428E+02 |
| 2.800E-01 | 43 | 0 | 4.101E-05 | 1 | 1.55436E+03 | 6.03163E+00 | 2.81383E+01 | 6.90000E+00 | 3.47853E+02 |
| 2.900E-01 | 44 | 0 | 4.184E-05 | 1 | 1.55990E+03 | 6.05490E+00 | 4.19421E+01 | 6.90000E+00 | 3.48241E+02 |
| 3.000E-01 | 45 | 0 | 4.271E-05 | 1 | 1.56534E+03 | 6.07862E+00 | 5.58406E+01 | 6.90000E+00 | 3.48592E+02 |
| 3.100E-01 | 46 | 0 | 5.693E-05 | 2 | 1.57011E+03 | 6.08945E+00 | 5.06760E+01 | 6.90000E+00 | 3.48895E+02 |
| 3.200E-01 | 47 | 0 | 2.777E-05 | 2 | 1.57343E+03 | 6.09799E+00 | 4.70136E+01 | 6.90000E+00 | 3.49084E+02 |
| 3.300E-01 | 48 | 0 | 1.977E-05 | 2 | 1.57586E+03 | 6.10414E+00 | 4.44689E+01 | 6.90000E+00 | 3.49208E+02 |
| 3.400E-01 | 49 | 0 | 1.447E-05 | 2 | 1.57766E+03 | 6.10876E+00 | 4.25883E+01 | 6.90000E+00 | 3.49295E+02 |
| 3.500E-01 | 50 | 0 | 1.068E-05 | 2 | 1.57901E+03 | 6.11239E+00 | 4.11790E+01 | 6.90000E+00 | 3.49359E+02 |
| 3.600E-01 | 51 | 0 | 8.064E-06 | 2 | 1.58006E+03 | 6.11534E+00 | 4.00932E+01 | 6.90000E+00 | 3.49407E+02 |
| 3.700E-01 | 52 | 0 | 6.248E-06 | 2 | 1.58090E+03 | 6.11780E+00 | 3.92326E+01 | 6.90000E+00 | 3.49445E+02 |
| 3.800E-01 | 53 | 0 | 4.980E-06 | 2 | 1.58158E+03 | 6.11993E+00 | 3.85299E+01 | 6.90000E+00 | 3.49475E+02 |

Tab.1-continuation.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3.900E-01 | 54 | 0 | 4.089E-06 | 2 | 1.58216E+03 | 6.12183E+00 | 3.79388E+01 | 6.90000E+00 | 3.49500E+02 |
| 4.000E-01 | 55 | 0 | 3.460E-06 | 2 | 1.58266E+03 | 6.12355E+00 | 3.74267E+01 | 6.90000E+00 | 3.49522E+02 |
| 4.100E-01 | 56 | 0 | 3.013E-06 | 2 | 1.58311E+03 | 6.12515E+00 | 3.69711E+01 | 6.90000E+00 | 3.49541E+02 |
| 4.200E-01 | 57 | 0 | 2.695E-06 | 2 | 1.58352E+03 | 6.12666E+00 | 3.65559E+01 | 6.90000E+00 | 3.49558E+02 |
| 4.300E-01 | 58 | 0 | 2.466E-06 | 2 | 1.58390E+03 | 6.12810E+00 | 3.61698E+01 | 6.90000E+00 | 3.49574E+02 |
| 4.400E-01 | 59 | 0 | 2.301E-06 | 2 | 1.58426E+03 | 6.12949E+00 | 3.58049E+01 | 6.90000E+00 | 3.49583E+02 |
| 4.500E-01 | 60 | 0 | 2.182E-06 | 2 | 1.58461E+03 | 6.13084E+00 | 3.54553E+01 | 6.90000E+00 | 3.49604E+02 |
| 4.600E-01 | 61 | 0 | 2.095E-06 | 2 | 1.58495E+03 | 6.13217E+00 | 3.51170E+01 | 6.90000E+00 | 3.49617E+02 |
| 4.700E-01 | 62 | 0 | 2.031E-06 | 2 | 1.58528E+03 | 6.13347E+00 | 3.47871E+01 | 6.90000E+00 | 3.49631E+02 |
| 4.800E-01 | 63 | 0 | 1.984E-06 | 2 | 1.58560E+03 | 6.13475E+00 | 3.44634E+01 | 6.90000E+00 | 3.49644E+02 |
| 4.900E-01 | 64 | 0 | 1.949E-06 | 2 | 1.58592E+03 | 6.13602E+00 | 3.41444E+01 | 6.90000E+00 | 3.49656E+02 |
| 5.000E-01 | 65 | 0 | 1.923E-06 | 2 | 1.58623E+03 | 6.13727E+00 | 3.38290E+01 | 6.90000E+00 | 3.49669E+02 |
| 5.100E-01 | 66 | 0 | 1.903E-06 | 2 | 1.58654E+03 | 6.13851E+00 | 3.35164E+01 | 6.90000E+00 | 3.49681E+02 |
| 5.200E-01 | 67 | 0 | 1.887E-06 | 2 | 1.58685E+03 | 6.13974E+00 | 3.32059E+01 | 6.90000E+00 | 3.49693E+02 |
| 5.300E-01 | 68 | 0 | 1.876E-06 | 2 | 1.58716E+03 | 6.14096E+00 | 3.28972E+01 | 6.90000E+00 | 3.49705E+02 |
| 5.400E-01 | 69 | 0 | 1.866E-06 | 2 | 1.58747E+03 | 6.14217E+00 | 3.25899E+01 | 6.90000E+00 | 3.49716E+02 |
| 5.500E-01 | 70 | 0 | 1.859E-06 | 2 | 1.58777E+03 | 6.14337E+00 | 3.22837E+01 | 6.90000E+00 | 3.49728E+02 |
| 5.600E-01 | 71 | 0 | 1.853E-06 | 2 | 1.58808E+03 | 6.14457E+00 | 3.19786E+01 | 6.90000E+00 | 3.49739E+02 |

NITIAL PRESSURE, TEMPERATURE = 6.900, 900.0

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 6.90000E+00 | 1.20000E+03 | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 6.90000E+00 | 1.07813E+03 | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 6.90000E+00 | 9.31875E+02 | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 6.90000E+00 | 7.85625E+02 | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 6.90000E+00 | 6.39375E+02 | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 6.90000E+00 | 4.93125E+02 | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 6.90000E+00 | 3.46875E+02 | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 6.90000E+00 | 2.00625E+02 | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 6.90000E+00 | 5.43750E+01 | 0. |
| 9.000E-02 | 16 | 0 | 9.971E-04 | 1 | 1.22493E+03 | 5.39996E+00 | 6.90000E+00 | -2.22933E+00 | 3.01691E+02 |
| 1.000E-01 | 18 | 0 | 1.638E-06 | 1 | 1.22506E+03 | 5.39994E+00 | 6.90000E+00 | 7.86749E+00 | 3.01705E+02 |
| 1.100E-01 | 19 | 0 | 1.125E-06 | 1 | 1.22519E+03 | 5.40029E+00 | 6.90000E+00 | 2.70948E+01 | 3.01720E+02 |
| 1.200E-01 | 20 | 0 | 2.831E-06 | 1 | 1.22540E+03 | 5.40103E+00 | 6.90000E+00 | 4.62634E+01 | 3.01743E+02 |
| 1.300E-01 | 21 | 0 | 4.517E-06 | 1 | 1.22573E+03 | 5.40214E+00 | 6.90000E+00 | 6.53148E+01 | 3.01761E+02 |
| 1.400E-01 | 22 | 0 | 6.165E-06 | 1 | 1.22624E+03 | 5.40364E+00 | 6.90000E+00 | 8.41960E+01 | 3.01840E+02 |
| 1.500E-01 | 23 | 0 | 7.761E-06 | 1 | 1.22698E+03 | 5.40552E+00 | 6.90000E+00 | 1.02856E+02 | 3.01927E+02 |
| 1.600E-01 | 24 | 0 | 9.287E-06 | 1 | 1.22800E+03 | 5.40776E+00 | 6.90000E+00 | 1.21247E+02 | 3.02047E+02 |
| 1.700E-01 | 25 | 0 | 1.077E-05 | 1 | 1.22934E+03 | 5.41037E+00 | 6.90000E+00 | 1.39323E+02 | 3.02206E+02 |
| 1.800E-01 | 26 | 0 | 1.201E-05 | 1 | 1.23104E+03 | 5.41334E+00 | 6.90000E+00 | 1.57047E+02 | 3.02409E+02 |

Tab.2-Program DEC results,ref.RAMP input.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.900E-01 | 27 | 0 | 1.315E-05 | 1 | 1.23313E+03 | 5.41666E+00 | 6.90000E+00 | 1.74385E+02 | 3.02663E+02 |
| 2.000E-01 | 28 | 0 | 1.412E-05 | 1 | 1.23563E+03 | 5.42032E+00 | 6.90000E+00 | 1.91313E+02 | 3.02971E+02 |
| 2.100E-01 | 29 | 0 | 1.495E-05 | 1 | 1.23856E+03 | 5.42432E+00 | 6.90000E+00 | 2.07812E+02 | 3.03337E+02 |
| 2.200E-01 | 30 | 0 | 1.569E-05 | 1 | 1.24193E+03 | 5.42864E+00 | 6.90000E+00 | 2.23871E+02 | 3.03765E+02 |
| 2.300E-01 | 31 | 0 | 1.641E-05 | 1 | 1.24576E+03 | 5.43329E+00 | 6.90000E+00 | 2.39478E+02 | 3.04258E+02 |
| 2.400E-01 | 32 | 0 | 1.724E-05 | 1 | 1.25006E+03 | 5.43823E+00 | 6.90000E+00 | 2.54618E+02 | 3.04823E+02 |
| 2.500E-01 | 33 | 0 | 1.838E-05 | 1 | 1.25486E+03 | 5.44348E+00 | 6.90000E+00 | 2.69264E+02 | 3.05466E+02 |
| 2.600E-01 | 34 | 0 | 2.006E-05 | 1 | 1.26021E+03 | 5.44902E+00 | 6.90000E+00 | 2.83367E+02 | 3.06197E+02 |
| 2.700E-01 | 35 | 0 | 2.252E-05 | 1 | 1.26622E+03 | 5.45483E+00 | 6.90000E+00 | 2.96838E+02 | 3.07035E+02 |
| 2.800E-01 | 36 | 0 | 2.533E-05 | 1 | 1.27301E+03 | 5.46090E+00 | 6.90000E+00 | 3.09548E+02 | 3.08003E+02 |
| 2.900E-01 | 37 | 0 | 2.783E-05 | 1 | 1.28071E+03 | 5.46722E+00 | 6.90000E+00 | 3.21366E+02 | 3.09126E+02 |
| 3.000E-01 | 38 | 0 | 2.942E-05 | 1 | 1.28942E+03 | 5.47377E+00 | 6.90000E+00 | 3.32186E+02 | 3.10427E+02 |
| 3.100E-01 | 39 | 0 | 2.844E-05 | 1 | 1.29915E+03 | 5.48033E+00 | 6.90000E+00 | 3.22621E+02 | 3.11916E+02 |
| 3.200E-01 | 40 | 0 | 2.235E-05 | 1 | 1.30970E+03 | 5.48669E+00 | 6.90000E+00 | 3.12182E+02 | 3.13568E+02 |
| 3.300E-01 | 41 | 0 | 1.001E-05 | 1 | 1.32054E+03 | 5.49283E+00 | 6.90000E+00 | 3.01322E+02 | 3.15305E+02 |
| 3.400E-01 | 42 | 0 | 5.967E-06 | 1 | 1.33091E+03 | 5.49876E+00 | 6.90000E+00 | 2.90788E+02 | 3.16994E+02 |
| 3.500E-01 | 43 | 0 | 1.418E-05 | 1 | 1.34022E+03 | 5.50449E+00 | 6.90000E+00 | 2.81252E+02 | 3.18520E+02 |
| 3.600E-01 | 44 | 0 | 1.170E-05 | 1 | 1.34847E+03 | 5.51004E+00 | 6.90000E+00 | 2.72838E+02 | 3.19870E+02 |
| 3.700E-01 | 45 | 0 | 4.901E-06 | 1 | 1.35595E+03 | 5.51543E+00 | 6.90000E+00 | 2.65267E+02 | 3.21092E+02 |
| 3.800E-01 | 46 | 0 | 8.570E-07 | 1 | 1.36300E+03 | 5.52067E+00 | 6.90000E+00 | 2.58179E+02 | 3.22247E+02 |

Tab.2-continuation.

2. <u>DEC</u> : RAMP: 1200., 1620., 0.083, 0.3

Constant velocity error: $E_v = \Delta_s$ = 58.235(RPM)

Error coefficient : $K_v = \dfrac{1}{E_v}$ = 0.01717

After compensation the steady-state error (constant velocity error) decreases from 301.76 RPM to 58.235 (RPM) only, and at the same time the steady-state error coefficient $K_v$ increases from : 0.003313 to 0.01717.

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By Benjamin F. Kuo 1963

2. Introduction to Control Theory for Engineers
Blackie & Son Ltd.
By Allan Sensicle 1968

## 8.4. DISCUSSION

Basic considerations for the design of a suitable control system in terms of BODE diagrams have been discussed in Chapter 6 and 7. The root-locus method is very helpful in the determination of compensation network structures and stability, and it is generally regarded that this method is complementary to the BODE diagrams when determining the behaviour of the systems. In this chapter Program RLT (Root Locus) is used in a further comparison of performance and analysis of both PCS and DEC models. Also evaluation of the steady-state error (constant velocity) are discussed using standard RAMP function input signal. In addition to this completely automatic program (RLT), a classical method of root loci analysis is also applied. Both methods' results lead to practically the same conclusions, and in the case of compensated systems, it is proved that this system is unconditionally stable for any practical gain values. It is also shown in section 8.3. that the steady-state error coefficient $K_v$ increases from 0.003313 to 0.01717, when comparing both compensated and uncompensated systems, in exactly the same operational conditions, using SYSTRAN's RAMP input. It has been found that the reduction of earlier indicated PHASE/GAIN margins does not lead to any practical problems in the system after compensation.

# 9. FURTHER SYSTEM ADJUSTMENTS TO ACHIEVE OPTIMAL DERIVATIVE ERROR CONTROL

## 9.1. SUMMARY AND DISCUSSION OF DEC PROGRAM RESULTS

Let us again consider the DEC program run at specific temperature/pressure conditions - 900°C and 6.90 bar with the appropriate fuel level (as specified by WALKER), and then investigate the accuracy requirements of the Derivative Error Compensator TD1 coefficient (for details see Chapter 7) with respect to YSET/YDOTD interrelation. In order to determine the TD1 tolerance necessary to obtain the relevant type of accuracy, it is wise to examine the error produced in the worst case. With TD1 (shown graphically in Fig.1.), the speed error increases dramatically with any YSET values below 1750 RPM. As proved earlier to obtain minimum speed error runs, the TD1 value should be decreased substantially for the various YSET runs, as TD1 acts exactly in the opposite direction to the damping ratio, so called in "classical" second order systems analysis. Increased value of TD1 leads to an effect similar to the underdamping of the system, and decreased values of TD1 cause overdamping.

According to previously described theory, too large a TD1 value may give an oscillating output (system underdamped), consequently the system becomes unstable. In our situation TD1 should be adjusted in a analogous manner, to meet the approximate critically damped conditions, when the output signal rises to the required

| YSET (RPM) | 1800 | 1780 | 1750 | 1620 | 1440 | 1230 | 1200 |
|---|---|---|---|---|---|---|---|
| TD1 | 1.110 | 1.105 | 1.100 | 0.80 | 0.20 | 0.075 | 0.070 |

T= 900 [°C]
p= 6.90 [bar]
f = 5.40 [g/min]

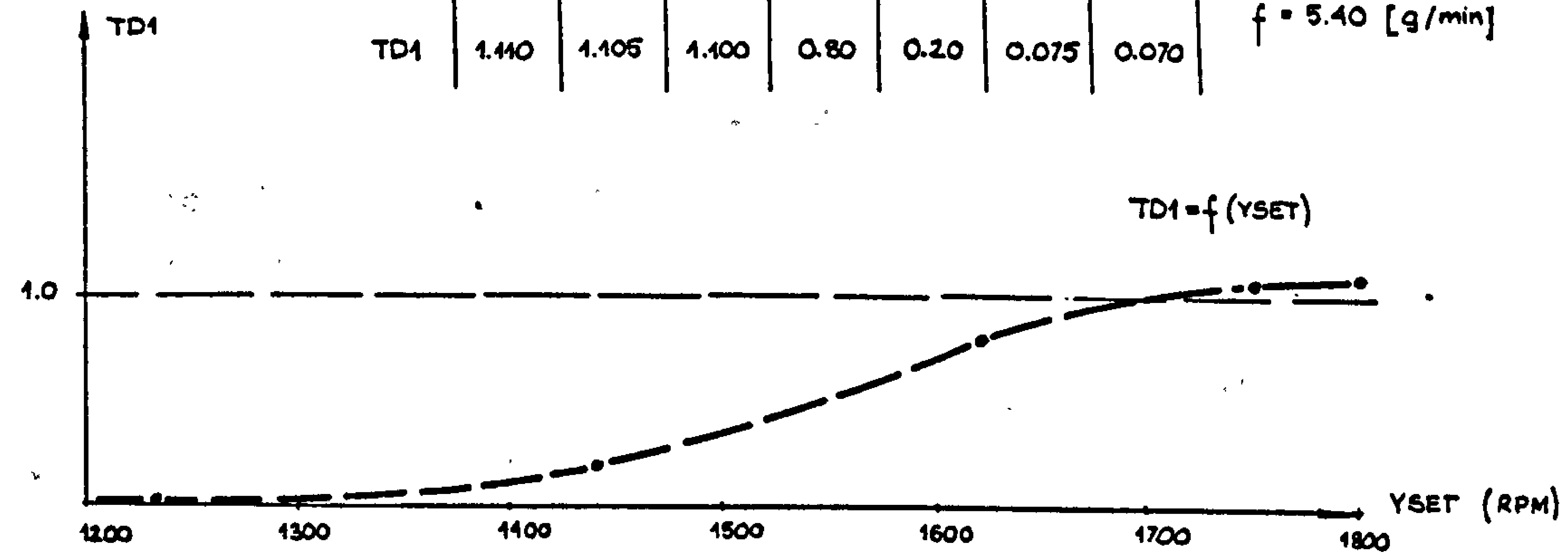



Fig.2 TD1=$f_2$(YSET) plot Ref.T=900$^{o}$C,p=6.90bar,f=5.40g/min.

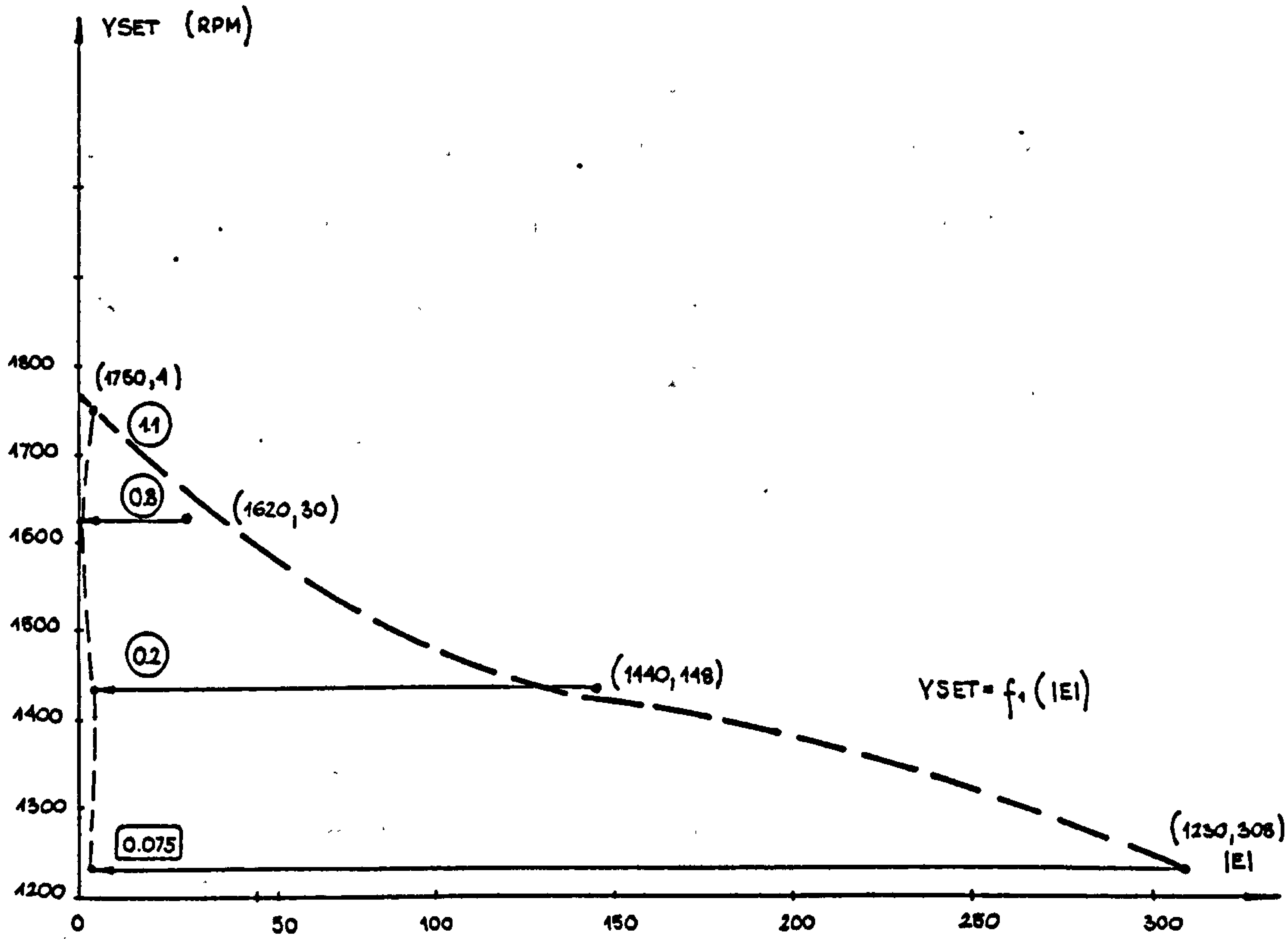



Fig.1 YSET=$f_1$(|E|) plot for modified TD1 parameter range.

level, but with small or no overshoot. It will be shown later, that a slight overshoot may occur (less than 2% of the nominal value) but the advantage of this as compared with a "critical" state is that the system responds much faster. The optimal criterion evaluating this effect is widely recognised as ITAE (Integral of Time Multiplied by the Absolute Value of Error). The trial and error method gives immediate results. With TD1 values adjusted as shown in Fig.1. (values in circles), the YDOT steady-state values are virtually equal to those called by YSET.
In order to incorporate TD1 parameter value modifications for various YSET levels, NAG subroutine EØ2ACF (see Lit.1.) calculates a minimax polynomial fit to a set of data points (sic) as a series of Chebyshev polynomials. Given a set of data points (see Fig.2) in the two arrays form, it computes on the Mth order of Chebyshev polynomial:

D(Y) =E(1)+E(2)*Y(1)+E(3)+Y(1)**2+E(4)*Y(1)**3

such that $\max/D(Y)_i - TD1_i/$ is minimum.

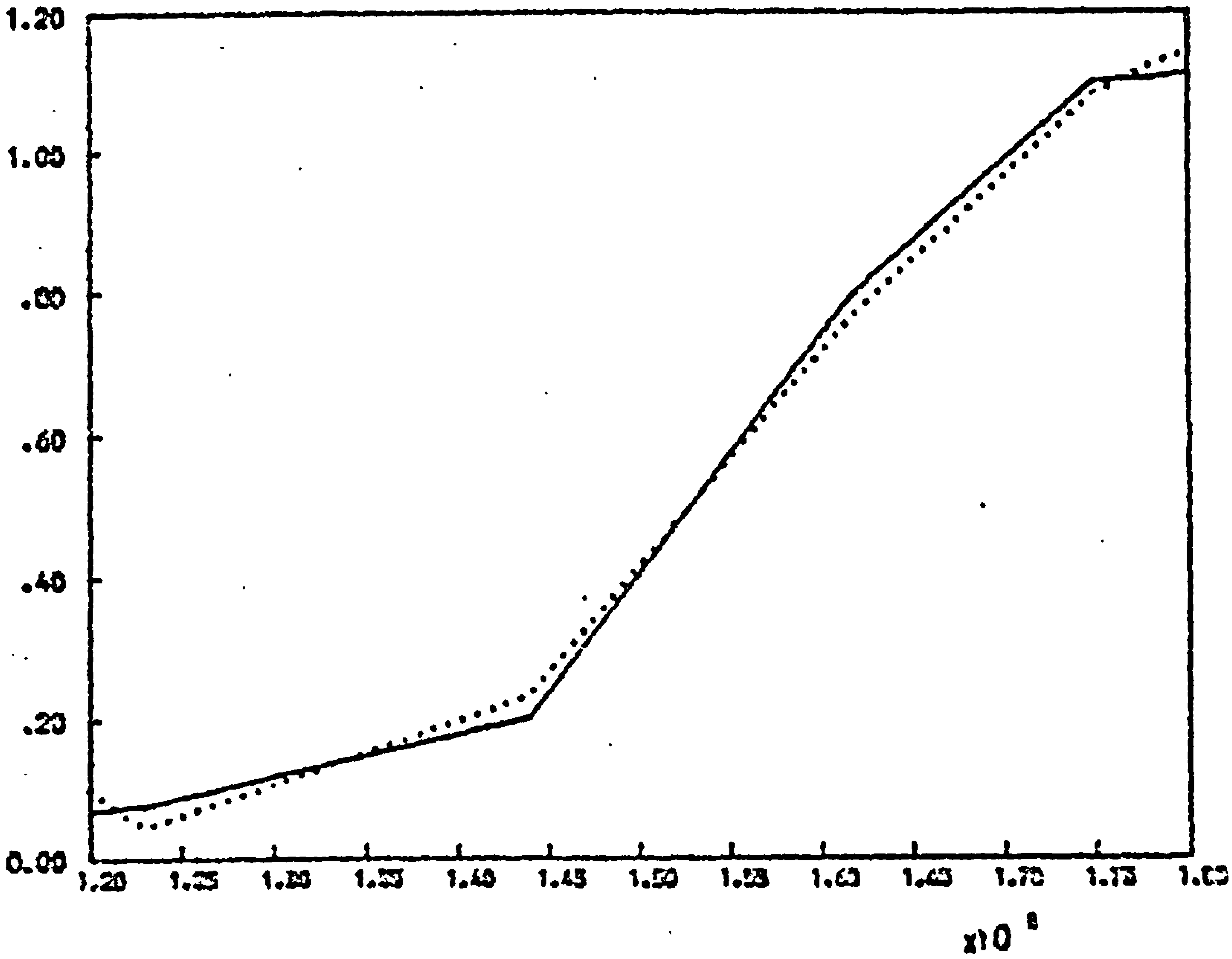
Program APPROX (Program No.1.) with exact arithmetic should terminate after a finite number of steps, also results (apart from calculated Chebyshev polynomial co-efficients and the function value) include a plot of TD1 = f(YSET) as shown in Fig.3. In order to obtain more accurate results more input data should be provided (compare with Fig.2), and a higher polynomial order should be tried. At this stage the TD1=F(YSET) as a polynomial function form can easily be incorporated in a less discrete form of Program DEC "EQNS" as shown fragmentally in Program No.2 listing.

```
:RSITY OF MINNESOTA 6600 FORTRAN COMPILER SCOPE  3.4.1 VER4.5     15
3=LFN)
      PROGRAM APPROX(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT
     1TAPE27=PLOT)
      DIMENSION T(20),Y(20),E(20),S(20)
      T(1)=0.070
      T(2)=0.075
      T(3)=0.20
      T(4)=0.80
      T(5)=1.10
      T(6)=1.105
      T(7)=1.110
      Y(1)=1200.
      Y(2)=1230.
      Y(3)=1440.
      Y(4)=1620.
      Y(5)=1750.
      Y(6)=1780.
      Y(7)=1800.
      N=7
      M1=4
      CALL E02ACF(Y,T,7,E,M1,REF)
      WRITE(6,200)
  200 FORMAT(1H ,24H POLYNOMIAL COEFFICIENTS//)
      WRITE(6,201) (E(I),I=1,M1)
  201 FORMAT(1H,5X,1PE20.8)
      DO10 I=1,7
      D=E(1)+E(2)*Y(I)+E(3)*Y(I)**2+E(4)*Y(I)**3
      S(I)=D
      WRITE(6,202) D
  202 FORMAT (1H , F10.4)
   10 CONTINUE
      CALL CAM35MM
      CALL GRSLIDE
      CALL XAXIS(1200.,1800.)
      CALL YAXIS(0.0,1.2)
      CALL GRAPHIC(Y,T,N)
      CALL DOT
      CALL GRAPHIC(Y,S,N)
      CALL DASHOFF
      CALL LXTICK
      CALL LYTICK
      CALL LXVAL
      CALL LYVAL
      CALL GRFRAME
      CALL ENDFILM
      STOP
      END
```

```
 POLYNOMIAL COEFFICIENTS

           4.90520799E+01
          -9.96179444E-02
           6.59503096E-05
          -1.41081361E-08
     .1001
     .0449
     .2301
     .7699
    1.0827
    1.1227
    1.1401
 2366 CHARACTERS OUTPUT
```

Progr.1 Program APPROX




[*EOF*]

Fig.3. TD1 parameter vs RPM speed @ p = 6.90 bar.

```
      SUBROUTINE EQNS
CVERSION
CVERSION              THIS VERSION   28/02/77
CVERSION
      COMMON /VBLES/ YDOT,FUEL,ERROR,TEMPER,PRSSUR,YDOTD,POWER,TORQUE,
     1FUELD,DERROR,ERROR0,DUM(29),E,DFUEL
      COMMON /CONTRL/TP,TE,TF,ICON
      COMMON/DFHK/ G1,G2,G3,TD1,TD2,TD3,X1,X2,X3
      DIMENSION PRESS(10),FUELS(10),C(25),TEMP(10),SPEEDS(25),CURR(10),
     *  FEEDB1(21,6),FEEDB2(21,6),FEEDB3(21,6),FEEDB4(21,6),DTEMP(10),
     *  WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4)
      DATA N,NT,N2,NP,NS,IG1,IFAIL/3,4,6,6,21,21,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      E1=4.90520799E+01
      E2=-9.96179444E-02
      E3=6.59503096E-05
      E4=-1.41081361E-08
      TD2=0.8
      TD3=0.2
      W=0.01
      Y=100.
      V=50.
      Z=14625.0
      DELAY=Y/60000.
      DELAYV=V/60000.
      TD=0.0
      YDOT=0.
      TEMPER=900.
      YSET=1750.
      TD1=E1+E2*YSET+E3*YSET**2+E4*YSET**3
      PRSSUR=6.90
      CALL SLGAIN (PRSSUR,TEMPER,DELAY,DELAYV,ICON)
      FUFL=5.4
      DFUEL=0.0
C
```

Progr.2 Program DEC(modified)
-EQNS part only

The modified DEC program results are presented for a range of YSET values i.e. 1750 RPM, 1620 RPM, 1440 RPM, 1230 RPM. For each program run the output file includes the following : tabulated with results, usual SYSTRAN plot; TORQUE/SPEED SCM characteristic :

YSET = 1750 RPM

| | | |
|---|---|---|
| Tab.1. | Tabulated results | |
| Fig.4. | Systran Plot | |
| Fig.5. | TORQUE/SPEED | Ch - k |

YSET = 162o RPM

| | | |
|---|---|---|
| Tab.2. | Tabulated Results | |
| Fig.6. | Systran Plot | |
| Fig.7. | TORQUE/SPEED | Ch - k |

YSET = 1440 RPM

| | | |
|---|---|---|
| Tab.3. | Tabulated results | |
| Fig.8. | Systran Plot | |
| Fig.9. | TORQUE/SPEED | Ch - k |

YSET = 1230 RPM

| | | |
|---|---|---|
| Tab. 4. | Tabulated Results | |
| Fig.10. | Systran Plot | |
| Fig. 11. | TORQUE/SPEED | Ch - k |

Remaining speed error better than 1.5% can be regarded as satisfactory and can be minimised to the required speed accuracy tolerance by means of simple manipulation of the EØ2ACF subroutine input data.

One general observation is noticed, in using minimax fit of Chebyshev polynomials in the DEC program, it is possible to achieve quite a remarkable degree of fit accuracy on the

INITIAL PRESSURE, TEMPERATURE = 6.900; 900.0

| Time | R-K | N-R | N.ERROR | X | x1 | x2 | x3 | x4 | x5 | x6 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 1.75000E+03 | 6.90000E+00 | 0. | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 1.62813E+03 | 6.90000E+00 | 0. | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 1.48188E+03 | 6.90000E+00 | 0. | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 1.33563E+03 | 6.90000E+00 | 0. | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 1.18938E+03 | 6.90000E+00 | 0. | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 1.04313E+03 | 6.90000E+00 | 0. | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 8.96875E+02 | 6.90000E+00 | 0. | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 7.50625E+02 | 6.90000E+00 | 0. | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 6.04375E+02 | 6.90000E+00 | 0. | 0. |
| 9.000E-02 | 16 | 0 | 9.825E-04 | 1 | 1.23205E+03 | 5.55941E+00 | 5.28658E+02 | 6.90000E+00 | 3.02488E+02 | 2.34534E+00 |
| 1.000E-01 | 19 | 0 | 1.220E-04 | 1 | 1.26393E+03 | 5.69519E+00 | 4.92016E+02 | 6.90000E+00 | 3.06627E+02 | 2.31803E+00 |
| 1.100E-01 | 21 | 0 | 1.627E-04 | 2 | 1.31116E+03 | 5.79037E+00 | 4.47231E+02 | 6.90000E+00 | 3.13723E+02 | 2.28596E+00 |
| 1.200E-01 | 23 | 0 | 2.019E-04 | 2 | 1.36998E+03 | 5.90769E+00 | 3.90026E+02 | 6.90000E+00 | 3.23357E+02 | 2.25433E+00 |
| 1.300E-01 | 25 | 0 | 8.396E-05 | 1 | 1.43647E+03 | 6.03326E+00 | 3.24708E+02 | 6.90000E+00 | 3.33998E+02 | 2.22051E+00 |
| 1.400E-01 | 26 | 0 | 4.418E-04 | 2 | 1.49809E+03 | 6.14802E+00 | 2.61380E+02 | 6.90000E+00 | 3.42538E+02 | 2.18205E+00 |
| 1.500E-01 | 27 | 0 | 3.758E-04 | 2 | 1.54797E+03 | 6.25009E+00 | 2.09269E+02 | 6.90000E+00 | 3.47495E+02 | 2.14188E+00 |
| 1.600E-01 | 28 | 0 | 2.982E-04 | 2 | 1.58583E+03 | 6.33161E+00 | 1.69182E+02 | 6.90000E+00 | 3.49714E+02 | 2.10378E+00 |
| 1.700E-01 | 29 | 0 | 1.202E-04 | 1 | 1.61365E+03 | 6.38636E+00 | 1.40188E+02 | 6.90000E+00 | 3.50310E+02 | 2.07184E+00 |
| 1.800E-01 | 30 | 0 | 1.355E-04 | 1 | 1.63390E+03 | 6.39934E+00 | 1.18564E+02 | 6.90000E+00 | 3.50148E+02 | 2.04506E+00 |
| 1.900E-01 | 31 | 0 | 1.077E-04 | 1 | 1.64693E+03 | 6.40886E+00 | 1.04506E+02 | 6.90000E+00 | 3.49757E+02 | 2.02689E+00 |
| 2.000E-01 | 32 | 0 | 5.001E-05 | 1 | 1.65601E+03 | 6.41560E+00 | 9.51600E+01 | 6.90000E+00 | 3.49363E+02 | 2.01409E+00 |
| 2.100E-01 | 33 | 0 | 2.183E-05 | 1 | 1.66318E+03 | 6.42083E+00 | 8.78683E+01 | 6.90000E+00 | 3.48967E+02 | 2.00341E+00 |
| 2.200E-01 | 34 | 0 | 1.077E-05 | 1 | 1.66933E+03 | 6.42531E+00 | 8.16242E+01 | 6.90000E+00 | 3.48561E+02 | 1.99382E+00 |
| 2.300E-01 | 35 | 0 | 8.059E-06 | 2 | 1.67486E+03 | 6.42933E+00 | 7.60186E+01 | 6.90000E+00 | 3.48144E+02 | 1.98490E+00 |

Tab.1 Program DEC(modified) results,Ref: YSET= 1750 RPM

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.600E-01 | 38 | 0 | 6.940E-06 | 2 | 1.68920E+03 | 6.43966E+00 | 6.15234E+01 | 6.90000E+00 | 3.46833E+02 | 1.96067E+00 |
| 2.700E-01 | 39 | 0 | 6.472E-06 | 2 | 1.69334E+03 | 6.44265E+00 | 5.73279E+01 | 6.90000E+00 | 3.46391E+02 | 1.95338E+00 |
| 2.800E-01 | 40 | 0 | 6.010E-06 | 2 | 1.69717E+03 | 6.44543E+00 | 5.34445E+01 | 6.90000E+00 | 3.45958E+02 | 1.94653E+00 |
| 2.900E-01 | 41 | 0 | 5.541E-06 | 2 | 1.70068E+03 | 6.44801E+00 | 4.98804E+01 | 6.90000E+00 | 3.45539E+02 | 1.94016E+00 |
| 3.000E-01 | 42 | 0 | 5.062E-06 | 2 | 1.70386E+03 | 6.45038E+00 | 4.66415E+01 | 6.90000E+00 | 3.45140E+02 | 1.93430E+00 |
| 3.100E-01 | 43 | 0 | 4.577E-06 | 2 | 1.70672E+03 | 6.45256E+00 | 4.37288E+01 | 6.90000E+00 | 3.44768E+02 | 1.92898E+00 |
| 3.200E-01 | 44 | 0 | 4.094E-06 | 2 | 1.70926E+03 | 6.45455E+00 | 4.11371E+01 | 6.90000E+00 | 3.44426E+02 | 1.92420E+00 |
| 3.300E-01 | 45 | 0 | 3.625E-06 | 2 | 1.71149E+03 | 6.45636E+00 | 3.88533E+01 | 6.90000E+00 | 3.44115E+02 | 1.91996E+00 |
| 3.400E-01 | 46 | 0 | 3.231E-06 | 1 | 1.71344E+03 | 6.45800E+00 | 3.68573E+01 | 6.90000E+00 | 3.43837E+02 | 1.91623E+00 |
| 3.500E-01 | 47 | 0 | 2.805E-06 | 1 | 1.71514E+03 | 6.45950E+00 | 3.51205E+01 | 6.90000E+00 | 3.43590E+02 | 1.91296E+00 |
| 3.600E-01 | 48 | 0 | 2.407E-06 | 2 | 1.71662E+03 | 6.46086E+00 | 3.36098E+01 | 6.90000E+00 | 3.43371E+02 | 1.91010E+00 |
| 3.700E-01 | 49 | 0 | 2.093E-06 | 2 | 1.71791E+03 | 6.46211E+00 | 3.22922E+01 | 6.90000E+00 | 3.43177E+02 | 1.90759E+00 |
| 3.800E-01 | 50 | 0 | 1.826E-06 | 2 | 1.71904E+03 | 6.46326E+00 | 3.11375E+01 | 6.90000E+00 | 3.43004E+02 | 1.90538E+00 |
| 3.900E-01 | 51 | 0 | 1.602E-06 | 2 | 1.72004E+03 | 6.46433E+00 | 3.01188E+01 | 6.90000E+00 | 3.42849E+02 | 1.90341E+00 |
| 4.000E-01 | 52 | 0 | 1.415E-06 | 2 | 1.72093E+03 | 6.46533E+00 | 2.92133E+01 | 6.90000E+00 | 3.42710E+02 | 1.90166E+00 |
| 4.100E-01 | 53 | 0 | 1.260E-06 | 2 | 1.72173E+03 | 6.46627E+00 | 2.84016E+01 | 6.90000E+00 | 3.42584E+02 | 1.90008E+00 |
| 4.200E-01 | 54 | 0 | 1.131E-06 | 2 | 1.72245E+03 | 6.46716E+00 | 2.76676E+01 | 6.90000E+00 | 3.42469E+02 | 1.89865E+00 |
| 4.300E-01 | 55 | 0 | 1.025E-06 | 2 | 1.72311E+03 | 6.46800E+00 | 2.69979E+01 | 6.90000E+00 | 3.42362E+02 | 1.89733E+00 |
| 4.400E-01 | 56 | 0 | 9.371E-07 | 2 | 1.72372E+03 | 6.46881E+00 | 2.63816E+01 | 6.90000E+00 | 3.42264E+02 | 1.89612E+00 |
| 4.500E-01 | 57 | 0 | 8.641E-07 | 2 | 1.72428E+03 | 6.46958E+00 | 2.58097E+01 | 6.90000E+00 | 3.42172E+02 | 1.89499E+00 |
| 4.600E-01 | 58 | 0 | 8.035E-07 | 2 | 1.72481E+03 | 6.47033E+00 | 2.52746E+01 | 6.90000E+00 | 3.42085E+02 | 1.89393E+00 |
| 4.700E-01 | 59 | 0 | 7.530E-07 | 2 | 1.72531E+03 | 6.47105E+00 | 2.47704E+01 | 6.90000E+00 | 3.42002E+02 | 1.89292E+00 |
| 4.800E-01 | 60 | 0 | 7.108E-07 | 2 | 1.72579E+03 | 6.47175E+00 | 2.42920E+01 | 6.90000E+00 | 3.41924E+02 | 1.89197E+00 |
| 4.900E-01 | 61 | 0 | 6.755E-07 | 2 | 1.72624E+03 | 6.47244E+00 | 2.38353E+01 | 6.90000E+00 | 3.41848E+02 | 1.89105E+00 |
| 5.000E-01 | 62 | 0 | 6.457E-07 | 2 | 1.72668E+03 | 6.47310E+00 | 2.33970E+01 | 6.90000E+00 | 3.41775E+02 | 1.89017E+00 |
| 5.100E-01 | 63 | 0 | 6.206E-07 | 2 | 1.72710E+03 | 6.47375E+00 | 2.29743E+01 | 6.90000E+00 | 3.41704E+02 | 1.88932E+00 |
| 5.200E-01 | 64 | 0 | 5.992E-07 | 2 | 1.72751E+03 | 6.47439E+00 | 2.25649E+01 | 6.90000E+00 | 3.41635E+02 | 1.88849E+00 |
| 5.300E-01 | 65 | 0 | 5.810E-07 | 2 | 1.72790E+03 | 6.47501E+00 | 2.21670E+01 | 6.90000E+00 | 3.41568E+02 | 1.88769E+00 |
| 5.400E-01 | 66 | 0 | 5.653E-07 | 2 | 1.72829E+03 | 6.47562E+00 | 2.17788E+01 | 6.90000E+00 | 3.41502E+02 | 1.88690E+00 |

Tab.1 continuation

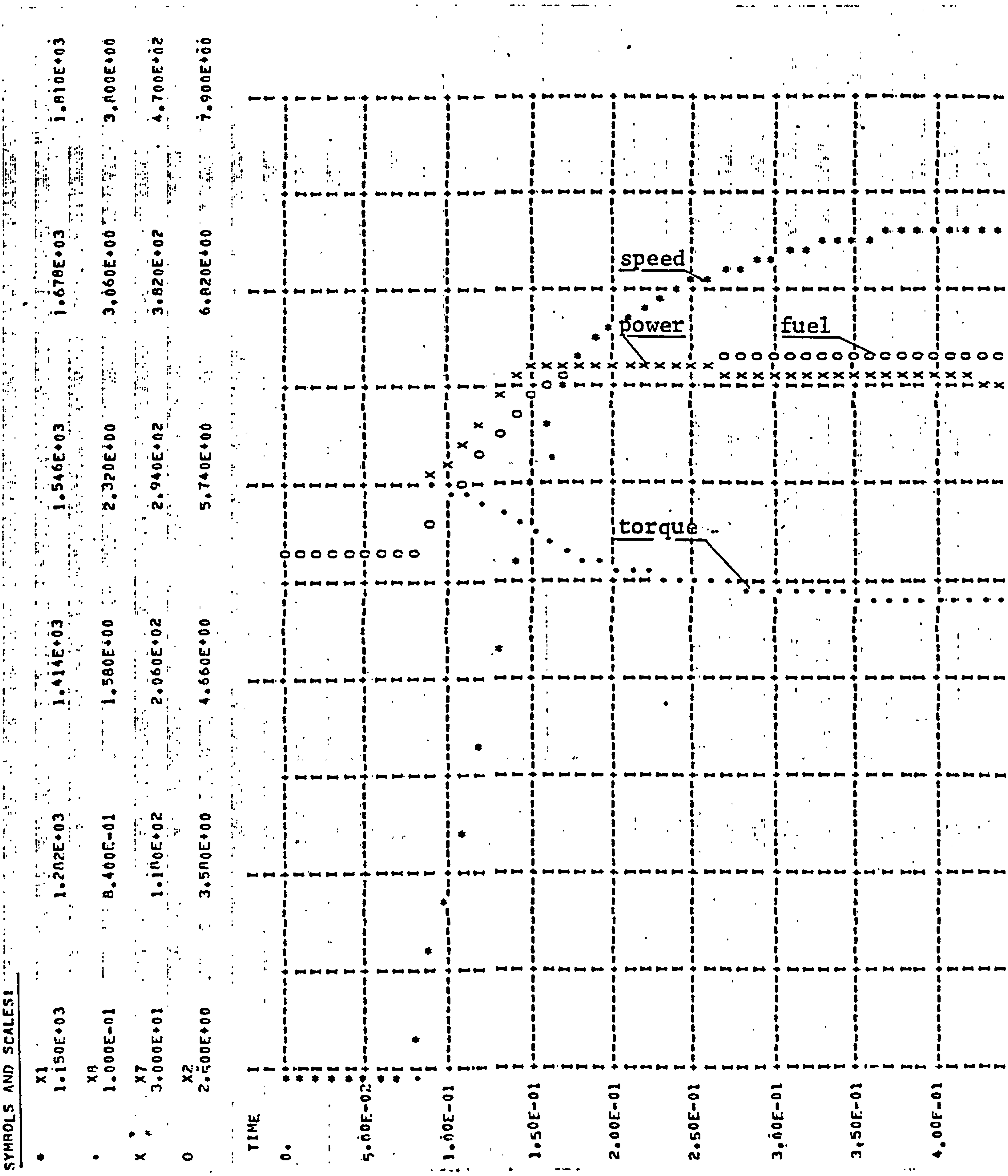



Fig.4 Program DEC(modified) results,Ref:YSET=1750 RPM

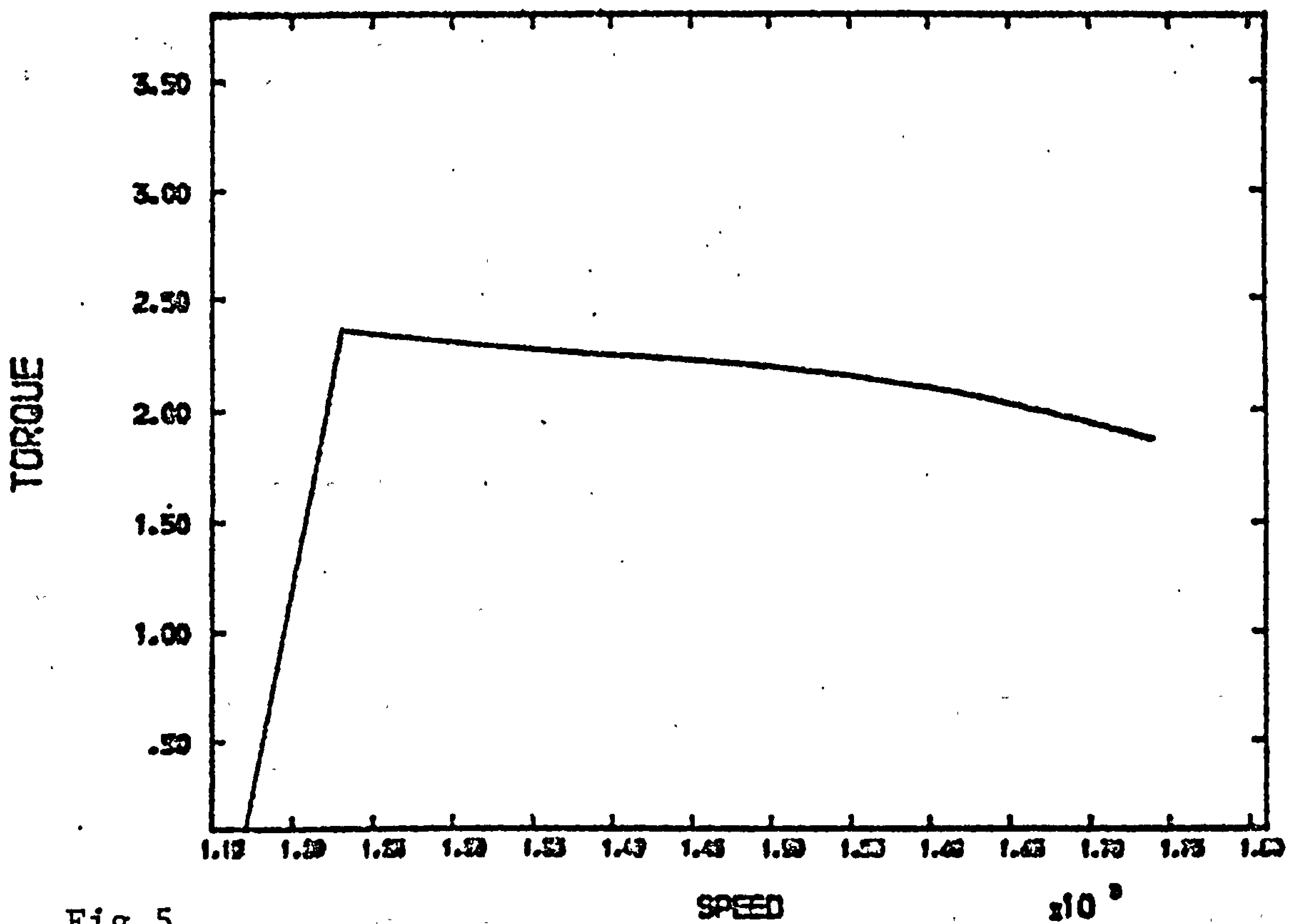



Fig.5.
[*EOF*]

SCM TORQUE/SPEED CHARACTERISTIC

Ref. YSET = 1750 RPM

INITIAL PRESSURE, TEMPERATURE = 6.900, 900.0

| Time | R-K | N-R | N.ERROR | K | X1 | X2 | X3 | X4 | X5 | X6 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 1.62000E+03 | 6.90000E+00 | 0. | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 1.49813E+03 | 6.90000E+00 | 0. | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 1.35188E+03 | 6.90000E+00 | 0. | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 1.20563E+03 | 6.90000E+00 | 0. | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 1.05938E+03 | 6.90000E+00 | 0. | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 9.13125E+02 | 6.90000E+00 | 0. | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 7.66875E+02 | 6.90000E+00 | 0. | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 6.20625E+02 | 6.90000E+00 | 0. | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 4.74375E+02 | 6.90000E+00 | 0. | 0. |
| 9.000E-02 | 16 | 0 | 9.867E-04 | 1 | 1.23000E+03 | 5.51313E+00 | 4.00263E+02 | 6.90000E+00 | 3.02255E+02 | 2.34720E+00 |
| 1.000E-01 | 18 | 0 | 8.105E-05 | 1 | 1.25245E+03 | 5.60361E+00 | 3.71714E+02 | 6.90000E+00 | 3.05089E+02 | 2.32709E+00 |
| 1.100E-01 | 20 | 0 | 7.850E-05 | 2 | 1.28387E+03 | 5.65501E+00 | 3.41637E+02 | 6.90000E+00 | 3.09573E+02 | 2.30315E+00 |
| 1.200E-01 | 22 | 0 | 9.737E-05 | 2 | 1.32294E+03 | 5.71286E+00 | 3.03825E+02 | 6.90000E+00 | 3.15652E+02 | 2.27897E+00 |
| 1.300E-01 | 24 | 0 | 1.076E-04 | 2 | 1.36652E+03 | 5.78052E+00 | 2.60783E+02 | 6.90000E+00 | 3.22820E+02 | 2.25596E+00 |
| 1.400E-01 | 26 | 0 | 1.311E-04 | 2 | 1.41294E+03 | 5.85395E+00 | 2.14865E+02 | 6.90000E+00 | 3.30356E+02 | 2.23283E+00 |
| 1.500E-01 | 28 | 0 | 1.221E-04 | 2 | 1.45905E+03 | 5.93540E+00 | 1.68329E+02 | 6.90000E+00 | 3.37374E+02 | 2.20752E+00 |
| 1.600E-01 | 30 | 0 | 9.827E-05 | 2 | 1.49724E+03 | 6.01142E+00 | 1.28809E+02 | 6.90000E+00 | 3.42369E+02 | 2.18305E+00 |
| 1.700E-01 | 32 | 0 | 7.960E-05 | 2 | 1.52840E+03 | 6.07452E+00 | 9.65496E+01 | 6.90000E+00 | 3.45714E+02 | 2.15957E+00 |
| 1.800E-01 | 34 | 0 | 6.317E-05 | 2 | 1.55329E+03 | 6.12571E+00 | 7.05762E+01 | 6.90000E+00 | 3.47823E+02 | 2.13788E+00 |
| 1.900E-01 | 36 | 0 | 4.571E-05 | 2 | 1.57151E+03 | 6.16561E+00 | 5.12810E+01 | 6.90000E+00 | 3.48989E+02 | 2.12024E+00 |
| 2.000E-01 | 37 | 0 | 1.066E-04 | 2 | 1.58424E+03 | 6.19464E+00 | 3.73775E+01 | 6.90000E+00 | 3.49614E+02 | 2.10656E+00 |
| 2.100E-01 | 38 | 0 | 7.493E-05 | 2 | 1.59354E+03 | 6.21495E+00 | 2.77152E+01 | 6.90000E+00 | 3.49936E+02 | 2.09653E+00 |
| 2.200E-01 | 39 | 0 | 5.559E-05 | 2 | 1.60070E+03 | 6.22976E+00 | 2.02439E+01 | 6.90000E+00 | 3.50123E+02 | 2.08834E+00 |
| 2.300E-01 | 40 | 0 | 8.765E-05 | 2 | 1.60630E+03 | 6.23437E+00 | 1.44318E+01 | 6.90000E+00 | 3.50226E+02 | 2.08177E+00 |

Tab.2 Program DEC(modified) results,Ref: YSET=1620 RPM

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.400E-01 | 41 | 0 | 2.845E-05 | 1 | 1.61009E+03 | 6.23666E+00 | 1.03405E+01 | 6.90000E+00 | 3.50275E+02 | 2.07716E+00 |
| 2.500E-01 | 42 | 0 | 1.921E-05 | 1 | 1.61256E+03 | 6.23820E+00 | 7.72263E+00 | 6.90000E+00 | 3.50298E+02 | 2.07420E+00 |
| 2.600E-01 | 43 | 0 | 1.291E-05 | 1 | 1.61420E+03 | 6.23922E+00 | 5.98288E+00 | 6.90000E+00 | 3.50308E+02 | 2.07222E+00 |
| 2.700E-01 | 44 | 0 | 8.597E-06 | 1 | 1.61529E+03 | 6.23991E+00 | 4.83088E+00 | 6.90000E+00 | 3.50313E+02 | 2.07090E+00 |
| 2.800E-01 | 45 | 0 | 5.671E-06 | 1 | 1.61601E+03 | 6.24039E+00 | 4.06897E+00 | 6.90000E+00 | 3.50316E+02 | 2.07002E+00 |
| 2.900E-01 | 46 | 0 | 3.712E-06 | 1 | 1.61649E+03 | 6.24072E+00 | 3.56440E+00 | 6.90000E+00 | 3.50318E+02 | 2.06944E+00 |
| 3.000E-01 | 47 | 0 | 2.410E-06 | 1 | 1.61681E+03 | 6.24096E+00 | 3.22883E+00 | 6.90000E+00 | 3.50318E+02 | 2.06905E+00 |
| 3.100E-01 | 48 | 0 | 1.551E-06 | 1 | 1.61702E+03 | 6.24114E+00 | 3.00385E+00 | 6.90000E+00 | 3.50319E+02 | 2.06879E+00 |
| 3.200E-01 | 49 | 0 | 9.865E-07 | 1 | 1.61717E+03 | 6.24127E+00 | 2.85108E+00 | 6.90000E+00 | 3.50319E+02 | 2.06861E+00 |
| 3.300E-01 | 50 | 0 | 6.177E-07 | 1 | 1.61727E+03 | 6.24138E+00 | 2.74537E+00 | 6.90000E+00 | 3.50319E+02 | 2.06849E+00 |
| 3.400E-01 | 51 | 0 | 3.772E-07 | 1 | 1.61734E+03 | 6.24147E+00 | 2.67033E+00 | 6.90000E+00 | 3.50320E+02 | 2.06840E+00 |
| 3.500E-01 | 52 | 0 | 2.207E-07 | 1 | 1.61739E+03 | 6.24155E+00 | 2.61527E+00 | 6.90000E+00 | 3.50320E+02 | 2.06833E+00 |
| 3.600E-01 | 53 | 0 | 1.191E-07 | 1 | 1.61743E+03 | 6.24163E+00 | 2.57322E+00 | 6.90000E+00 | 3.50320E+02 | 2.06828E+00 |
| 3.700E-01 | 54 | 0 | 5.665E-08 | 2 | 1.61747E+03 | 6.24170E+00 | 2.53968E+00 | 6.90000E+00 | 3.50320E+02 | 2.06824E+00 |
| 3.800E-01 | 55 | 0 | 4.558E-08 | 2 | 1.61749E+03 | 6.24176E+00 | 2.51170E+00 | 6.90000E+00 | 3.50320E+02 | 2.06821E+00 |
| 3.900E-01 | 56 | 0 | 3.833E-08 | 2 | 1.61752E+03 | 6.24182E+00 | 2.48737E+00 | 6.90000E+00 | 3.50320E+02 | 2.06818E+00 |
| 4.000E-01 | 57 | 0 | 3.451E-08 | 1 | 1.61754E+03 | 6.24188E+00 | 2.46546E+00 | 6.90000E+00 | 3.50320E+02 | 2.06815E+00 |
| 4.100E-01 | 58 | 0 | 4.581E-08 | 1 | 1.61756E+03 | 6.24194E+00 | 2.44517E+00 | 6.90000E+00 | 3.50320E+02 | 2.06812E+00 |
| 4.200E-01 | 59 | 0 | 5.298E-08 | 1 | 1.61758E+03 | 6.24200E+00 | 2.42597E+00 | 6.90000E+00 | 3.50320E+02 | 2.06810E+00 |
| 4.300E-01 | 60 | 0 | 5.747E-08 | 1 | 1.61760E+03 | 6.24205E+00 | 2.40751E+00 | 6.90000E+00 | 3.50320E+02 | 2.06808E+00 |
| 4.400E-01 | 61 | 0 | 6.021E-08 | 1 | 1.61761E+03 | 6.24211E+00 | 2.38959E+00 | 6.90000E+00 | 3.50320E+02 | 2.06805E+00 |
| 4.500E-01 | 62 | 0 | 6.183E-08 | 1 | 1.61763E+03 | 6.24217E+00 | 2.37206E+00 | 6.90000E+00 | 3.50320E+02 | 2.06803E+00 |
| 4.600E-01 | 63 | 0 | 6.272E-08 | 1 | 1.61765E+03 | 6.24222E+00 | 2.35482E+00 | 6.90000E+00 | 3.50320E+02 | 2.06801E+00 |
| 4.700E-01 | 64 | 0 | 6.313E-08 | 1 | 1.61767E+03 | 6.24228E+00 | 2.33782E+00 | 6.90000E+00 | 3.50320E+02 | 2.06799E+00 |
| 4.800E-01 | 65 | 0 | 6.324E-08 | 1 | 1.61768E+03 | 6.24233E+00 | 2.32101E+00 | 6.90000E+00 | 3.50320E+02 | 2.06797E+00 |
| 4.900E-01 | 66 | 0 | 6.315E-08 | 1 | 1.61770E+03 | 6.24239E+00 | 2.30437E+00 | 6.90000E+00 | 3.50320E+02 | 2.06794E+00 |
| 5.000E-01 | 67 | 0 | 6.294E-08 | 1 | 1.61772E+03 | 6.24244E+00 | 2.28780E+00 | 6.90000E+00 | 3.50320E+02 | 2.06792E+00 |
| 5.100E-01 | 68 | 0 | 6.265E-08 | 1 | 1.61773E+03 | 6.24249E+00 | 2.27153E+00 | 6.90000E+00 | 3.50320E+02 | 2.06790E+00 |
| 5.200E-01 | 69 | 0 | 6.230E-08 | 1 | 1.61775E+03 | 6.24255E+00 | 2.25531E+00 | 6.90000E+00 | 3.50320E+02 | 2.06788E+00 |
| 5.300E-01 | 70 | 0 | 6.192E-08 | 1 | 1.61776E+03 | 6.24260E+00 | 2.23922E+00 | 6.90000E+00 | 3.50320E+02 | 2.06786E+00 |
| 5.400E-01 | 71 | 0 | 6.152E-08 | 1 | 1.61778E+03 | 6.24265E+00 | 2.22326E+00 | 6.90000E+00 | 3.50320E+02 | 2.06784E+00 |

Tab.2 continuation

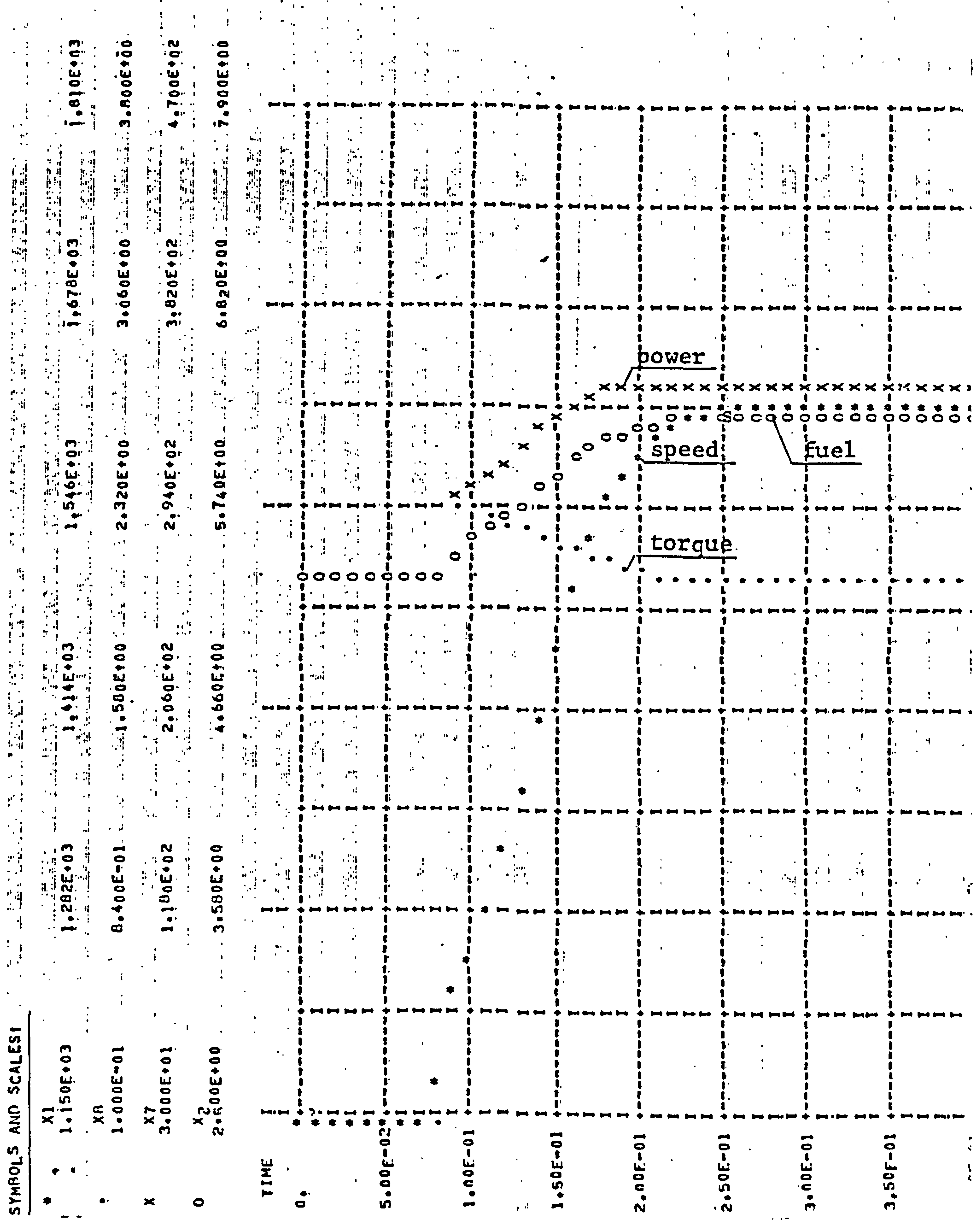



Fig.6 Program DEC(modified) results,Ref: YSET=1620 RPM

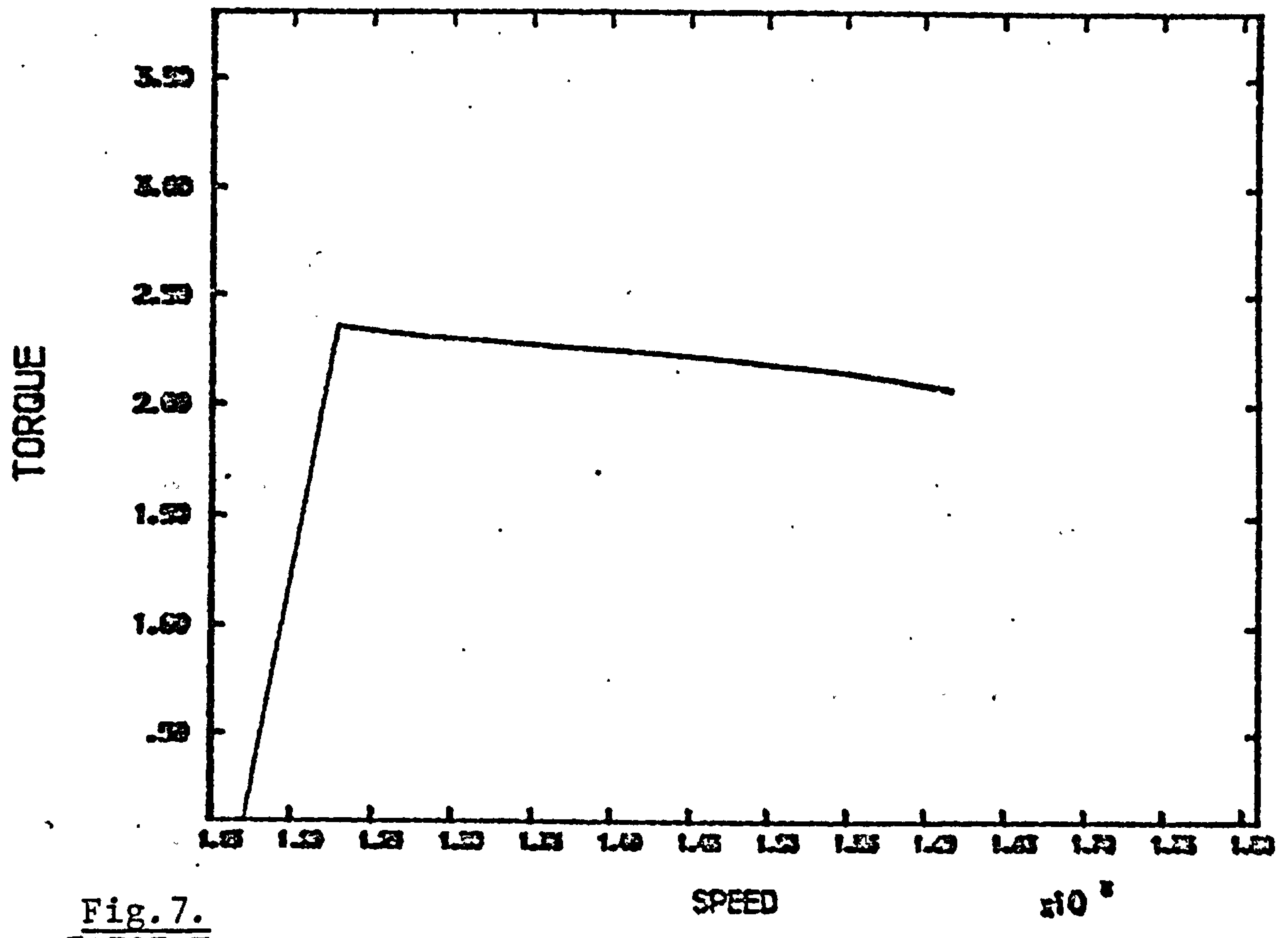



Fig.7.
[*EOF*]
SCM TORQUE/SPEED CHARACTERISTIC

Ref. YSET = 1620 RPM

INITIAL PRESSURE, TEMPERATURE = 6.900 900.0

| Time | e-k | N-R | U.ERROR | x | x1 | x2 | x3 | x4 | x5 | x6 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 1.44000E+03 | 6.90000E+00 | 0. | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 1.31813E+03 | 6.90000E+00 | -0. | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 1.17188E+03 | 6.90000E+00 | 0. | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 1.02563E+03 | 6.90000E+00 | 0. | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 8.79375E+02 | 6.90000E+00 | 0. | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 7.33125E+02 | 6.90000E+00 | 0. | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 5.86875E+02 | 6.90000E+00 | 0. | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 4.40625E+02 | 6.90000E+00 | 0. | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 2.94375E+02 | 6.90000E+00 | 0. | 0. |
| 9.000E-02 | 16 | 0 | 9.939E-04 | 1 | 1.22650E+03 | 5.43489E+00 | 2.22999E+02 | 6.90000E+00 | 3.01863E+02 | 2.35044E+00 |
| 1.000E-01 | 18 | 0 | 2.472E-05 | 1 | 1.23347E+03 | 5.46064E+00 | 2.07816E+02 | 6.90000E+00 | 3.02697E+02 | 2.34370E+00 |
| 1.100E-01 | 19 | 0 | 4.247E-05 | 1 | 1.24264E+03 | 5.47022E+00 | 1.99105E+02 | 6.90000E+00 | 3.03834E+02 | 2.33536E+00 |
| 1.200E-01 | 20 | 0 | 2.132E-05 | 1 | 1.25261E+03 | 5.47844E+00 | 1.89164E+02 | 6.90000E+00 | 3.05160E+02 | 2.32665E+00 |
| 1.300E-01 | 21 | 0 | 2.244E-05 | 1 | 1.26328E+03 | 5.48686E+00 | 1.78616E+02 | 6.90000E+00 | 3.06622E+02 | 2.31805E+00 |
| 1.400E-01 | 22 | 0 | 2.797E-05 | 1 | 1.27492E+03 | 5.49541E+00 | 1.67165E+02 | 6.90000E+00 | 3.08278E+02 | 2.30035E+00 |
| 1.500E-01 | 23 | 0 | 3.229E-05 | 1 | 1.28785E+03 | 5.50420E+00 | 1.54474E+02 | 6.90000E+00 | 3.10187E+02 | 2.30038E+00 |
| 1.600E-01 | 24 | 0 | 3.123E-05 | 1 | 1.30218E+03 | 5.51334E+00 | 1.40379E+02 | 6.90000E+00 | 3.12381E+02 | 2.29115E+00 |
| 1.700E-01 | 25 | 0 | 2.888E-05 | 2 | 1.31751E+03 | 5.52278E+00 | 1.25145E+02 | 6.90000E+00 | 3.14812E+02 | 2.28196E+00 |
| 1.800E-01 | 26 | 0 | 3.039E-05 | 2 | 1.33267E+03 | 5.53222E+00 | 1.09814E+02 | 6.90000E+00 | 3.17282E+02 | 2.27341E+00 |
| 1.900E-01 | 27 | 0 | 2.923E-05 | 2 | 1.34638E+03 | 5.54110E+00 | 9.57550E+01 | 6.90000E+00 | 3.19543E+02 | 2.26608E+00 |
| 2.000E-01 | 28 | 0 | 2.617E-05 | 2 | 1.35867E+03 | 5.54902E+00 | 8.32703E+01 | 6.90000E+00 | 3.21556E+02 | 2.25981E+00 |
| 2.100E-01 | 29 | 0 | 2.362E-05 | 2 | 1.37013E+03 | 5.55611E+00 | 7.17278E+01 | 6.90000E+00 | 3.23433E+02 | 2.25410E+00 |
| 2.200E-01 | 30 | 0 | 2.221E-05 | 2 | 1.38106E+03 | 5.56265E+00 | 6.07208E+01 | 6.90000E+00 | 3.25226E+02 | 2.24869E+00 |
| 2.300E-01 | 31 | 0 | 2.114E-05 | 2 | 1.39136E+03 | 5.56872E+00 | 5.02936E+01 | 6.90000E+00 | 3.26913E+02 | 2.24357E+00 |

Tab.3 Program DEC(modified) results,Ref:YSET=1440RPM

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.500E-01 | 33 | 0 | 2.004E-04 | 2 | 1.40954E+03 | 5.58051E+00 | 3.17947E+01 | 6.90000E+00 | 3.29853E+02 | 2.23444E+00 |
| 2.600E-01 | 34 | 0 | 6.288E-05 | 2 | 1.41834E+03 | 5.60368E+00 | 2.31382E+01 | 6.90000E+00 | 3.31219E+02 | 2.23003E+00 |
| 2.700E-01 | 35 | 0 | 6.219E-05 | 2 | 1.42679E+03 | 5.61853E+00 | 1.45951E+01 | 6.90000E+00 | 3.32541E+02 | 2.22560E+00 |
| 2.800E-01 | 36 | 0 | 5.882E-05 | 2 | 1.43455E+03 | 5.63259E+00 | 6.68225E+00 | 6.90000E+00 | 3.33736E+02 | 2.22145E+00 |
| 2.900E-01 | 37 | 0 | 5.338E-05 | 2 | 1.44140E+03 | 5.64521E+00 | -3.43400E-01 | 6.90000E+00 | 3.34771E+02 | 2.21771E+00 |
| 3.000E-01 | 38 | 0 | 4.643E-05 | 2 | 1.44719E+03 | 5.65606E+00 | -6.32288E+00 | 6.90000E+00 | 3.35630E+02 | 2.21448E+00 |
| 3.100E-01 | 39 | 0 | 3.885E-05 | 2 | 1.45193E+03 | 5.66502E+00 | -1.12377E+01 | 6.90000E+00 | 3.36321E+02 | 2.21179E+00 |
| 3.200E-01 | 40 | 0 | 3.159E-05 | 2 | 1.45572E+03 | 5.67217E+00 | -1.51826E+01 | 6.90000E+00 | 3.36865E+02 | 2.20961E+00 |
| 3.300E-01 | 41 | 0 | 2.518E-05 | 2 | 1.45872E+03 | 5.67773E+00 | -1.82980E+01 | 6.90000E+00 | 3.37289E+02 | 2.20787E+00 |
| 3.400E-01 | 42 | 0 | 1.980E-05 | 2 | 1.46105E+03 | 5.68196E+00 | -2.07271E+01 | 6.90000E+00 | 3.37615E+02 | 2.20651E+00 |
| 3.500E-01 | 43 | 0 | 1.539E-05 | 2 | 1.46284E+03 | 5.68511E+00 | -2.25993E+01 | 6.90000E+00 | 3.37865E+02 | 2.20545E+00 |
| 3.600E-01 | 44 | 0 | 1.182E-05 | 2 | 1.46420E+03 | 5.68737E+00 | -2.40238E+01 | 6.90000E+00 | 3.38053E+02 | 2.20464E+00 |
| 3.700E-01 | 45 | 0 | 8.967E-06 | 2 | 1.46522E+03 | 5.68894E+00 | -2.50902E+01 | 6.90000E+00 | 3.38192E+02 | 2.20403E+00 |
| 3.800E-01 | 46 | 0 | 6.687E-06 | 2 | 1.46596E+03 | 5.68994E+00 | -2.58711E+01 | 6.90000E+00 | 3.38293E+02 | 2.20359E+00 |
| 3.900E-01 | 47 | 0 | 4.870E-06 | 2 | 1.46648E+03 | 5.69051E+00 | -2.64244E+01 | 6.90000E+00 | 3.38365E+02 | 2.20328E+00 |
| 4.000E-01 | 48 | 0 | 3.700E-06 | 1 | 1.46683E+03 | 5.69072E+00 | -2.67965E+01 | 6.90000E+00 | 3.38412E+02 | 2.20307E+00 |
| 4.100E-01 | 49 | 0 | 3.174E-06 | 1 | 1.46704E+03 | 5.69065E+00 | -2.70246E+01 | 6.90000E+00 | 3.38440E+02 | 2.20295E+00 |
| 4.200E-01 | 50 | 0 | 2.745E-06 | 1 | 1.46714E+03 | 5.69036E+00 | -2.71383E+01 | 6.90000E+00 | 3.38453E+02 | 2.20289E+00 |
| 4.300E-01 | 51 | 0 | 2.399E-06 | 1 | 1.46715E+03 | 5.68990E+00 | -2.71611E+01 | 6.90000E+00 | 3.38454E+02 | 2.20288E+00 |
| 4.400E-01 | 52 | 0 | 2.071E-06 | 1 | 1.46709E+03 | 5.68937E+00 | -2.71138E+01 | 6.90000E+00 | 3.38446E+02 | 2.20292E+00 |
| 4.500E-01 | 53 | 0 | 1.384E-06 | 1 | 1.46700E+03 | 5.68893E+00 | -2.70230E+01 | 6.90000E+00 | 3.38432E+02 | 2.20298E+00 |
| 4.600E-01 | 54 | 0 | 9.379E-07 | 1 | 1.46689E+03 | 5.68854E+00 | -2.69179E+01 | 6.90000E+00 | 3.38418E+02 | 2.20304E+00 |
| 4.700E-01 | 55 | 0 | 7.918E-07 | 1 | 1.46679E+03 | 5.68816E+00 | -2.68106E+01 | 6.90000E+00 | 3.38403E+02 | 2.20311E+00 |
| 4.800E-01 | 56 | 0 | 7.572E-07 | 1 | 1.46668E+03 | 5.68779E+00 | -2.67032E+01 | 6.90000E+00 | 3.38388E+02 | 2.20317E+00 |
| 4.900E-01 | 57 | 0 | 7.472E-07 | 1 | 1.46657E+03 | 5.68742E+00 | -2.65961E+01 | 6.90000E+00 | 3.38373E+02 | 2.20324E+00 |
| 5.000E-01 | 58 | 0 | 7.417E-07 | 1 | 1.46647E+03 | 5.68705E+00 | -2.64893E+01 | 6.90000E+00 | 3.38358E+02 | 2.20330E+00 |
| 5.100E-01 | 59 | 0 | 7.376E-07 | 1 | 1.46636E+03 | 5.68669E+00 | -2.63830E+01 | 6.90000E+00 | 3.38344E+02 | 2.20337E+00 |
| 5.200E-01 | 60 | 0 | 7.343E-07 | 1 | 1.46625E+03 | 5.68632E+00 | -2.62770E+01 | 6.90000E+00 | 3.38329E+02 | 2.20343E+00 |
| 5.300E-01 | 61 | 0 | 7.313E-07 | 1 | 1.46615E+03 | 5.68596E+00 | -2.61715E+01 | 6.90000E+00 | 3.38315E+02 | 2.20350E+00 |
| 5.400E-01 | 62 | 0 | 7.284E-07 | 1 | 1.46604E+03 | 5.68559E+00 | -2.60664E+01 | 6.90000E+00 | 3.38300E+02 | 2.20356E+00 |

Tab.3 continuation

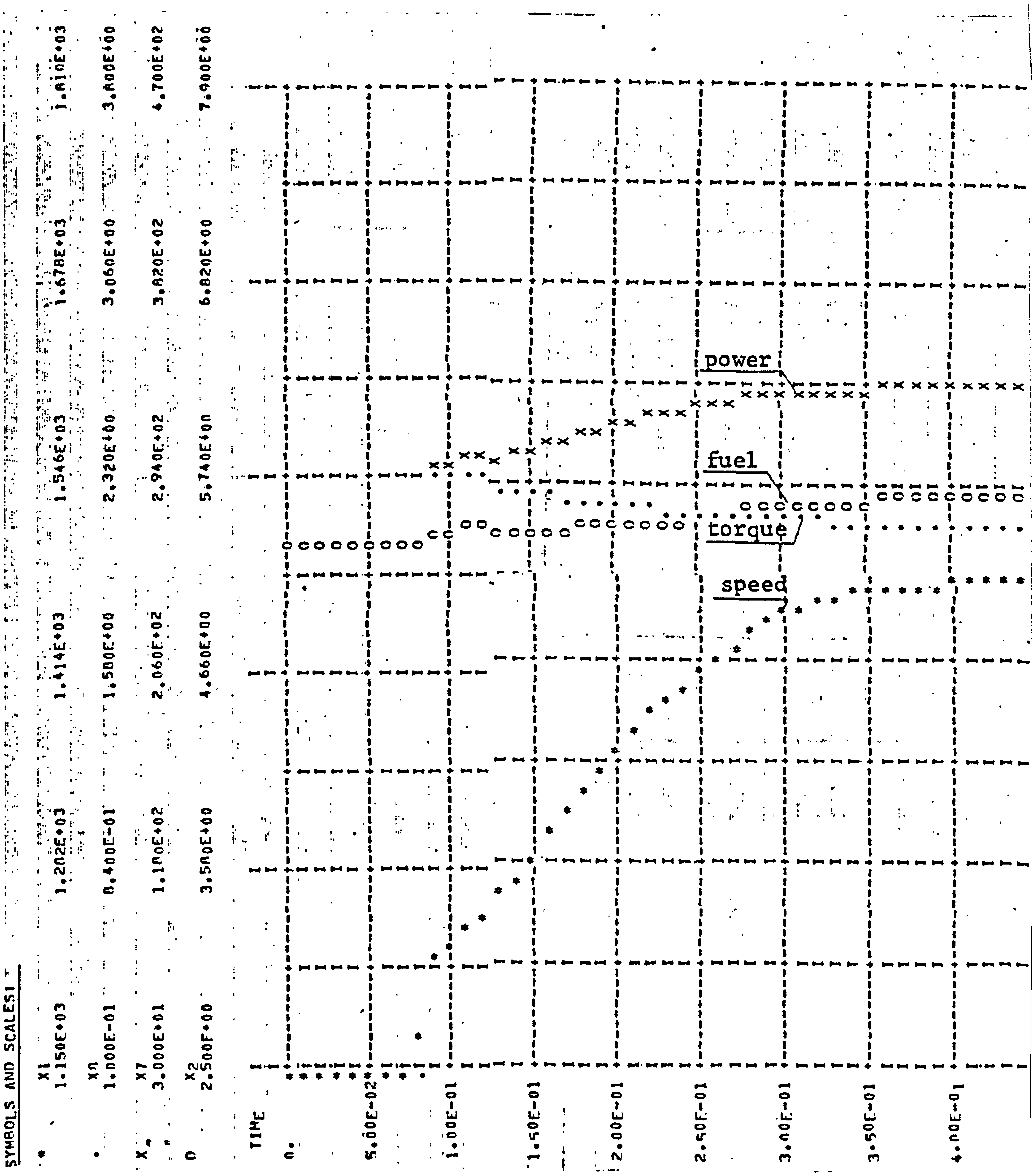



Fig.8 Program DEC(modified) results,Ref:YSET=1440 RPM

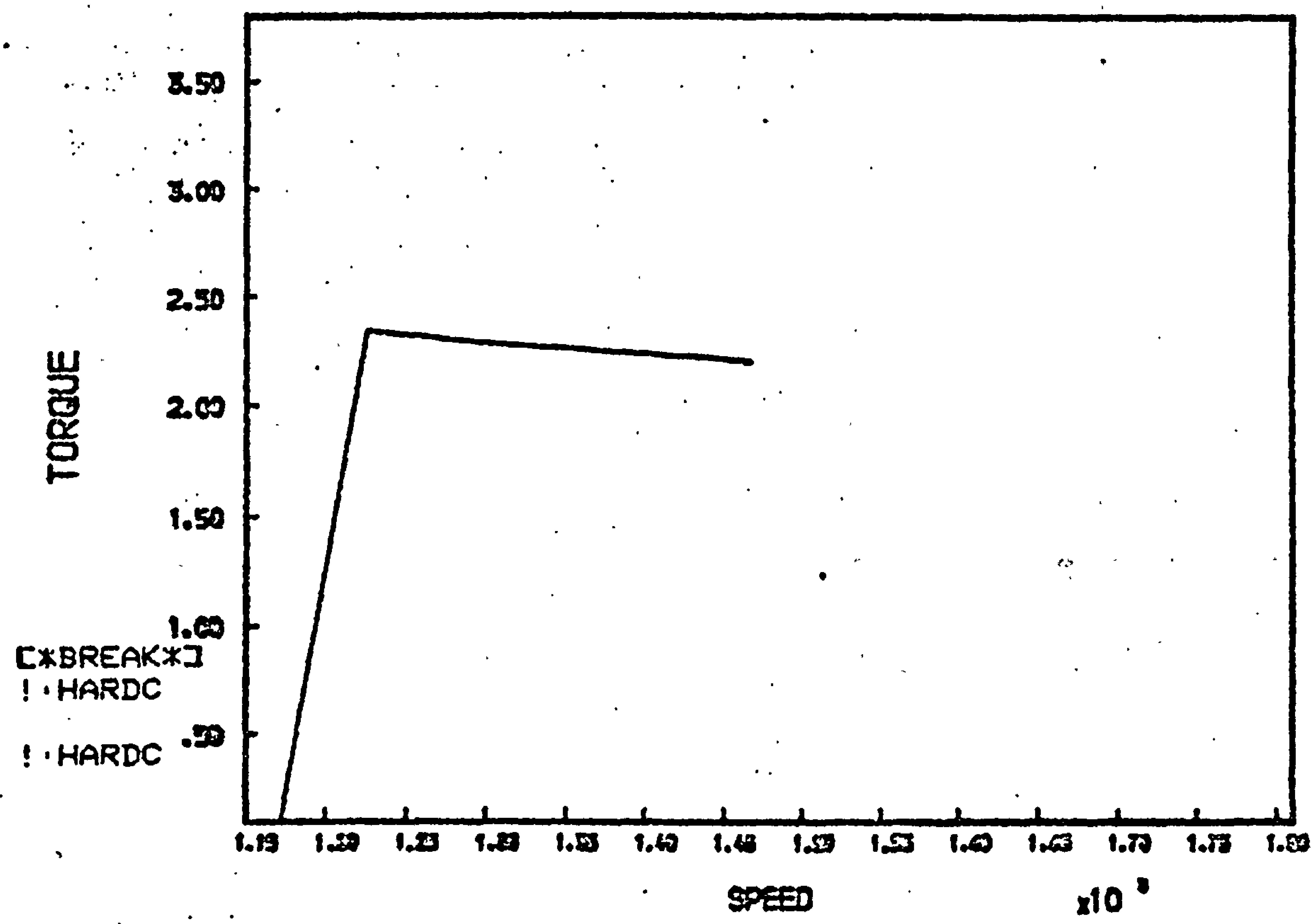



Fig.9. SCM TORQUE/SPEED CHARACTERISTIC

Ref. YSET = 1440 RPM

| Time | R-K | N-R | N.ERROR | x | x1 | x2 | x3 | x4 | x5 | x6 |
|---|---|---|---|---|---|---|---|---|---|---|
| INITIAL PRESSURE; TEMPERATURE = 6.900; 900.0 | | | | | | | | | | |
| 0. | 0 | 0 | 0. | 0 | 0. | 5.40000E+00 | 1.23000E+03 | 6.90000E+00 | 0. | 0. |
| 1.000E-02 | 1 | 0 | 0. | 0 | 1.46250E+02 | 5.40000E+00 | 1.10813E+03 | 6.90000E+00 | 0. | 0. |
| 2.000E-02 | 2 | 0 | 0. | 0 | 2.92500E+02 | 5.40000E+00 | 9.61875E+02 | 6.90000E+00 | 0. | 0. |
| 3.000E-02 | 3 | 0 | 0. | 0 | 4.38750E+02 | 5.40000E+00 | 8.15625E+02 | 6.90000E+00 | 0. | 0. |
| 4.000E-02 | 4 | 0 | 0. | 0 | 5.85000E+02 | 5.40000E+00 | 6.69375E+02 | 6.90000E+00 | 0. | 0. |
| 5.000E-02 | 5 | 0 | 0. | 0 | 7.31250E+02 | 5.40000E+00 | 5.23125E+02 | 6.90000E+00 | 0. | 0. |
| 6.000E-02 | 6 | 0 | 0. | 0 | 8.77500E+02 | 5.40000E+00 | 3.76875E+02 | 6.90000E+00 | 0. | 0. |
| 7.000E-02 | 7 | 0 | 0. | 0 | 1.02375E+03 | 5.40000E+00 | 2.30625E+02 | 6.90000E+00 | 0. | 0. |
| 8.000E-02 | 8 | 0 | 0. | 0 | 1.17000E+03 | 5.40000E+00 | 8.43750E+01 | 6.90000E+00 | 0. | 0. |
| 9.000E-02 | 16 | 0 | 9.965E-04 | 1 | 1.22523E+03 | 5.40646E+00 | 1.39923E+01 | 6.90000E+00 | 3.01723E+02 | 2.35163E+00 |
| 1.000E-01 | 18 | 0 | 3.885E-06 | 2 | 1.22660E+03 | 5.41053E+00 | 3.65218E+00 | 6.90000E+00 | 3.01884E+02 | 2.35027E+00 |
| 1.100E-01 | 19 | 0 | 4.435E-06 | 1 | 1.22825E+03 | 5.41112E+00 | 2.04506E+00 | 6.90000E+00 | 3.02080E+02 | 2.34863E+00 |
| 1.200E-01 | 20 | 0 | 5.381E-07 | 2 | 1.22993E+03 | 5.41130E+00 | 3.50759E-01 | 6.90000E+00 | 3.02284E+02 | 2.34697E+00 |
| 1.300E-01 | 21 | 0 | 6.394E-07 | 1 | 1.23157E+03 | 5.41144E+00 | -1.30153E+00 | 6.90000E+00 | 3.02484E+02 | 2.34537E+00 |
| 1.400E-01 | 22 | 0 | 9.401E-07 | 1 | 1.23317E+03 | 5.41154E+00 | -2.91204E+00 | 6.90000E+00 | 3.02679E+02 | 2.34384E+00 |
| 1.500E-01 | 23 | 0 | 1.198E-06 | 1 | 1.23472E+03 | 5.41161E+00 | -4.47072E+00 | 6.90000E+00 | 3.02870E+02 | 2.34237E+00 |
| 1.600E-01 | 24 | 0 | 1.414E-06 | 1 | 1.23621E+03 | 5.41164E+00 | -5.96843E+00 | 6.90000E+00 | 3.03055E+02 | 2.34098E+00 |
| 1.700E-01 | 25 | 0 | 1.591E-06 | 1 | 1.23763E+03 | 5.41164E+00 | -7.39798E+00 | 6.90000E+00 | 3.03232E+02 | 2.33966E+00 |
| 1.800E-01 | 26 | 0 | 1.731E-06 | 1 | 1.23897E+03 | 5.41161E+00 | -8.75309E+00 | 6.90000E+00 | 3.03401E+02 | 2.33843E+00 |
| 1.900E-01 | 27 | 0 | 1.840E-06 | 1 | 1.24023E+03 | 5.41154E+00 | -1.00322E+01 | 6.90000E+00 | 3.03562E+02 | 2.33728E+00 |
| 2.000E-01 | 28 | 0 | 1.972E-06 | 1 | 1.24142E+03 | 5.41143E+00 | -1.12305E+01 | 6.90000E+00 | 3.03713E+02 | 2.33621E+00 |
| 2.100E-01 | 29 | 0 | 1.981E-06 | 1 | 1.24252E+03 | 5.41130E+00 | -1.23470E+01 | 6.90000E+00 | 3.03854E+02 | 2.33522E+00 |
| 2.200E-01 | 30 | 0 | 2.022E-06 | 1 | 1.24354E+03 | 5.41114E+00 | -1.33813E+01 | 6.90000E+00 | 3.03985E+02 | 2.33432E+00 |
| 2.300E-01 | 31 | 0 | 2.047E-06 | 1 | 1.24448E+03 | 5.41095E+00 | -1.43333E+01 | 6.90000E+00 | 3.04107E+02 | 2.33349E+00 |

Tab.4 ProgramDEC(modified) results,Ref:1230 RPM

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.400E-01 | 32 | 0 | 2.061E-06 | 1 | 1.24534E+03 | 5.41074E+00 | -1.52038E+01 | 6.90000E+00 | 3.04218E+02 | 2.33273E+00 |
| 2.500E-01 | 33 | 0 | 2.064E-06 | 1 | 1.24611E+03 | 5.41050E+00 | -1.59936E+01 | 6.90000E+00 | 3.04319E+02 | 2.33206E+00 |
| 2.600E-01 | 34 | 0 | 2.061E-06 | 1 | 1.24681E+03 | 5.41024E+00 | -1.67041E+01 | 6.90000E+00 | 3.04410E+02 | 2.33145E+00 |
| 2.700E-01 | 35 | 0 | 2.051E-06 | 1 | 1.24743E+03 | 5.40996E+00 | -1.73368E+01 | 6.90000E+00 | 3.04492E+02 | 2.33092E+00 |
| 2.800E-01 | 36 | 0 | 2.036E-06 | 1 | 1.24797E+03 | 5.40966E+00 | -1.78934E+01 | 6.90000E+00 | 3.04563E+02 | 2.33045E+00 |
| 2.900E-01 | 37 | 0 | 2.017E-06 | 1 | 1.24844E+03 | 5.40934E+00 | -1.83757E+01 | 6.90000E+00 | 3.04625E+02 | 2.33004E+00 |
| 3.000E-01 | 38 | 0 | 1.994E-06 | 1 | 1.24884E+03 | 5.40901E+00 | -1.87855E+01 | 6.90000E+00 | 3.04678E+02 | 2.32970E+00 |
| 3.100E-01 | 39 | 0 | 1.968E-06 | 1 | 1.24917E+03 | 5.40866E+00 | -1.91248E+01 | 6.90000E+00 | 3.04721E+02 | 2.32942E+00 |
| 3.200E-01 | 40 | 0 | 1.938E-06 | 1 | 1.24943E+03 | 5.40831E+00 | -1.93956E+01 | 6.90000E+00 | 3.04756E+02 | 2.32920E+00 |
| 3.300E-01 | 41 | 0 | 1.906E-06 | 1 | 1.24962E+03 | 5.40794E+00 | -1.96000E+01 | 6.90000E+00 | 3.04781E+02 | 2.32904E+00 |
| 3.400E-01 | 42 | 0 | 1.870E-06 | 1 | 1.24975E+03 | 5.40756E+00 | -1.97400E+01 | 6.90000E+00 | 3.04798E+02 | 2.32893E+00 |
| 3.500E-01 | 43 | 0 | 1.831E-06 | 1 | 1.24982E+03 | 5.40717E+00 | -1.98178E+01 | 6.90000E+00 | 3.04807E+02 | 2.32887E+00 |
| 3.600E-01 | 44 | 0 | 1.790E-06 | 1 | 1.24983E+03 | 5.40678E+00 | -1.98357E+01 | 6.90000E+00 | 3.04808E+02 | 2.32886E+00 |
| 3.700E-01 | 45 | 0 | 1.740E-06 | 1 | 1.24978E+03 | 5.40638E+00 | -1.97960E+01 | 6.90000E+00 | 3.04802E+02 | 2.32891E+00 |
| 3.800E-01 | 46 | 0 | 1.670E-06 | 1 | 1.24968E+03 | 5.40599E+00 | -1.97018E+01 | 6.90000E+00 | 3.04788E+02 | 2.32899E+00 |
| 3.900E-01 | 47 | 0 | 1.589E-06 | 1 | 1.24952E+03 | 5.40561E+00 | -1.95569E+01 | 6.90000E+00 | 3.04768E+02 | 2.32912E+00 |
| 4.000E-01 | 48 | 0 | 1.508E-06 | 1 | 1.24932E+03 | 5.40524E+00 | -1.93651E+01 | 6.90000E+00 | 3.04741E+02 | 2.32929E+00 |
| 4.100E-01 | 49 | 0 | 1.428E-06 | 1 | 1.24908E+03 | 5.40487E+00 | -1.91301E+01 | 6.90000E+00 | 3.04709E+02 | 2.32950E+00 |
| 4.200E-01 | 50 | 0 | 1.349E-06 | 1 | 1.24880E+03 | 5.40451E+00 | -1.88555E+01 | 6.90000E+00 | 3.04672E+02 | 2.32974E+00 |
| 4.300E-01 | 51 | 0 | 1.270E-06 | 1 | 1.24849E+03 | 5.40416E+00 | -1.85448E+01 | 6.90000E+00 | 3.04630E+02 | 2.33001E+00 |
| 4.400E-01 | 52 | 0 | 1.192E-06 | 1 | 1.24814E+03 | 5.40383E+00 | -1.82013E+01 | 6.90000E+00 | 3.04584E+02 | 2.33031E+00 |
| 4.500E-01 | 53 | 0 | 1.116E-06 | 1 | 1.24776E+03 | 5.40350E+00 | -1.78284E+01 | 6.90000E+00 | 3.04534E+02 | 2.33064E+00 |
| 4.600E-01 | 54 | 0 | 1.041E-06 | 1 | 1.24736E+03 | 5.40318E+00 | -1.74293E+01 | 6.90000E+00 | 3.04481E+02 | 2.33098E+00 |
| 4.700E-01 | 55 | 0 | 9.684E-07 | 1 | 1.24693E+03 | 5.40287E+00 | -1.70069E+01 | 6.90000E+00 | 3.04425E+02 | 2.33135E+00 |
| 4.800E-01 | 56 | 0 | 8.977E-07 | 1 | 1.24649E+03 | 5.40257E+00 | -1.65642E+01 | 6.90000E+00 | 3.04367E+02 | 2.33174E+00 |
| 4.900E-01 | 57 | 0 | 8.292E-07 | 1 | 1.24602E+03 | 5.40229E+00 | -1.61039E+01 | 6.90000E+00 | 3.04307E+02 | 2.33214E+00 |
| 5.000E-01 | 58 | 0 | 7.633E-07 | 1 | 1.24555E+03 | 5.40201E+00 | -1.56287E+01 | 6.90000E+00 | 3.04244E+02 | 2.33256E+00 |
| 5.100E-01 | 59 | 0 | 7.000E-07 | 1 | 1.24506E+03 | 5.40175E+00 | -1.51411E+01 | 6.90000E+00 | 3.04180E+02 | 2.33299E+00 |
| 5.200E-01 | 60 | 0 | 6.395E-07 | 1 | 1.24456E+03 | 5.40149E+00 | -1.46433E+01 | 6.90000E+00 | 3.04116E+02 | 2.33343E+00 |
| 5.300E-01 | 61 | 0 | 5.817E-07 | 1 | 1.24405E+03 | 5.40125E+00 | -1.41375E+01 | 6.90000E+00 | 3.04050E+02 | 2.33387E+00 |
| 5.400E-01 | 62 | 0 | 5.269E-07 | 1 | 1.24354E+03 | 5.40102E+00 | -1.36257E+01 | 6.90000E+00 | 3.03984E+02 | 2.33433E+00 |

Tab.4 continuation.

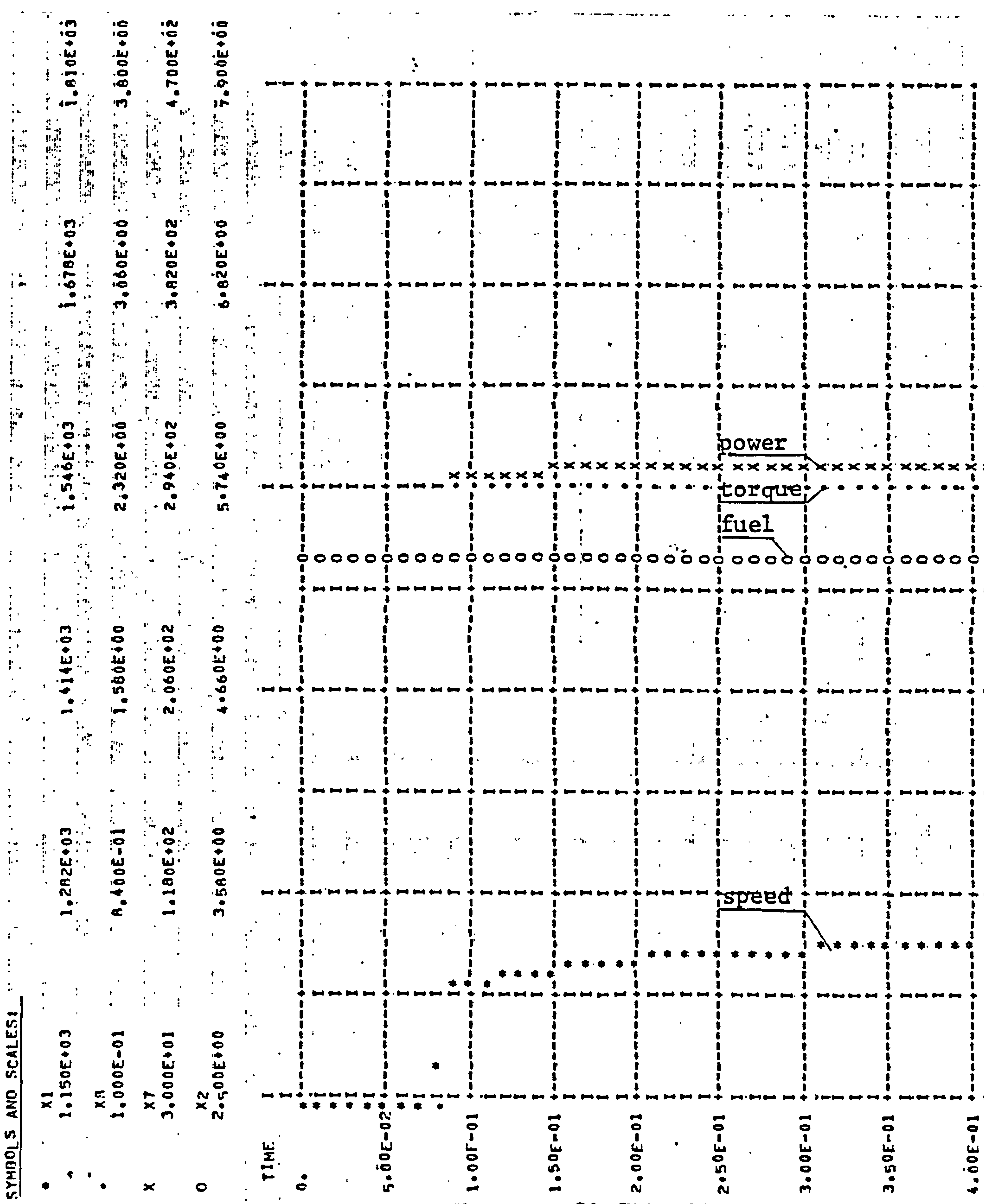
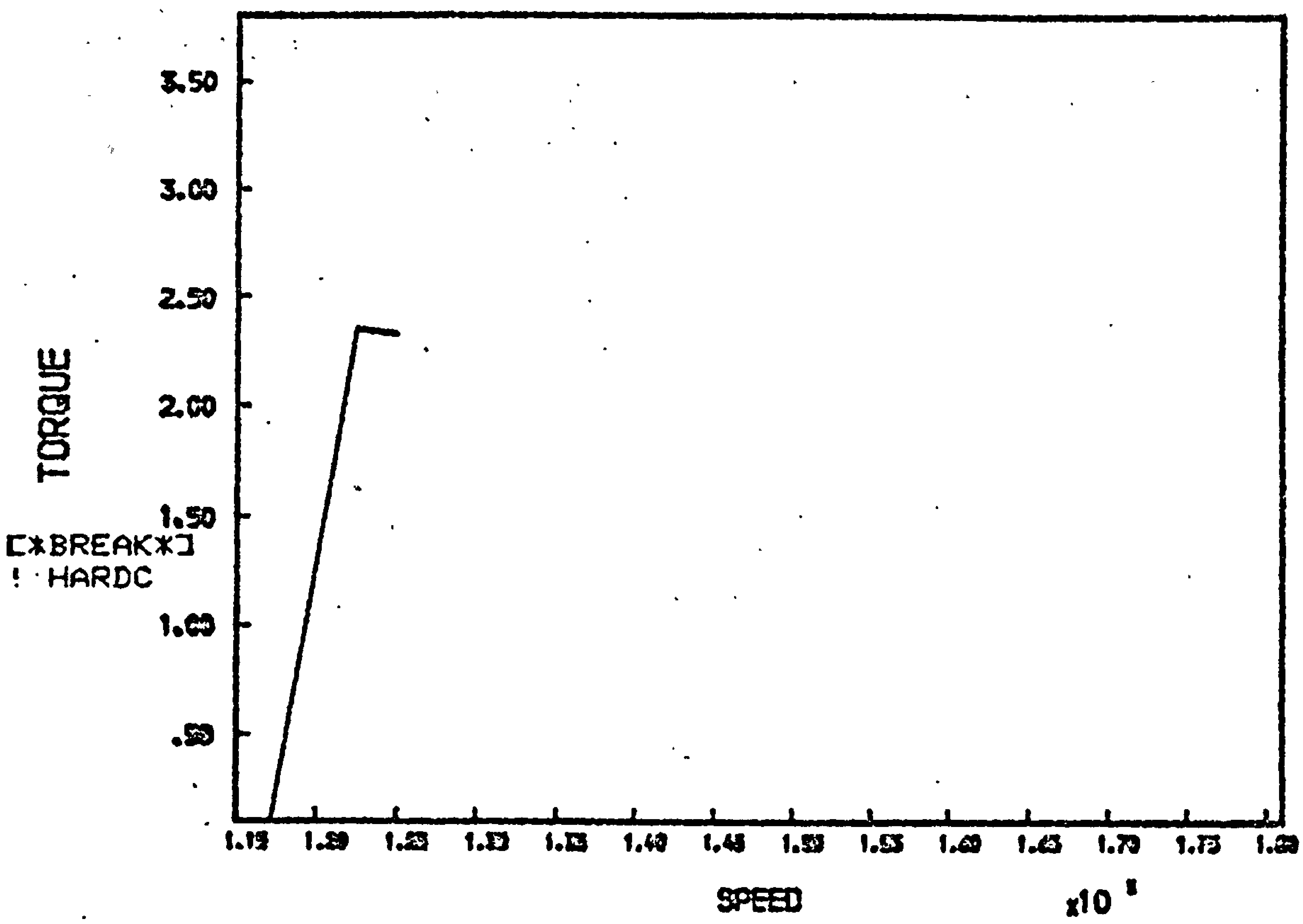



Fig.10 Program DEC(modified) results, Ref:YSET=1230 RPM




<u>Fig.11</u> SCM TORQUE/SPEED CHARACTERISTIC

Ref. YSET = 1230 RPM

considered parameter swing range, however, experimental runs with higher polynomials degree values defined closely to the "edge" conditions lead sometimes to quite unexpected results. Namely, absolute value of the calculated function is accurate only within a given YSET range, outside this range EØ1 ACF subroutine calculates practically any random values, especially when a very close fit is used.

BIBLIOGRAPHY

1. Subroutine EØ2 ACF - Nottingham Algorithms Group, ICL 1900 System, N.A.G. Library Manual. Document No. 189, 30th Sept. 1971

2. Linear Optimal Control, Prentice Hall Inc. By Brian D.O. Anderson and John B. Moore 1971.

3. An Exposition of Adaptive Control, Pergamon Press. By J.H. Westcott, 1969

## 9.2. PRACTICAL EXTENSION OF PRECEEDING RESULTS

As indicated in the previous section, all the computational work was carried out at a fixed temperature point, the choice of temperature was purely arbitrary. However, bearing in mind the fact that in the real 'WALKER' Stirling Cycle Engine, the temperature was adjusted for its maximum value for the best fuel utilisation, in this section, the temperature will be kept constant at a maximum of 900°C, with an automatic "fuel loop" adjusting the fuel supply at minimal required level. The main reason for incorporating this automatic fuel supply is because of a large number of program runs at various working medium pressure levels, where fuel initially was individually adjusted according to present requirements. This is possible with a fixed temperature condition where fuel supplied is a strong function of operational pressure. The above "fuel loop" is based on the WALKER statistical set of data points represented functionally in the form of Chebyshev polynomials (EØ2ACF). The main difficulty in applying this method into our program depends on the fact that the fuel input adjustment is extremely critical on overall model performance, i.e. engine's speed, power, torque and therefore the fuel rate should be maintained as close as possible to the original levels. The calculated polynomial form is shown in Program No.1. listing and also in Fig.1, Fig.2, and Fig.3, which correspond to the third, fifth and seventh power of its pressure parameter components. As predicted the last fit (see Fig.3.) gives the closest function

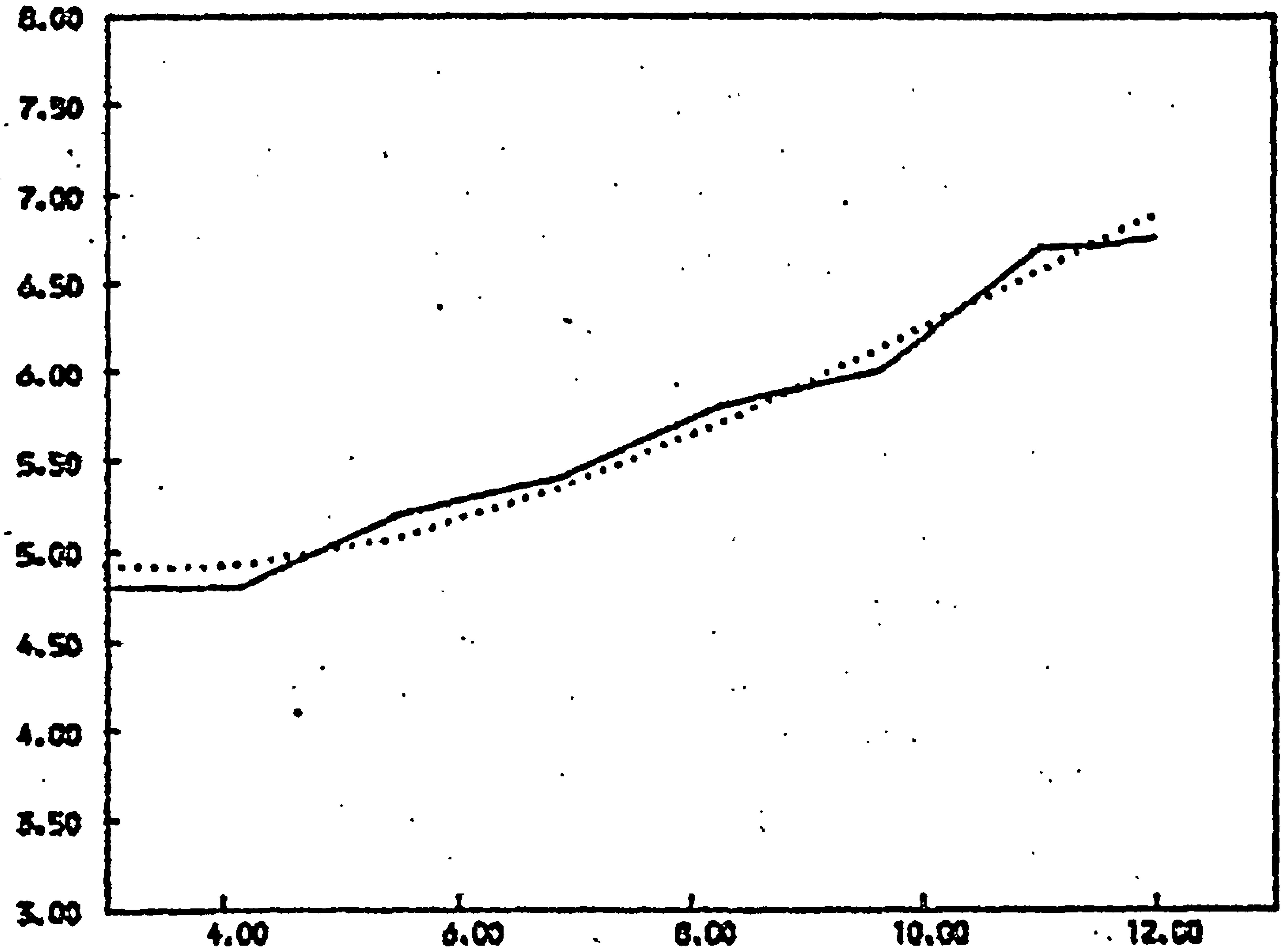
```
05/06/77 UNIVERSITY OF MINNESOTA 6600 FORTRAN COMPILER NOSBE 1.1  VER4.5   13/01/78  15.33.25.
         MNF(EB=LFN)
000000B          PROGRAM APPROX(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
                1TAPE27=PLOT)
004152B          DIMENSION F(20),P(20),E(20),S(20)
004152B          F(1)=4.795
004272B          F(2)=4.798
004274B          F(3)=4.80
004275B          F(4)=5.20
004277B          F(5)=5.40
004300B          F(6)=5.80
004302B          F(7)=6.00
004303B          F(8)=6.70
004305B          F(9)=6.71
004306B          F(10)=6.75
004310B          P(1)=3.0
004311B          P(2)=3.5
004313B          P(3)=4.14
004314B          P(4)=5.52
004316B          P(5)=6.90
004317B          P(6)=8.28
004321B          P(7)=9.66
004322B          P(8)=11.03
004324B          P(9)=11.5
004325B          P(10)=12.0
004327B          N=10
004330B          M1=8
004331B          CALL E02ACF(P,F,10,E,M1,REF)
004334B          WRITE(6,200)
004337B      200 FORMAT(1H ,24H POLYNOMIAL COEFFICIENTS//)
004337B          WRITE(6,201) (E(I),I=1,M1)
004347B      201 FORMAT(1H,5X,1PE20.8)
004347B          DO10 I=1,10
004350B          D=E(1)+E(2)*P(I)+E(3)*P(I)**2+E(4)*P(I)**3+E(5)*P(I)**4+E(6)*
                1P(I)**5+E(7)*P(I)**6+E(8)*P(I)**7
004365B          S(I)=D
004366B          WRITE(6,202) D
004374B      202 FORMAT (1H , F10.4)
004374B       10 CONTINUE
004376B          CALL CAM35MM
004400B          CALL GRSLIDE
004402B          CALL XAXIS(3.0,13.0)
004404B          CALL YAXIS(3.0,8.0)
004406B          CALL GRAPHIC(P,F,N)
004410B          CALL DOT
004412B          CALL GRAPHIC(P,S,N)
004414B          CALL DASHOFF
004416B          CALL LXTICK
004420B          CALL LYTICK
004422B          CALL LXVAL
004424B          CALL LYVAL
004426B          CALL GRFRAME
004430B          CALL ENDFILM
004432B          STOP
```

```
POLYNOMIAL COEFFICIENTS

      8.49495918E+00
     -5.18210543E+00
      2.92511147E+00
     -8.92535854E-01
      1.60006194E-01

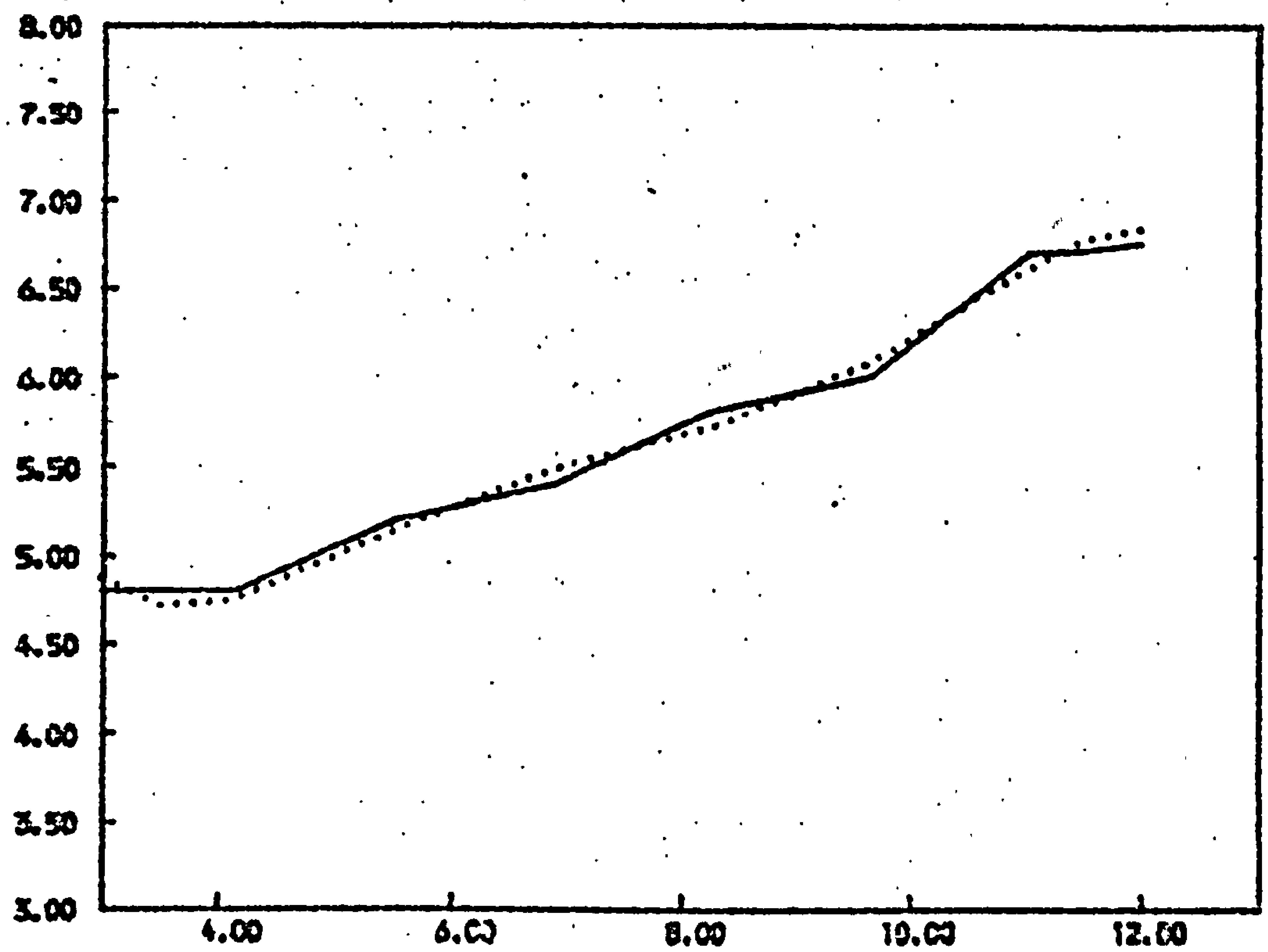
     -1.65613100E-02
      9.10516495E-04
     -2.04910064E-05
    4.7312
    4.7773
    4.8638
    5.1362
    5.4638
    5.7362
    6.0638
    6.6362
    6.7738
    6.6862
2029 CHARACTERS OUTPUT
```

Progr.1 Program APPROX.



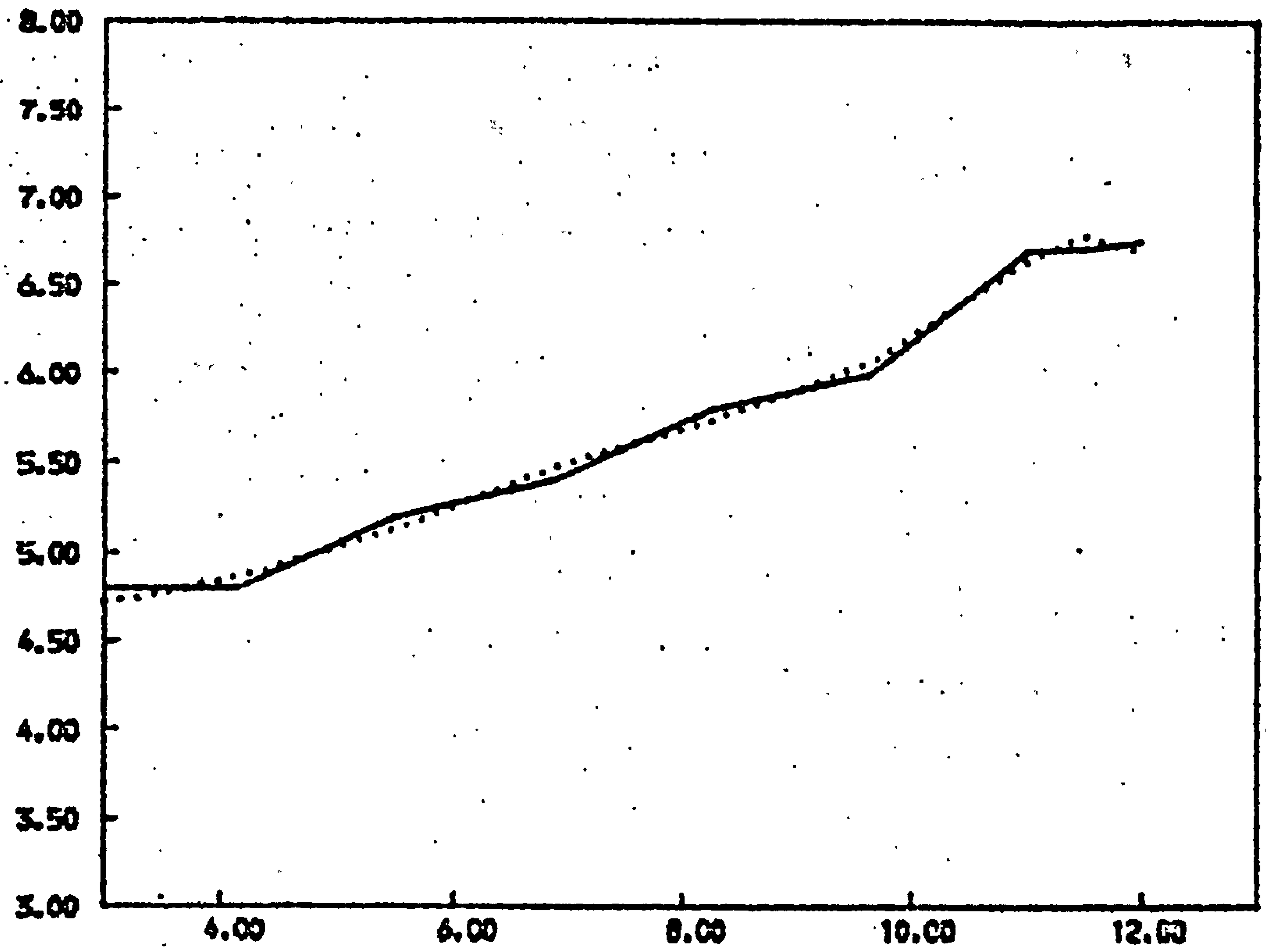

[*EOF*]

Fig.1. Specific Fuel Level vs Operational Pressure, Chebyshev polynomial fit (thrid order).




[*EOF*]

Fig.2. Specific Fuel Level vs Operational Pressure, Chebyshev polynomial fit (fifth order).




[*EOF*]

Fig. 3. Specific Fuel Level vs Operational Pressure, Chebyshev polynomial fit (seventh order).

approximation to the given set of data points (compare with Fig.1 and Fig. 2). A comparison of calculated values and data for the best fit is shown in Tab.1. Even the best polynomial fit is loaded with some remaining fuel errors (see Tab.1.) and therefore the approximation accuracy was found generally to be unsatisfactory. As a result a decision was taken to interpolate at a given pressure point on a plane, by fitting a cubic spline function. The computer library N.A.G. interpolative subroutine EØ1 ADF was applied for this and found to be superior to the earlier polynomial approximations.

TAB.1. "Fuel Loop" Details as Calculated by EØ2ACF Routine

F1 = 8.49495918E+00
F2 = -5.18210543E+00
F3 = 2.92511147E+00
F4 = -8.92535854E-01    Pressure = PRSSUR = P
F5 = 1.60006194E-01
F6 = -1.62513100E-02
F7 = 9.10516495E-04
F8 = -2.04910064E-05

FUEL=F1+F2*P+F3*P**2+F4*P**3+F5*P**4+F6*P**5+F7*P**6+F8*P**7

Accuracy Evaluation

| | | | | | | |
|---|---|---|---|---|---|---|
| Data (Fuel values): | 4.795 | 4.798 | 4.800 | 5.200 | 5.400 | 5.800 |
| | 6.000 | 6.700 | 6.710 | 6.750 | | |
| Chebyshev NAG 5F : | 4.7312 | 4.7773 | 4.8638 | 5.1362 | 5.4638 | |
| | 5.7362 | 6.0638 | 6.6362 | 6.7738 | 6.6862 | |
| Error (Ref. $P^7$) : | 0.0638 | 0.o207 | 0.0638 | 0.0638 | 0.0638 | |
| (g/min) | 0.0638 | 0.0638 | 0.0638 | 0.0638 | 0.0638 | |

The program INTERP employing external EØ 1ADF subroutine is presented as Program No.2. with tabulation of interpolated values (see Tab.2) or in the plotted form in Fig.4. Program INTERP calculates one hundred fuel values for the system's overall flexibility and the accuracy of the final result depends on the choice of given pressure/fuel values and the position relative to these values at the interpolation point. The EØ1ADF during the Program INTERP execution calls the other NAG Library routines such as :EØ1ACY and EØ1 ACZ which generally smooth the calculated cubic spline function.

The method described here of "automatisation" of the fuel supply allows for a run of large numbers of computer programs on the DEC model at various pressure levels without additional fuel adjustements. So the next step is to incorporate the TD1 parameter modification across the whole operational pressure range, in order to achieve a practical extension of the preceeding results. Similarly (see section 9.1 Chapter 9), the EØ2ACF routine is once more employed for the calculation of the minimax Chebyshev polynomial fit to a set of data points corresponding to various pressure levels i.e. 4.14 bar (Fig.5), 5.52 bar (Fig.6), 8.28 bar (Fig.7), 9.66 bar (Fig.8),11.03 bar (Fig.9). All these plots are achieved from a cubic fit and the relevant range of polynomial coefficients are presented in Tab.3.

With six pressure points and with fixed temperature conditions (calculated previously), the polynomial coefficients E1,E2,E3,E4, may be approximated functionally in order to achieve some form of optimal TD1 parameter continuity across the considered pressure range.

```
05/06/77 UNIVERSITY OF MINNESOTA 6600 FORTRAN COMPILER NOSBE  1.1   VER4.5      17/03/78   12.04.10.
         MNF(CB=LFN)
000000B            PROGRAM INTERP(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
                  1TAPE27=PLOT)
004152B            DIMENSION F(100),P(100),W(100),D(100),AX(110),VALY(110)
004152B            FMIN=4.5
005326B            FMAX=7.0
005330B            PMIN=6.5
005331B            PMAX=12.5
005333B            F(1)=4.795
005334B            F(2)=4.798
005336B            F(3)=4.80
005337B            F(4)=5.20
005341B            F(5)=5.40
005342B            F(6)=5.80
005344B            F(7)=6.00
005345B            F(8)=6.70
005347B            F(9)=6.71
005350B            F(10)=6.75
005352B            P(1)=3.0
005353B            P(2)=3.5
005355B            P(3)=4.14
005356B            P(4)=5.52
005360B            P(5)=6.90
005361B            P(6)=8.28
005363B            P(7)=9.66
005364B            P(8)=11.03
005366B            P(9)=11.5
005367B            P(10)=12.0
005371B            A=4.0
005372B            DO 10 I=1,101
005375B            CALL E01ADF(9,A,P,F,D,W,100,VAL)
005400B            AX(I)=A
005400B            VALY(I)=VAL
005402B            WRITE(6,100) A,VAL
005411B        100 FORMAT(10X,E14.7,10X,E14.7)
005411B            A=A+0.0250
005412B         10 CONTINUE
005415B            CALL CAM35MM
005417B            CALL GRSLIDE
005421B            CALL XAXIS(PMIN,PMAX)
005423B            CALL YAXIS(FMIN,FMAX)
005425B            CALL GRAPHIC(P,F,10)
005427B            CALL DOT
005431B            CALL GRAPHIC(AX,VALY,101)
005433B            CALL DASHOFF
005435B            CALL LXTICK
005437B            CALL LYTICK
005441B            CALL LXVAL
005443B            CALL LYVAL
005445B            CALL GRFRAME
005447B            CALL ENDFILM
005451B            STOP
005452B            END
```

<u>Progr. 2</u>  Program INTERP
Specific Fuel vs Op. Pressr
(lo data points)

| | |
|---|---|
| .4595000E+01 | .4899927E+01 |
| .4680000E+01 | .4926671E+01 |
| .4765000E+01 | .4954[illegible] 01 |
| .4850000E+01 | .498412[illegible]01 |
| .4935000E+01 | .5013950E+01 |
| .5020000E+01 | .5043929E+01 |
| .5105000E+01 | .5073621E+01 |
| .5190000E+01 | .5102586E+01 |
| .5275000E+01 | .5130386E+01 |
| .5360000E+01 | .5156582E+01 |
| .5445000E+01 | .5180735E+01 |
| .5530000E+01 | .5202407E+01 |
| .5615000E+01 | .5221339E+01 |
| .5700000E+01 | .5237784E+01 |
| .5785000E+01 | .5252082E+01 |
| .5870000E+01 | .5264574E+01 |
| .5955000E+01 | .5275602E+01 |
| .6040000E+01 | .5285507E+01 |
| .6125000E+01 | .5294629E+01 |
| .6210000E+01 | .5303310E+01 |
| .6295000E+01 | .5311890E+01 |
| .6380000E+01 | .5320711E+01 |
| .6465000E+01 | .5330113E+01 |
| .6550000E+01 | .5340438E+01 |
| .6635000E+01 | .5352026E+01 |
| .6720000E+01 | .5365220E+01 |
| .6805000E+01 | .5380358E+01 |
| .6890000E+01 | .5397784E+01 |
| .6975000E+01 | .5417756E+01 |
| .7060000E+01 | .5440075E+01 |
| .7145000E+01 | .5464375E+01 |
| .7230000E+01 | .5490292E+01 |
| .7315000E+01 | .5517462E+01 |
| .7400000E+01 | .5545520E+01 |
| .7485000E+01 | .5574102E+01 |
| .7570000E+01 | .5602843E+01 |
| .7655000E+01 | .5631379E+01 |
| .7740000E+01 | .5659345E+01 |
| .7825000E+01 | .5686377E+01 |
| .7910000E+01 | .5712110E+01 |
| .7995000E+01 | .5736179E+01 |
| .8080000E+01 | .5758221E+01 |
| .8165000E+01 | .5777871E+01 |
| .8250000E+01 | .5794764E+01 |
| .8335000E+01 | .5808578E+01 |
| .8420000E+01 | .5819508E+01 |
| .8505000E+01 | .5828104E+01 |
| .8590000E+01 | .5834922E+01 |
| .8675000E+01 | .5840519E+01 |
| .8760000E+01 | .5845450E+01 |
| .8845000E+01 | .5850272E+01 |
| .8930000E+01 | .5855540E+01 |
| .9015000E+01 | .5861812E+01 |
| .9100000E+01 | .5869643E+01 |
| .9185000E+01 | .5879580E+01 |
| .9270000E+01 | .5892206E+01 |
| .9355000E+01 | .5908050E+01 |
| .9440000E+01 | .5927678E+01 |
| .9525000E+01 | .5951646E+01 |
| .9610000E+01 | .5980510E+01 |
| .9695000E+01 | .6014809E+01 |
| .9780000E+01 | .6054478E+01 |
| .9865000E+01 | .6098607E+01 |
| .9950000E+01 | .6146565E+01 |
| .1003500E+02 | .61972[illegible]E+01 |
| .1012000E+02 | .62498[illegible] 01 |
| .1020500E+02 | .630342[illegible]E+01 |
| .1029000E+02 | .6357200E+01 |
| .1037500E+02 | .6410259E+01 |
| .1046000E+02 | .6461726E+01 |
| .1054500E+02 | .6510723E+01 |
| .1063000E+02 | .6556376E+01 |
| .1071500E+02 | .6597807E+01 |
| .1080000E+02 | .6634140E+01 |
| .1088500E+02 | .6664498E+01 |
| .1097000E+02 | .6688007E+01 |
| .1105500E+02 | .6703800E+01 |
| .1114000E+02 | .6712045E+01 |
| .1122500E+02 | .6714678E+01 |
| .1131000E+02 | .6713810E+01 |
| .1139500E+02 | .6711556E+01 |
| .1148000E+02 | .6710026E+01 |
| .1156500E+02 | .6711133E+01 |
| .1165000E+02 | .6715133E+01 |
| .1173500E+02 | .6721472E+01 |
| .1182000E+02 | .6729575E+01 |
| .1190500E+02 | .6738879E+01 |
| .1199000E+02 | .6748816E+01 |
| .1207500E+02 | .6758819E+01 |
| .1216000E+02 | .6768323E+01 |
| .1224500E+02 | .6776759E+01 |
| .1233000E+02 | .6783562E+01 |
| .1241500E+02 | .6788165E+01 |
| .1250000E+02 | .6790000E+01 |

2721 CHARACTERS OUTPUT

Tab. 2.

Interpolated results for Program 2.

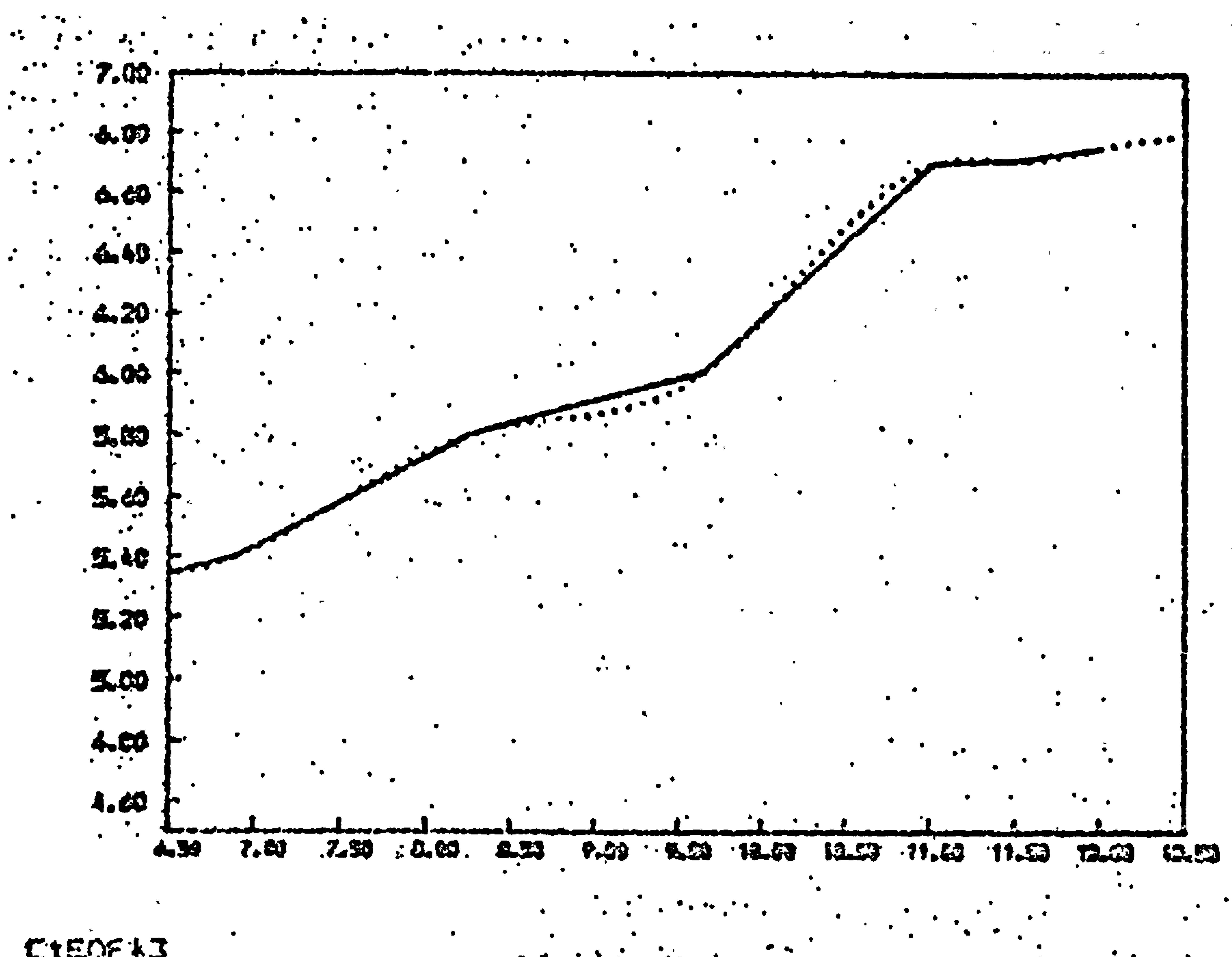
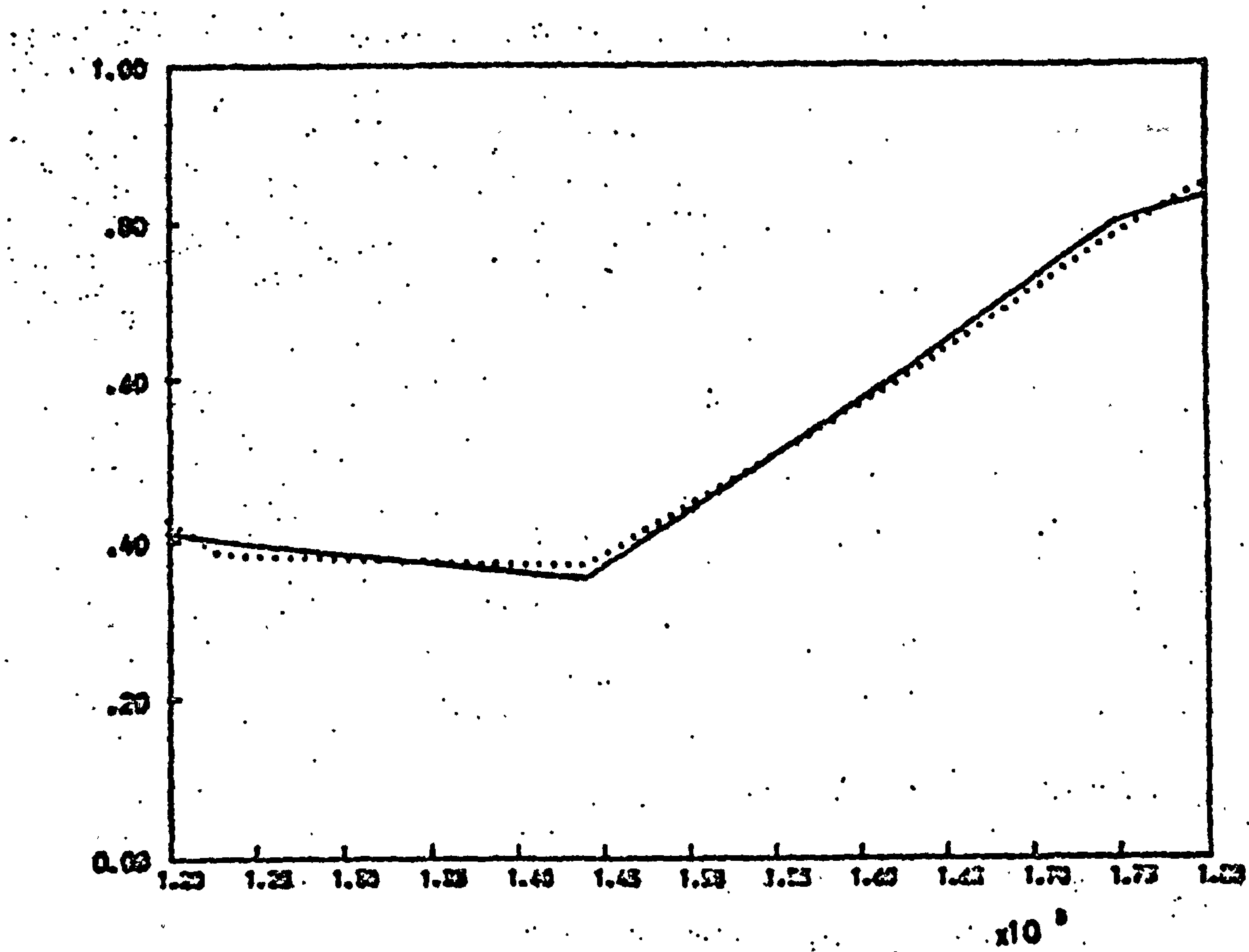
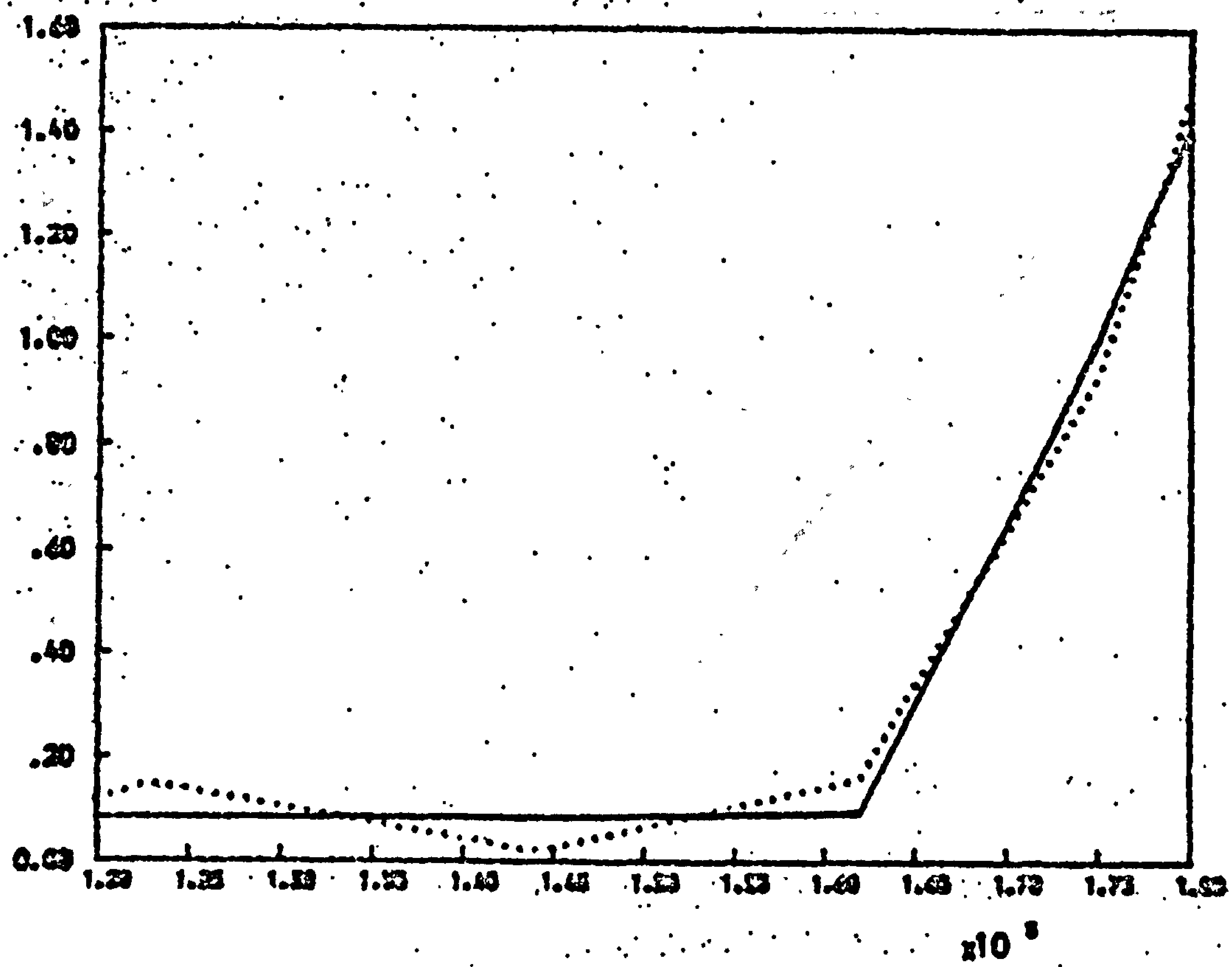


Fig.4. Specific Fuel Level vs Operational Pressure, Plotted by INTERP.




[*EOF*]

Fig. 5. TD1 parameter vs RPM speed @ p = 4.14 bar.



C*EOF*]

Fig. 6 . TD1 parameter vs RPM speed @ p = 5.52 bar.

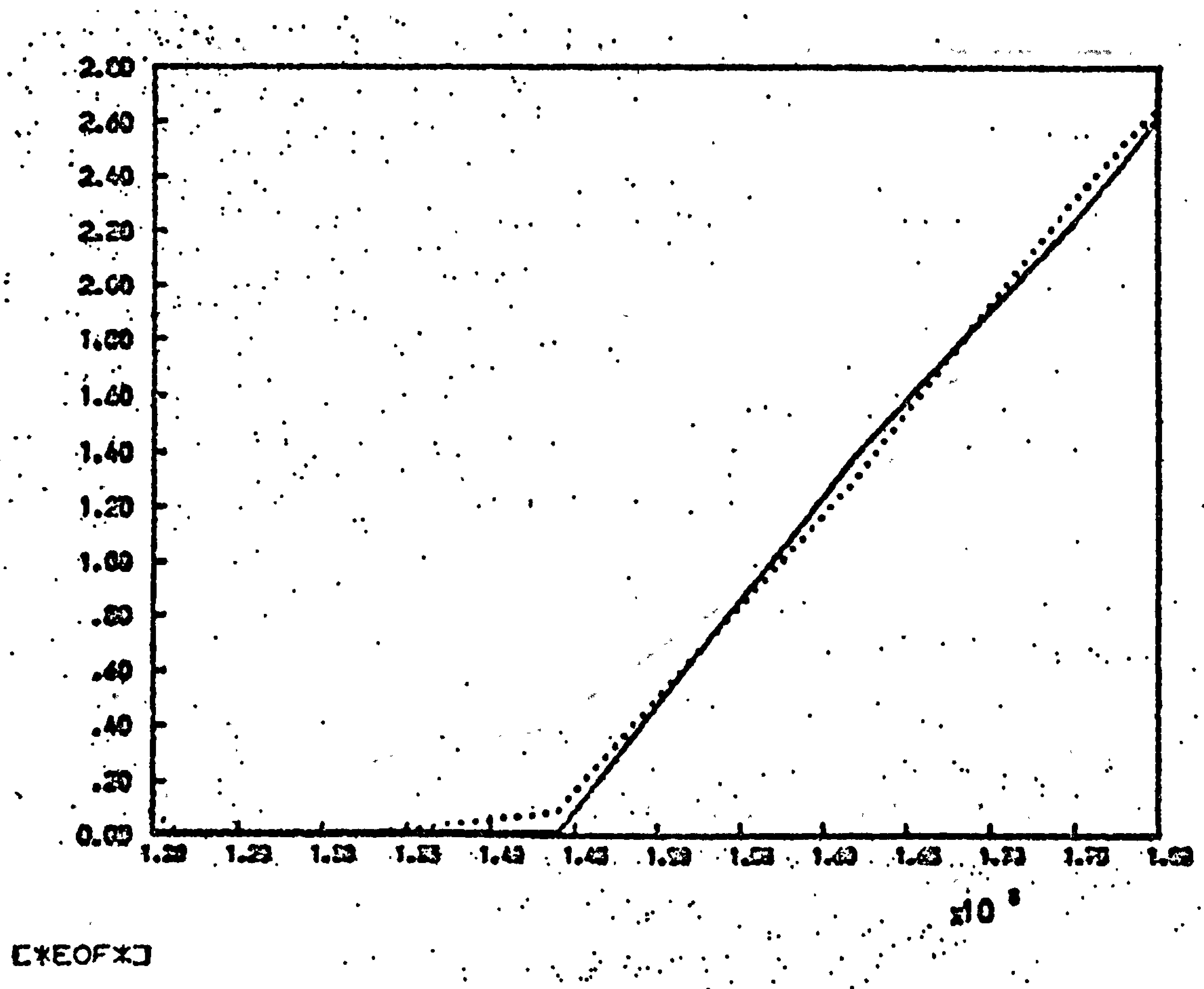



Fig. 7. TD1 parameter vs RPM speed @ p = 8.28 bar.

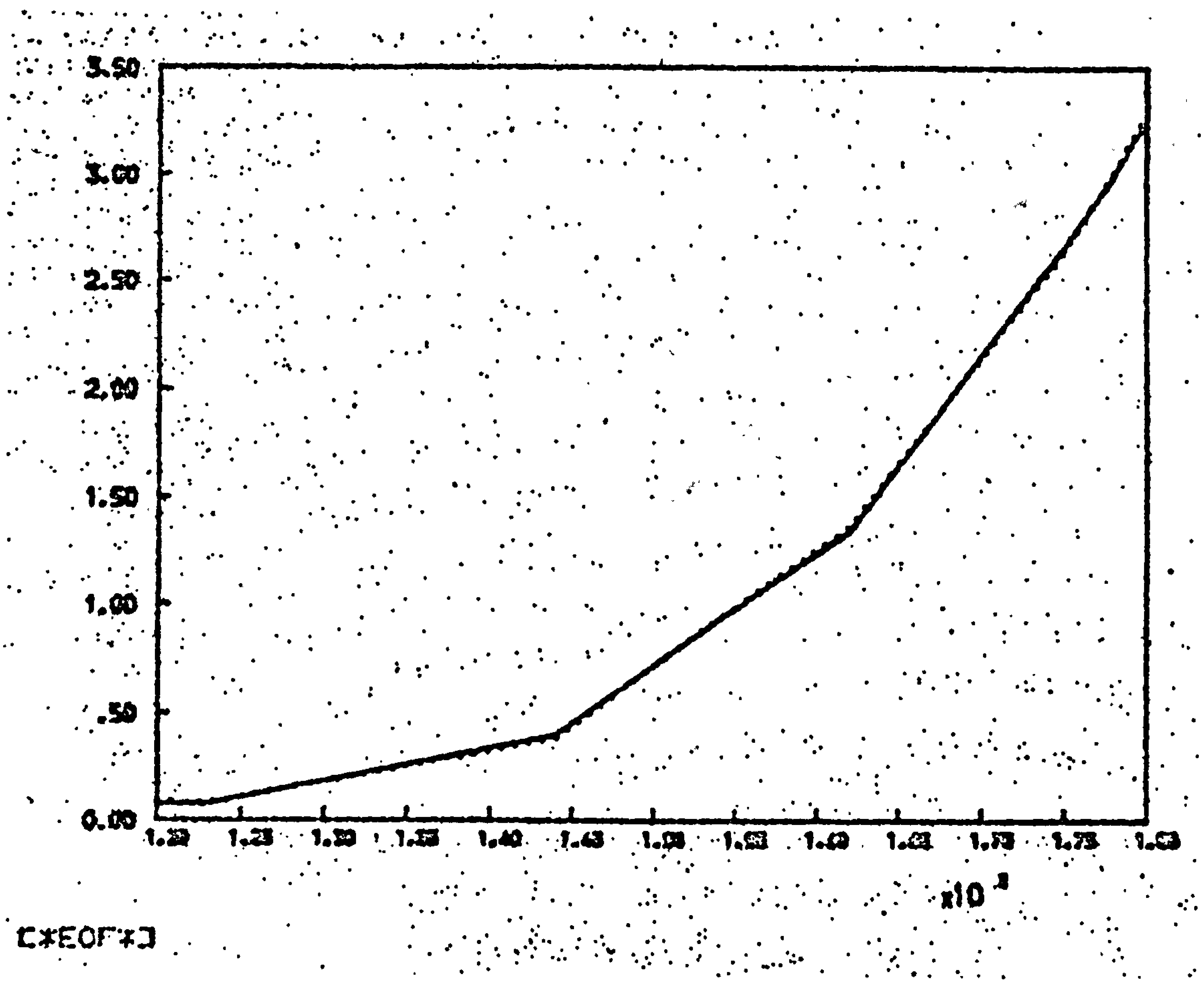



Fig. 8. TD1 parameter vs RPM speed @ p =9.66 bar.

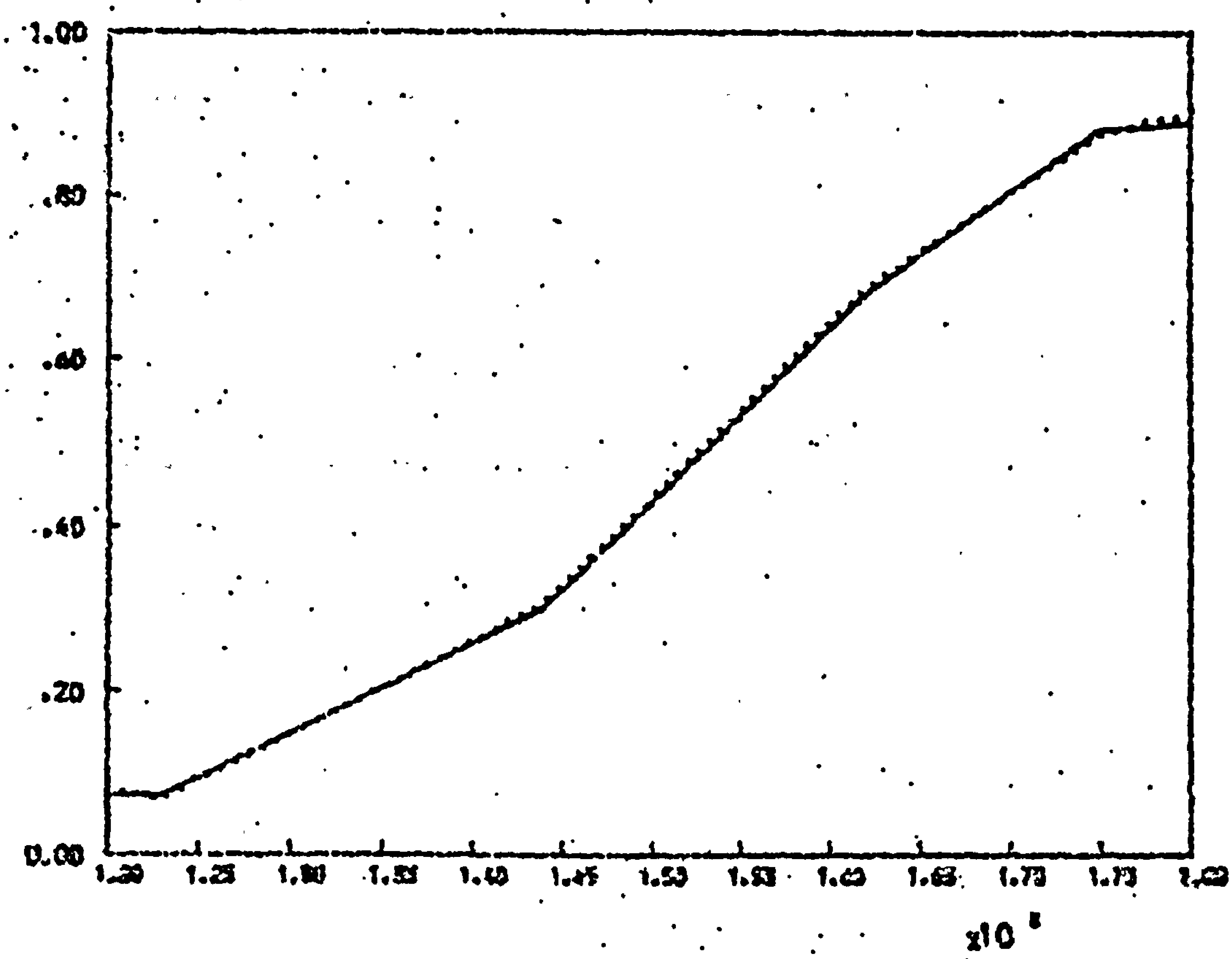



Fig.9 TD1 parameter vs RPM speed @ p=11.03 bar.

TABLE. 3.

| | $P_1=4.14$ | $P_2=5.52$ | $P_3=6.90$ | $P_4=8.28$ | $P_5=9.66$ | $P_6 = 11.03$ |
|---|---|---|---|---|---|---|
| $E_1$ | 2.15738201E+01 | -6.51637536E+01 | 4.90520799E+01 | 9.51298720E+01 | -2.77968893E+00 | 2.92259271E+01 |
| $E_2$ | -4.11486574E-02 | 1.43696841E-01 | -9.96179444E-02 | -1.86648345E-01 | 1.50139958E-02 | -6.09987140E-02 |
| $E_3$ | 2.58885863E-05 | -1.04703810E-04 | 6.59503096E-05 | 1.18321254E-04 | -1.86052006E-05 | 4.14880100E-05 |
| $E_4$ | -5.23641833E-09 | 2.52426073E-08 | -1.41081361E-08 | -2.39848391E-08 | 6.73902622E-09 | -9.07972456E-09 |

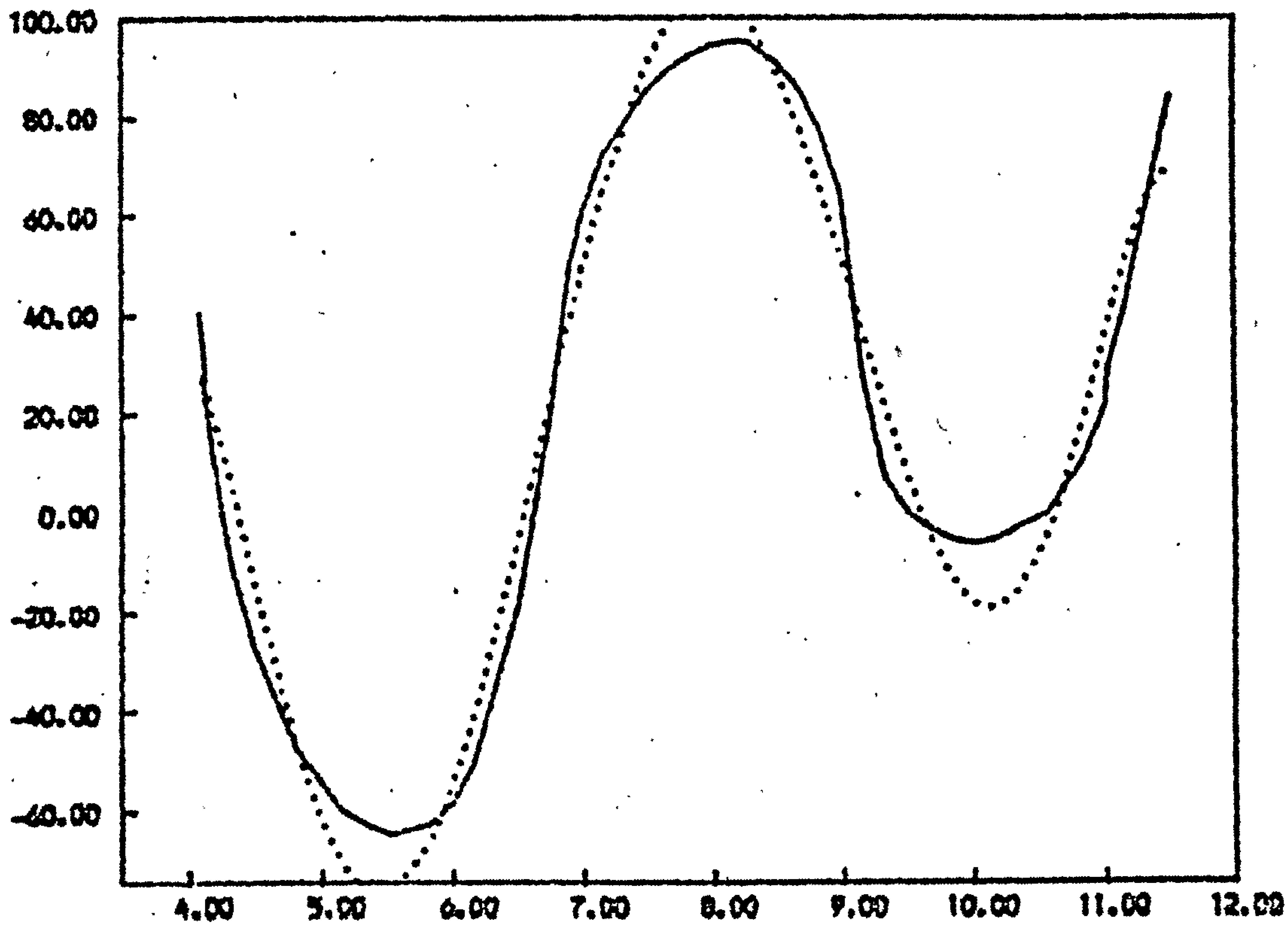
Range of EØ2 ACF Routine Polynomial Coefficients (as discussed in text)

In fact, it was virtually impossible to carry out work using only 6 data points and still maintain the relevant standard of accuracy. Therefore various numbers of additional data points were generated and a few typical results (related to the E1 parameter) are presented below.

51 data points (EØ2 ACF);E1plot(Fig.10) Polynomial order 7
51 data points (EØ2 ACF);E1plot(Fig.11) Polynomial order 11
51 data points (EØ2 ACF);E1plot(Fig.12) Polynomial order 16

It is clear from these results that a too high polynomial order does not necessarily improve the fit quality, shown in Fig.12, the "middle pressure" region's fit is quite good, but both low and high "ends" are substantially distorted.

Again interpolation of these given data points in terms of NAG EØ1 ADF routine, by the fitting of cubic spline functions was applied and found generally superior to previously discussed techniques. The "E1" coefficient INTERP program (Program No.3) uses 51 data points and the corresponding Tektronix plot is shown in Fig.13. The "E2" coefficient INTERP Program (Program No.4) uses 35 data points and the corresponding Tektronix plot is shown in Fig.14. The "E3" coefficient INTERP program (Program No.5) uses 49 data points and the corresponding Tektronix plot is shown in Fig.15. The "E4" coefficient INTERP program (Program No.6) uses 49 data points and the corresponding Tektronix plot is shown in Fig. 16.




[*EOF*]

Fig.10 Chebyshev polynomial fit Ref:p[7]

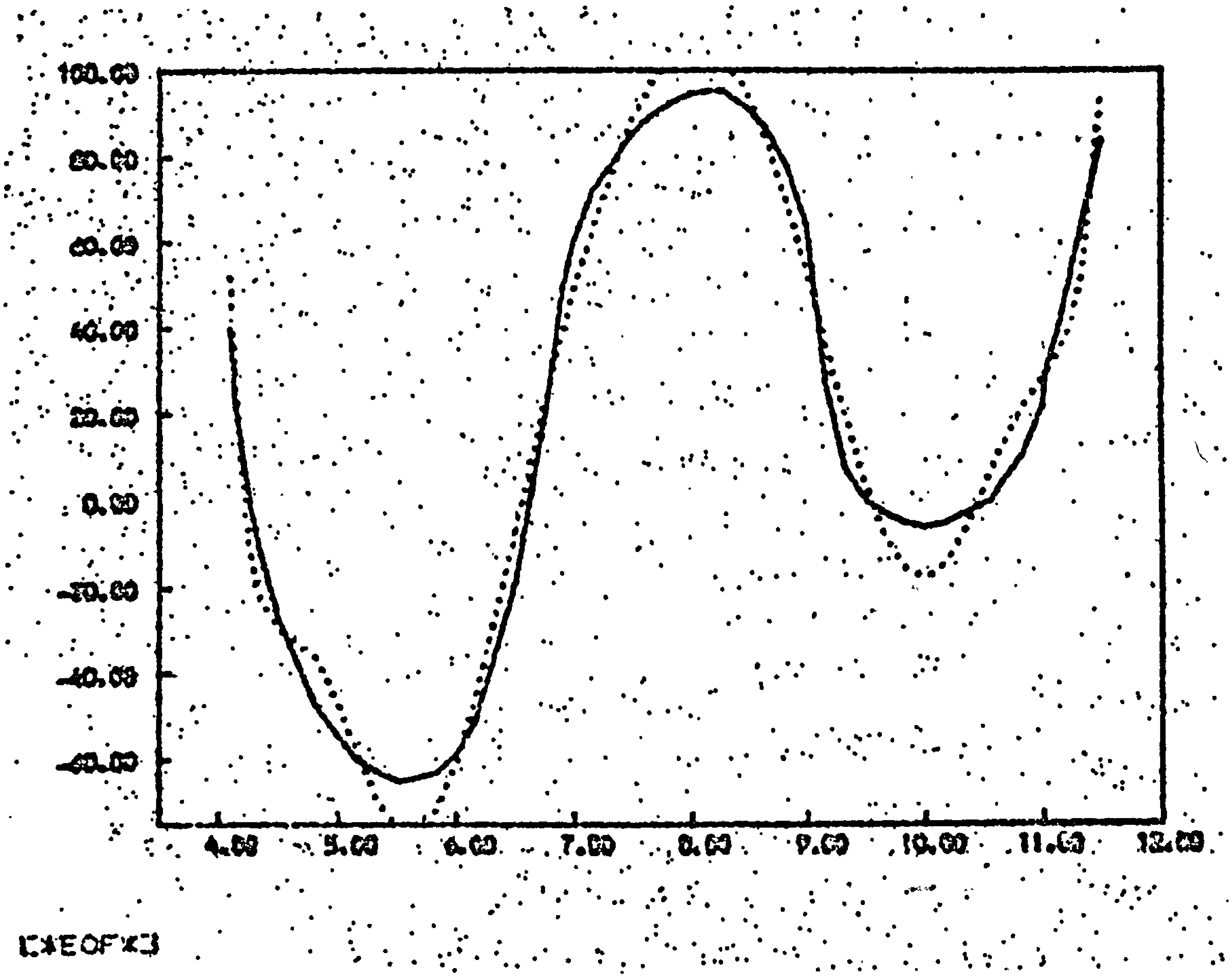



Fig.11 Chebyshev polynomial fit Ref:p [11]

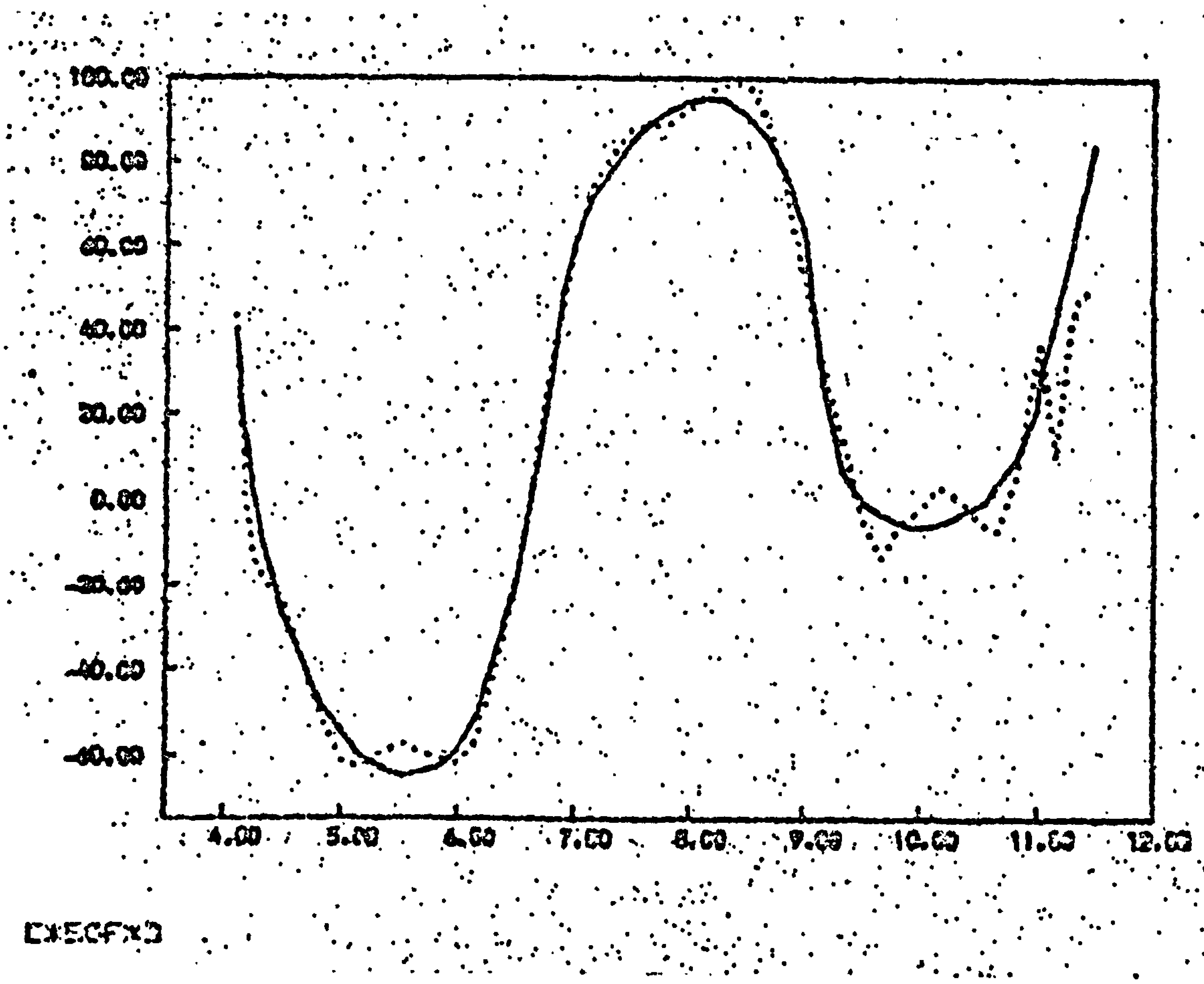



Fig.12 Chebyshev polynomial fit Ref:p[16]

```
RSITY OF MINNESOTA 6600 FORTRAN COMPILER NOSBE 1.1   VER4.5      07/03/78  09.58.53.
I=LFN)
      PROGRAM INTERP(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=PLOT)
      DIMENSION E(100),P(100),W(100),D(100),AX(110),VALY(110)
      EMIN=-75.
      EMAX=100.
      PMIN=3.5
      PMAX=12.0
      E(1)=40.0
      E(2)=30.0
      E(3)=25.0
      E(4)=2.15738201E+01
      E(5)=15.0
      E(6)=10.0
      E(7)=0.0
      E(8)=-10.0
      E(9)=-26.5
      E(10)=-48.0
      E(11)=-54.5
      E(12)=-60.0
      E(13)=-63.0
      E(14)=-6.51637536E+01
      E(15)=-62.2
      E(16)=-59.0
      E(17)=-51.0
      E(18)=-19.0
      E(19)=4.0
      E(20)=20.0
      E(21)=34.5
      E(22)=4.90520799E+01
      E(23)=60.0
      E(24)=72.0
      E(25)=85.0
      E(26)=89.5
      E(27)=92.5
      E(28)=94.5
      E(29)=95.2
      E(30)=9.51298720E+01
      E(31)=91.0
      E(32)=86.0
      E(33)=77.5
      E(34)=64.0
      E(35)=37.0
      E(36)=7.0
      E(37)=0.0
      E(38)=-2.77968893E+00
      E(39)=-5.0
      E(40)=-6.0
      E(41)=-5.0
      E(42)=-3.0
      E(43)=0.0
      E(44)=4.0
      E(45)=10.0
      E(46)=21.5
      E(47)=2.92259271E+01
      E(48)=41.0
      E(49)=53.0
      E(50)=63.0
      E(51)=84.0
      P(1)=4.10
      P(2)=4.13
      P(3)=4.135
      P(4)=4.16
      P(5)=4.183
      P(6)=4.20
      P(7)=4.26
      P(8)=4.33
      P(9)=4.50
      P(10)=4.83
      P(11)=5.0
      P(12)=5.16
      P(13)=5.33
      P(14)=5.52
      P(15)=5.83
      P(16)=6.0
      P(17)=6.16
      P(18)=6.50
      P(19)=6.66
      P(20)=6.76
      P(21)=6.83
      P(22)=6.90
      P(23)=7.0
      P(24)=7.16
      P(25)=7.50
      P(26)=7.66
      P(27)=7.83
      P(28)=8.0
      P(29)=8.16
      P(30)=8.28
      P(31)=8.50
      P(32)=8.66
      P(33)=8.83
      P(34)=9.0
      P(35)=9.16
      P(36)=9.33
      P(37)=9.50
      P(38)=9.66
      P(39)=9.83
      P(40)=10.0
      P(41)=10.16
      P(42)=10.33
      P(43)=10.55
      P(44)=10.66
      P(45)=10.83
      P(46)=11.0
      P(47)=11.03
      P(48)=11.16
      P(49)=11.25
      P(50)=11.33
      P(51)=11.50
      A=4.14
      DO 10 I=1,101
      CALL EDIADE(50,A,P,E,D,W,100,VAL)
      AX(I)=A
      VALY(I)=VAL
      WRITE(6,100) A,VAL
  100 FORMAT(10X,E14.7,10X,E14
      A=A+0.0689
   10 CONTINUE
      CALL CAM35MM
      CALL GRSLIDE
      CALL XAXIS(PMIN,PMAX)
      CALL YAXIS(EMIN,EMAX)
      CALL GRAPHIC(P,E,51)
      CALL DOT
      CALL GRAPHIC(AX,VALY,101)
      CALL DASHDEF
      CALL LXTICK
      CALL LYTICK
      CALL LXVAL
      CALL LYVAL
      CALL GRFRAME
      CALL ENDFICH
      STOP
      END
```

Progr. 3 Program INTERP

E1 (51 points)

2.380 CP SECONDS 71300B CM STORAGE USED

Program 3 cont....

| | |
|---|---|
| .4140000E+01 | .2157382E+02 |
| .4208900E+01 | .7531834E+01 |
| .4277800E+01 | -.2291723E+01 |
| .4346700E+01 | -.1226276E+02 |
| .4415600E+01 | -.1923447E+02 |
| .4484500E+01 | -.2534068E+02 |
| .4553400E+01 | -.3046907E+02 |
| .4622300E+01 | -.3545950E+02 |
| .4691200E+01 | -.4014828E+02 |
| .4760100E+01 | -.4436856E+02 |
| .4829000E+01 | -.4795347E+02 |
| .4897900E+01 | -.5082992E+02 |
| .4966800E+01 | -.5331639E+02 |
| .5035700E+01 | -.5581442E+02 |
| .5104600E+01 | -.5829993E+02 |
| .5173500E+01 | -.6034594E+02 |
| .5242400E+01 | -.6173249E+02 |
| .5311300E+01 | -.6274400E+02 |
| .5380200E+01 | -.6369761E+02 |
| .5449100E+01 | -.6457030E+02 |
| .5518000E+01 | -.6515329E+02 |
| .5586900E+01 | -.6528085E+02 |
| .5655800E+01 | -.6490337E+02 |
| .5724700E+01 | -.6434471E+02 |
| .5793600E+01 | -.6344871E+02 |
| .5862500E+01 | -.6237183E+02 |
| .5931400E+01 | -.6099515E+02 |
| .6000300E+01 | -.5898941E+02 |
| .6069200E+01 | -.5609247E+02 |
| .6138100E+01 | -.5234824E+02 |
| .6207000E+01 | -.4786424E+02 |
| .6275900E+01 | -.4261703E+02 |
| .6344800E+01 | -.3650205E+02 |
| .6413700E+01 | -.2941359E+02 |
| .6482600E+01 | -.2124591E+02 |
| .6551500E+01 | -.1194304E+02 |
| .6620400E+01 | -.1885873E+01 |
| .6689300E+01 | .8360872E+01 |
| .6758200E+01 | .1966720E+02 |
| .6827100E+01 | .3385560E+02 |
| .6896000E+01 | .4836175E+02 |
| .6964900E+01 | .5706659E+02 |
| .7033800E+01 | .6271215E+02 |
| .7102700E+01 | .6805526E+02 |
| .7171600E+01 | .7271465E+02 |
| .7240500E+01 | .7637288E+02 |
| .7309400E+01 | .7921610E+02 |
| .7378300E+01 | .8150582E+02 |
| .7447200E+01 | .8350353E+02 |
| .7516100E+01 | .8546958E+02 |
| .7585000E+01 | .8749946E+02 |
| .7653900E+01 | .8935370E+02 |
| .7722800E+01 | .9080959E+02 |
| .7791700E+01 | .9194408E+02 |
| .7860600E+01 | .9292757E+02 |
| .7929500E+01 | .9380719E+02 |
| .7998400E+01 | .9448740E+02 |
| .8067300E+01 | .9490100E+02 |
| .8136200E+01 | .9513843E+02 |
| .8205100E+01 | .9529757E+02 |
| .8274000E+01 | .9516885E+02 |
| .8342900E+01 | .9440604E+02 |
| .8411800E+01 | .9309875E+02 |
| .8480700E+01 | .9147845E+02 |
| .8549600E+01 | .8973317E+02 |
| .8618500E+01 | .8763195E+02 |
| .8687400E+01 | .8472406E+02 |
| .8756300E+01 | .8112773E+02 |
| .8825200E+01 | .7770811E+02 |
| .8894100E+01 | .7473544E+02 |
| .8963000E+01 | .6929682E+02 |
| .9031900E+01 | .5767113E+02 |
| .9100800E+01 | .4086322E+02 |
| .9169700E+01 | .2507948E+02 |
| .9238600E+01 | .1468466E+02 |
| .9307500E+01 | .8475622E+01 |
| .9376400E+01 | .4434014E+01 |
| .9445300E+01 | .1595073E+01 |
| .9514200E+01 | -.3384646E+00 |
| .9583100E+01 | -.1658526E+01 |
| .9652000E+01 | -.2668563E+01 |
| .9720900E+01 | -.3627593E+01 |
| .9789800E+01 | -.4533485E+01 |
| .9858700E+01 | -.5292048E+01 |
| .9927600E+01 | -.5811041E+01 |
| .9996500E+01 | -.6000782E+01 |
| .1006540E+02 | -.5798308E+01 |
| .1013430E+02 | -.5264407E+01 |
| .1020320E+02 | -.4499296E+01 |
| .1027210E+02 | -.3649129E+01 |
| .1034100E+02 | -.2892610E+01 |
| .1040990E+02 | -.2266004E+01 |
| .1047880E+02 | -.1447570E+01 |
| .1054770E+02 | -.6051483E-01 |
| .1061660E+02 | .2184055E+01 |
| .1068550E+02 | .5176157E+01 |
| .1075440E+02 | .8221292E+01 |
| .1082330E+02 | .9954275E+01 |
| .1089220E+02 | .1000337E+02 |
| .1096110E+02 | .1434424E+02 |
| .1103000E+02 | .2922593E+02 |

ABACTERS OUTPUT

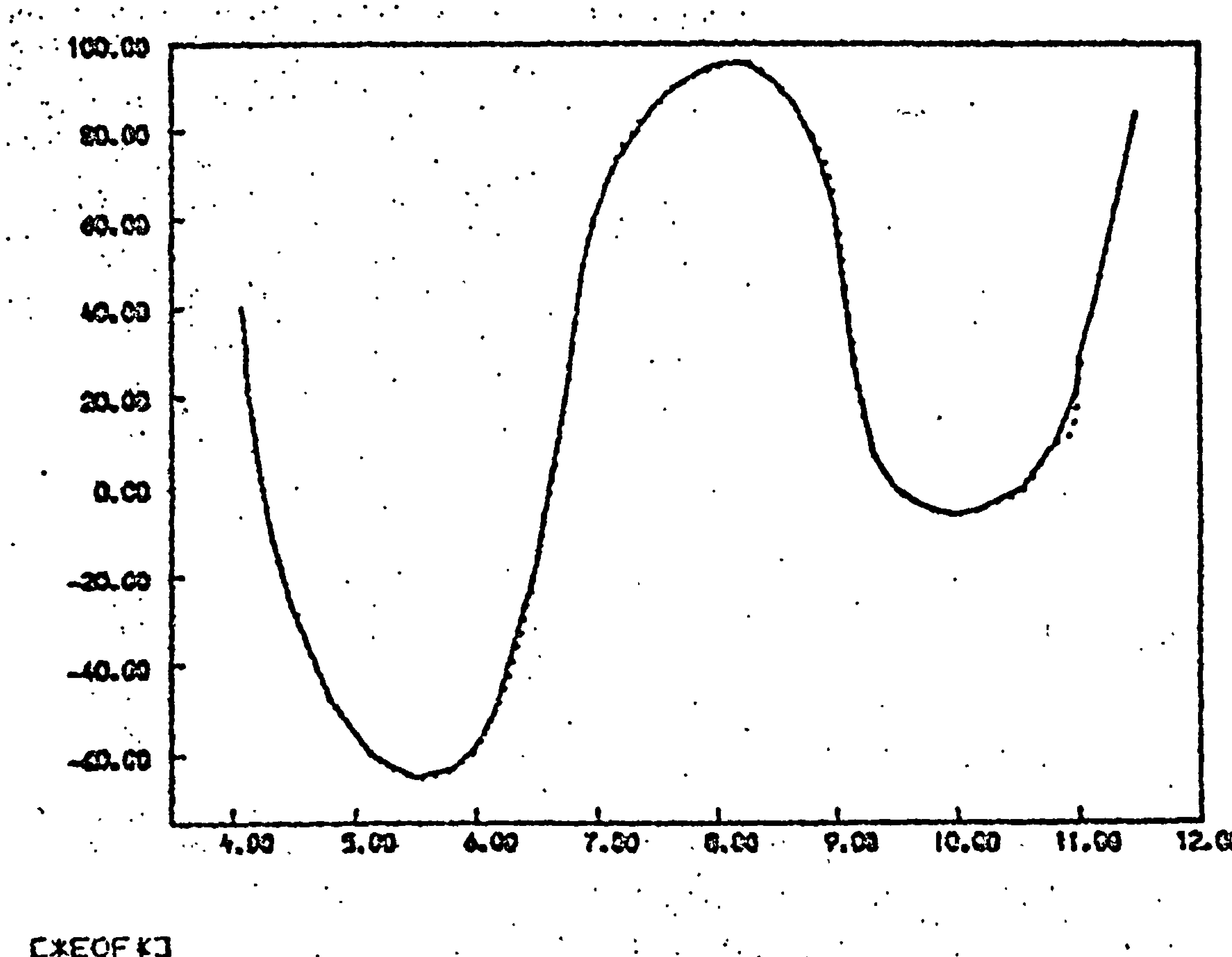



Fig.13 $El=f_1(p)$-EØ1ADF interpolation

RSITY OF MINNESOTA 6600 FORTRAN COMPILER MOSRE 1.1 VER4.5 08/03/70 10.40.54.

(=LFN)

```
      PROGRAM INTERP(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE27=PLOT)
      DIMENSION E(100),P(100),W(100),U(100),AX(110),VALY(110)
      EMIN=-0.3
      EMAX=0.25
      PMIN=3.5
      PMAX=12.0
      E(1)=-0.08
      E(2)=-4.11486574E-02
      E(3)=0.0
      E(4)=0.04
      E(5)=0.07
      E(6)=0.115
      E(7)=0.14
      E(8)=1.43696841E-01
      E(9)=0.14
      E(10)=0.10
      E(11)=0.035
      E(12)=-0.04
      E(13)=-0.08
      E(14)=-9.96179444E-02
      E(15)=-0.11
      E(16)=-0.15
      E(17)=-0.18
      E(18)=-0.19
      E(19)=-0.188
      E(20)=-1.86648345E-01
      E(21)=-0.17
      E(22)=-0.16
      E(23)=-0.12
      E(24)=-0.05
      E(25)=-0.02
      E(26)=-1.50139958E-02
      E(27)=0.02
      E(28)=0.03
      E(29)=0.04
      E(30)=0.01
      E(31)=-0.02
      E(32)=-0.05
      E(33)=-6.09907140E-02
      E(34)=-0.085
      E(35)=-0.13
      P(1)=4.0
      P(2)=4.14
      P(3)=4.33
      P(4)=4.50
      P(5)=4.66
      P(6)=5.0
      P(7)=5.33
      P(8)=5.52
      P(9)=5.66
      P(10)=6.0
      P(11)=6.33
      P(12)=6.66
      P(13)=6.83
      P(14)=6.90
      P(15)=7.0
      P(16)=7.33
      P(17)=7.66
      P(18)=8.0
      P(19)=8.16
      P(20)=8.28
      P(21)=8.50
      P(22)=8.66
      P(23)=9.0
      P(24)=9.33
      P(25)=9.50
      P(26)=9.66
      P(27)=9.83
      P(28)=10.0
      P(29)=10.33
      P(30)=10.66
      P(31)=10.83
      P(32)=11.0
      P(33)=11.03
      P(34)=11.16
      P(35)=10.33
      A=4.14
      DO 10 I=1,101
      CALL E01ADF(34,A,P,E,U,W,100,VAL)
      AX(I)=A
      VALY(I)=VAL
      WRITE(6,100) A,VAL
  100 FORMAT(10X,E14.7,10X,E14.7)
      A=A+0.0689
   10 CONTINUE
      CALL CAM35MM
      CALL GRSLIDE
      CALL XAXIS(PMIN,PMAX)
      CALL YAXIS(EMIN,EMAX)
      CALL GRAPHIC(P,E,35)
      CALL DOT
      CALL GRAPHIC(AX,VALY,101)
      CALL DASHOFF
      CALL LXTICK
      CALL LYTICK
      CALL LXVAL
      CALL LYVAL
      CALL GRFRAME
      CALL ENDFILM
      STOP
      END
```

Progr. 4 Program INTERP

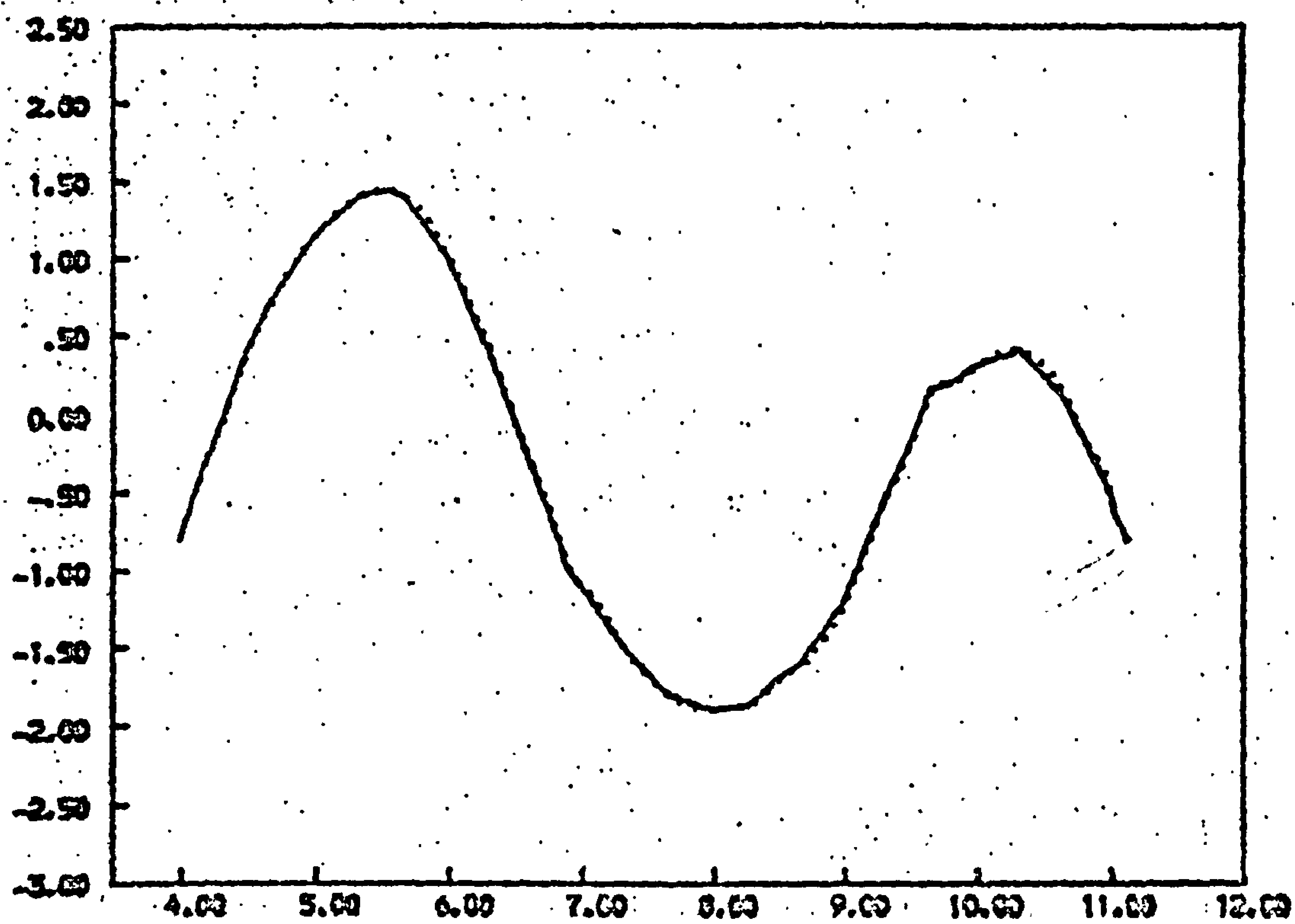
E2 (35 points)

Program 4 cont...

| | |
|---|---|
| .4140000E+01 | [illegible] |
| .4208900E+01 | -.2553015E-01 |
| .4277900E+01 | -.1120036E-01 |
| .4346700E+01 | .3795462E-02 |
| .4415600E+01 | .2023104E-01 |
| .4484500E+01 | .3653781E-01 |
| .4553400E+01 | .5106224E-01 |
| .4622400E+01 | .6368155E-01 |
| .4691200E+01 | .7439984E-01 |
| .4760100E+01 | .8517267E-01 |
| .4829000E+01 | .9455920E-01 |
| .4897900E+01 | .1035435E+00 |
| .4966800E+01 | .1114491E+00 |
| .5035700E+01 | .1186367E+00 |
| .5104600E+01 | .1250880E+00 |
| .5173500E+01 | .1307186E+00 |
| .5242400E+01 | .1354419E+00 |
| .5311300E+01 | .1391708E+00 |
| .5380200E+01 | .1418221E+00 |
| .5449100E+01 | .1433445E+00 |
| .5518000E+01 | .1437341E+00 |
| .5586900E+01 | .1427982E+00 |
| .5655800E+01 | .1402213E+00 |
| .5724700E+01 | .1355890E+00 |
| .5793500E+01 | .1285990E+00 |
| .5862500E+01 | .1206992E+00 |
| .5931400E+01 | .1129341E+00 |
| .6000300E+01 | .9994989E-01 |
| .6069200E+01 | .8796252E-01 |
| .6138100E+01 | .7507247E-01 |
| .6207000E+01 | .6134900E-01 |
| .6275900E+01 | .4696297E-01 |
| .6344800E+01 | .3168556E-01 |
| .6413700E+01 | .1596026E-01 |
| .6482600E+01 | -.1026555E-03 |
| .6551500E+01 | -.1570424E-01 |
| .6620400E+01 | -.3139259E-01 |
| .6689300E+01 | -.4621132E-01 |
| .6758200E+01 | -.6130182E-01 |
| .6827100E+01 | -.7914993E-01 |
| .6896000E+01 | -.9874585E-01 |
| .6964900E+01 | -.1077739E+00 |
| .7033800E+01 | -.1125722E+00 |
| .7102700E+01 | -.1196153E+00 |
| .7171500E+01 | -.1283439E+00 |
| .7240500E+01 | -.1378708E+00 |
| .7309400E+01 | -.1473209E+00 |

| | |
|---|---|
| .7378300E+01 | -.1558680E+00 |
| .7447200E+01 | -.1632750E+00 |
| .7516100E+01 | -.1696276E+00 |
| .7585000E+01 | -.1753492E+00 |
| .7653900E+01 | -.1796308E+00 |
| .7722800E+01 | -.1834825E+00 |
| .7791700E+01 | -.1864997E+00 |
| .7859600E+01 | -.1883535E+00 |
| .7929500E+01 | -.1898561E+00 |
| .7998400E+01 | -.1900695E+00 |
| .8067300E+01 | -.1891994E+00 |
| .8135200E+01 | -.1892082E+00 |
| .8205100E+01 | -.1878382E+00 |
| .8274000E+01 | -.1856431E+00 |
| .8342900E+01 | -.1831774E+00 |
| .8411800E+01 | -.1775315E+00 |
| .8480700E+01 | -.1715265E+00 |
| .8549600E+01 | -.1666335E+00 |
| .8618500E+01 | -.1625308E+00 |
| .8687400E+01 | -.1581533E+00 |
| .8756300E+01 | -.1526650E+00 |
| .8825200E+01 | -.1453428E+00 |
| .8894100E+01 | -.1371645E+00 |
| .8963000E+01 | -.1265902E+00 |
| .9031900E+01 | -.1137847E+00 |
| .9100800E+01 | -.9707973E-01 |
| .9169700E+01 | -.8395746E-01 |
| .9238600E+01 | -.6821987E-01 |
| .9307500E+01 | -.5414493E-01 |
| .9376400E+01 | -.4217553E-01 |
| .9445300E+01 | -.3074886E-01 |
| .9514200E+01 | -.1575660E-01 |
| .9583100E+01 | .7178396E-04 |
| .9652000E+01 | .1340730E-01 |
| .9720900E+01 | .1950967E-01 |
| .9789800E+01 | .1986923E-01 |
| .9858700E+01 | .2078046E-01 |
| .9927600E+01 | .2465533E-01 |
| .9996500E+01 | .2974712E-01 |
| .1006540E+02 | .3425473E-01 |
| .1013430E+02 | .3766699E-01 |
| .1020320E+02 | .3982649E-01 |
| .1027210E+02 | .4057507E-01 |
| .1034100E+02 | .3975772E-01 |
| .1040990E+02 | .3720048E-01 |
| .1047880E+02 | .3269714E-01 |
| .1054770E+02 | .2603528E-01 |
| .1061660E+02 | .1700245E-01 |
| .1068550E+02 | .5414278E-02 |
| .1075440E+02 | -.7603177E-02 |
| .1082330E+02 | -.1908175E-01 |
| .1089220E+02 | -.2739025E-01 |
| .1096110E+02 | -.3880903E-01 |
| .1103000E+02 | -.6099071E-01 |

RACTERS OUTPUT




[*EOF*]

Fig.14 $E2=f_2(p)$-EØ1ADF interpolation

```
ERSITY OF MINNESOTA 6600 FORTRAN COMPILER NOSBE 1.1   VER4.5   10/03/78  10.08.32.
B=LFN).
      PROGRAM INTERP(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE2=PLOT)
      DIMENSION E(100),P(100),W(100),U(100),AX(110),VALY(110)
      EMIN=-1.8E-04
      EMAX=2.5E-04
      PMIN=3.5
      PMAX=12.0
      E(1)=6.5E-05
      E(2)=4.0E-05
      E(3)=2.58885863E-05
      E(4)=0.85E-05
      E(5)=0.0
      E(6)=-3.0E-05
      E(7)=-5.0E-05
      E(8)=-6.9E-05
      E(9)=-8.0E-05
      E(10)=-9.0E-05
      E(11)=-1.01E-04
      E(12)=-1.04703810E-04
      E(13)=-9.5E-05
      E(14)=-9.0E-05
      E(15)=-8.0E-05
      E(16)=-5.0E-05
      E(17)=-1.5E-05
      E(18)=1.9E-05
      E(19)=5.2E-05
      E(20)=6.59503096E-05
      E(21)=9.5E-05
      E(22)=1.25E-04
      E(23)=1.44E-04
      E(24)=1.58E-04
      E(25)=1.40E-04
      E(26)=1.30E-04
      E(27)=1.18321254E-04
      E(28)=9.5E-05
      E(29)=7.9E-05
      E(30)=6.5E-05
      E(31)=4.1E-05
      E(32)=2.5E-05
      E(33)=0.0
      E(34)=-0.9E-05
      E(35)=-1.2E-05
      E(36)=-1.86052006E-05
      E(37)=-2.0E-05
      E(38)=-2.1E-05
      E(39)=-2.0E-05
      E(40)=-1.0E-05
      E(41)=0.9E-05
      E(42)=2.0E-05
      E(43)=3.0E-05
      E(44)=3.15E-05
      E(45)=4.14880100E-05
      E(46)=5.0E-05
      E(47)=6.0E-05
      E(48)=7.0E-05
      E(49)=9.5E-05
      P(1)=4.0
      P(2)=4.1
      P(3)=4.14
      P(4)=4.26
      P(5)=4.33
      P(6)=4.50
      P(7)=4.66
      P(8)=4.83
      P(9)=5.0
      P(10)=5.16
      P(11)=5.33
      P(12)=5.52
      P(13)=5.66
      P(14)=5.83
      P(15)=6.0
      P(16)=6.23
      P(17)=6.50
      P(18)=6.66
      P(19)=6.83
      P(20)=6.90
      P(21)=7.0
      P(22)=7.16
      P(23)=7.33
      P(24)=7.66
      P(25)=8.0
      P(26)=8.16
      P(27)=8.28
      P(28)=8.43
      P(29)=8.56
      P(30)=8.66
      P(31)=8.83
      P(32)=9.0
      P(33)=9.33
      P(34)=9.43
      P(35)=9.56
      P(36)=9.66
      P(37)=9.80
      P(38)=9.93
      P(39)=10.0
      P(40)=10.33
      P(41)=10.66
      P(42)=10.83
      P(43)=10.93
      P(44)=11.0
      P(45)=11.03
      P(46)=11.16
      P(47)=11.26
      P(48)=11.40
      P(49)=11.60
      A=4.14
      DO 10 I=1,101
      CALL E01ADF(49,A,P,E,D,W,100,VAL)
      AX(I)=A
      VALY(I)=VAL
      WRITE(6,100) A,VAL
  100 FORMAT(10X,E14.7,10X,E14.7)
      A=A+0.0689
   10 CONTINUE
      CALL CAM35MM
      CALL BRSLIDE
      CALL XAXIS(PMIN,PMAX)
      CALL YAXIS(EMIN,EMAX)
      CALL GRAPHIC(P,E,49)
      CALL DOT
      CALL GRAPHIC(AX,VALY,101)
      CALL DASHOFF
      CALL LXTICK
      CALL LYTICK
      CALL LXVAL
      CALL LYVAL
      CALL GRFRAME
      CALL ENDFILM
      STOP
      END
```

Progr. 5  Program INTERP E3 (49 points)

Program 5 cont.....

| | |
|---|---|
| .4140000E+01 | .2588859E-04 |
| .4208900E+01 | .1291484E-04 |
| .4277800E+01 | .6748037E-05 |
| .4346700E+01 | -.2629713E-05 |
| .4415600E+01 | -.1495461E-04 |
| .4484500E+01 | -.2746127E-04 |
| .4553400E+01 | -.3758542E-04 |
| .4622300E+01 | -.4572792E-04 |
| .4691200E+01 | -.5366630E-04 |
| .4760100E+01 | -.6179041E-04 |
| .4829000E+01 | -.6891036E-04 |
| .4897900E+01 | -.7413280E-04 |
| .4966800E+01 | -.7815806E-04 |
| .5035700E+01 | -.8207377E-04 |
| .5104600E+01 | -.8635771E-04 |
| .5173500E+01 | -.9090570E-04 |
| .5242400E+01 | -.9552921E-04 |
| .5311300E+01 | -.9990171E-04 |
| .5380200E+01 | -.1036154E-03 |
| .5449100E+01 | -.1056801E-03 |
| .5518000E+01 | -.1047071E-03 |
| .5586900E+01 | -.1003870E-03 |
| .5655800E+01 | -.9525473E-04 |
| .5724700E+01 | -.9231147E-04 |
| .5793600E+01 | -.9082252E-04 |
| .5862500E+01 | -.8897891E-04 |
| .5931400E+01 | -.8555830E-04 |
| .6000300E+01 | -.7997016E-04 |
| .6069200E+01 | -.7196851E-04 |
| .6138100E+01 | -.6254397E-04 |
| .6207000E+01 | -.5299236E-04 |
| .6275900E+01 | -.4448117E-04 |
| .6344800E+01 | -.3668364E-04 |
| .6413700E+01 | -.2830777E-04 |
| .6482600E+01 | -.1804538E-04 |
| .6551500E+01 | -.4829741E-05 |
| .6620400E+01 | .1028667E-04 |
| .6689300E+01 | .2518408E-04 |
| .6758200E+01 | .3082366E-04 |
| .6827100E+01 | .5148326E-04 |
| .6896000E+01 | .6498132E-04 |
| .6964900E+01 | .8468115E-04 |
| .7033800E+01 | .1034799E-03 |
| .7102700E+01 | .1166752E-03 |
| .7171600E+01 | .1265296E-03 |
| .7240500E+01 | .1349011E-03 |
| .7309400E+01 | .1420796E-03 |
| .7378300E+01 | .1480931E-03 |
| .7447200E+01 | .1528548E-03 |
| .7516100E+01 | .1562102E-03 |
| .7585000E+01 | .1580041E-03 |
| .7653900E+01 | .1580811E-03 |
| .7722800E+01 | .1563677E-03 |

| | |
|---|---|
| .7791700E+01 | .1532192E-03 |
| .7860600E+01 | .1491289E-03 |
| .7929500E+01 | .1445911E-03 |
| .7998400E+01 | .1400998E-03 |
| .8067300E+01 | .1359588E-03 |
| .8136200E+01 | .1316564E-03 |
| .8205100E+01 | .1264287E-03 |
| .8274000E+01 | .1191055E-03 |
| .8342900E+01 | .1088208E-03 |
| .8411800E+01 | .9769185E-04 |
| .8480700E+01 | .8842707E-04 |
| .8549600E+01 | .8029042E-04 |
| .8618500E+01 | .7107462E-04 |
| .8687400E+01 | .6071106E-04 |
| .8756300E+01 | .5078227E-04 |
| .8825200E+01 | .4158335E-04 |
| .8894100E+01 | .3401839E-04 |
| .8963000E+01 | .2787913E-04 |
| .9031900E+01 | .2267738E-04 |
| .9100800E+01 | .1794707E-04 |
| .9169700E+01 | .1325072E-04 |
| .9238600E+01 | .8147328E-05 |
| .9307500E+01 | .2199113E-05 |
| .9376400E+01 | -.4750134E-05 |
| .9445300E+01 | -.9673716E-05 |
| .9514200E+01 | -.1082844E-04 |
| .9583100E+01 | -.1330660E-04 |
| .9652000E+01 | -.1817067E-04 |
| .9720900E+01 | -.2011612E-04 |
| .9789800E+01 | -.1999558E-04 |
| .9858700E+01 | -.2052005E-04 |
| .9927600E+01 | -.2100803E-04 |
| .9996500E+01 | -.2006860E-04 |
| .1006540E+02 | -.1867598E-04 |
| .1013430E+02 | -.1708604E-04 |
| .1020320E+02 | -.1512903E-04 |
| .1027210E+02 | -.1263517E-04 |
| .1034100E+02 | -.4350298E-05 |
| .1040990E+02 | -.5506462E-05 |
| .1047880E+02 | -.1195564E-05 |
| .1054770E+02 | .3094947E-05 |
| .1061660E+02 | .6962351E-05 |
| .1068550E+02 | .1003122E-04 |
| .1075440E+02 | .1320143E-04 |
| .1082330E+02 | .1916555E-04 |
| .1089220E+02 | .2822041E-04 |
| .1096110E+02 | .2851837E-04 |
| .1103000E+02 | .4148801E-04 |

ACTERS OUTPUT

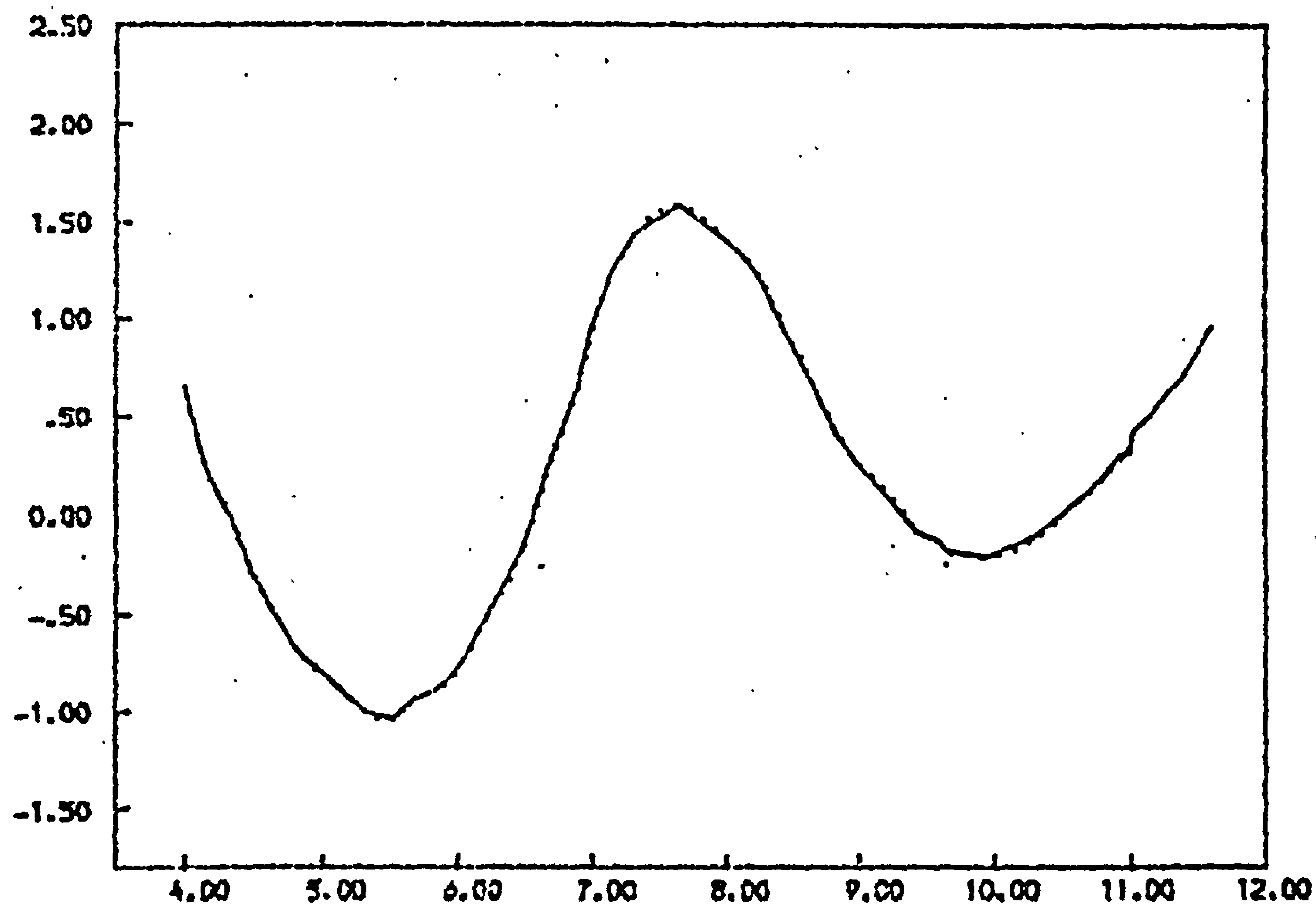



Fig.15 E3=$f_3$(p)-EØ1ADF interpolation

ERSITY OF MINNESOTA 6600 FORTRAN COMPILER MOSDE 1.1 VER4.5 10/03/78 15.52.23.
=LFN)

```
      PROGRAM INTERP(PLOT,INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,
     1TAPE7=PLOT)
      DIMENSION E(100),P(100),W(100),U(100),AX(110),VALY(110)
      EMIN=-3.5E-08
      EMAX=4.0E-08
      PMIN=3.5
      PMAX=12.0
      E(1)=-2.0E-08
      E(2)=-1.48E-08
      E(3)=-1.0E-08
      E(4)=-5.23941833E-09
      E(5)=-1.5E-09
      E(6)=3.0E-09
      E(7)=6.5E-09
      E(8)=1.4E-08
      E(9)=2.1E-08
      E(10)=2.4E-08
      E(11)=2.58E-08
      E(12)=2.6E-08
      E(13)=2.52426073E-08
      E(14)=2.35E-08
      E(15)=2.05E-08
      E(16)=1.65E-08
      E(17)=1.2E-08
      E(18)=6.1E-09
      E(19)=-1.2E-09
      E(20)=-7.0E-09
      E(21)=-1.2E-08
      E(22)=-1.41001361E-08
      E(23)=-1.6E-08
      E(24)=-1.98E-08
      E(25)=-2.25E-08
      E(26)=-2.48E-08
      E(27)=-2.5E-08
      E(28)=-2.47E-08
      E(29)=-2.4E-08
      E(30)=-2.39940391E-08
      E(31)=-2.0E-08
      E(32)=-1.6E-08
      E(33)=-8.0E-09
      E(34)=-2.0E-09
      E(35)=2.8E-09
      E(36)=4.9E-09
      E(37)=5.9E-09
      E(38)=6.73902622E-09
      E(39)=7.0E-09
      E(40)=6.5E-09
      E(41)=5.5E-09
      E(42)=3.5E-09
      E(43)=0.7E-09
      E(44)=-1.0E-09
      E(45)=-4.5E-09
      E(46)=-8.3E-09
      E(47)=-9.07972456E-09
      E(48)=-1.45E-08
      E(49)=-1.80E-08
      P(1)=4.0
      P(2)=4.06
      P(3)=4.1
      P(4)=4.14
      P(5)=4.2
      P(6)=4.26
      P(7)=4.33
      P(8)=4.50
      P(9)=4.76
      P(10)=5.0
      P(11)=5.16
      P(12)=5.33
      P(13)=5.22
      P(14)=5.66
      P(15)=5.83
      P(16)=6.0
      P(17)=6.16
      P(18)=6.33
      P(19)=6.50
      P(20)=6.66
      P(21)=6.83
      P(22)=6.90
      P(23)=7.0
      P(24)=7.16
      P(25)=7.33
      P(26)=7.56
      P(27)=7.83
      P(28)=8.0
      P(29)=8.16
      P(30)=8.28
      P(31)=8.50
      P(32)=8.66
      P(33)=8.83
      P(34)=9.0
      P(35)=9.16
      P(36)=9.33
      P(37)=9.50
      P(38)=9.66
      P(39)=9.83
      P(40)=10.0
      P(41)=10.16
      P(42)=10.36
      P(43)=10.56
      P(44)=10.66
      P(45)=10.83
      P(46)=11.0
      P(47)=11.03
      P(48)=11.16
      P(49)=11.23
      A=4.14
      DO 10 I=1,101
      CALL E01A F(48,A,P,E,U,W,100,VAL)
      AX(I)=A
      VALY(I)=VAL
      WRITE(6,100) A,VAL
  100 FORMAT(10X,E14.7,10X,E14.7)
      A=A+0.0689
   10 CONTINUE
      CALL CAM35MM
      CALL GRSLIDE
      CALL XAXIS(PMIN,PMAX)
      CALL YAXIS(EMIN,EMAX)
      CALL GRAPHIC(P,E,49)
      CALL DOT
      CALL GRAPHIC(AX,VALY,101)
      CALL DASHOFF
      CALL EXTICK
      CALL LYTICK
      CALL LXVAL
      CALL LYVAL
      CALL GRFRAME
      CALL ENDFILM
      STOP
      END
```

<u>Progr. 6</u> Program INTERP E4(49 points).

Program 6 cont....

| | |
|---|---|
| .4140000E+01 | -.5236418E-08 |
| .4208900E+01 | -.9124461E-09 |
| .4277800E+01 | .4103424E-08 |
| .4346700E+01 | .7216063E-08 |
| .4415600E+01 | .1031921E-07 |
| .4484500E+01 | .1336388E-07 |
| .4553400E+01 | .1598601E-07 |
| .4622300E+01 | .1810037E-07 |
| .4691200E+01 | .1975568E-07 |
| .4760100E+01 | .2100154E-07 |
| .4829000E+01 | .2191480E-07 |
| .4897900E+01 | .2268105E-07 |
| .4966800E+01 | .2351292E-07 |
| .5035700E+01 | .2459849E-07 |
| .5104600E+01 | .2560615E-07 |
| .5173500E+01 | .2571262E-07 |
| .5242400E+01 | .2495808E-07 |
| .5311300E+01 | .2538091E-07 |
| .5380200E+01 | .2426760E-07 |
| .5449100E+01 | .2425098E-07 |
| .5518000E+01 | .2423623E-07 |
| .5586900E+01 | .2405667E-07 |
| .5655800E+01 | .2354558E-07 |
| .5724700E+01 | .2258299E-07 |
| .5793600E+01 | .2127343E-07 |
| .5862500E+01 | .1978539E-07 |
| .5931400E+01 | .1819763E-07 |
| .6000300E+01 | .1649229E-07 |
| .6069200E+01 | .1464979E-07 |
| .6138100E+01 | .1266264E-07 |
| .6207000E+01 | .1052152E-07 |
| .6275900E+01 | .8164216E-08 |
| .6344800E+01 | .5496410E-08 |
| .6413700E+01 | .2534207E-08 |
| .6482600E+01 | -.4688775E-09 |
| .6551500E+01 | -.3248874E-08 |
| .6620400E+01 | -.5718850E-08 |
| .6689300E+01 | -.7886939E-08 |
| .6758200E+01 | -.9868433E-08 |
| .6827100E+01 | -.1179094E-07 |
| .6896000E+01 | -.1400601E-07 |
| .6964900E+01 | -.1537991E-07 |
| .7033800E+01 | -.1671900E-07 |
| .7102700E+01 | -.1892446E-07 |
| .7171600E+01 | -.2004682E-07 |
| .7240500E+01 | -.2127594E-07 |
| .7309400E+01 | -.2223656E-07 |
| .7378300E+01 | -.2310357E-07 |
| .7447200E+01 | -.2389125E-07 |
| .7516100E+01 | -.2451375E-07 |
| .7585000E+01 | -.2490347E-07 |
| .7653700E+01 | -.2505370E-07 |
| .7722800E+01 | -.2505014E-07 |

| | |
|---|---|
| .7791700E+01 | -.2500456E-07 |
| .7860600E+01 | -.2501809E-07 |
| .7929500E+01 | -.2505230E-0 |
| .7998400E+01 | -.2490718E-07 |
| .8067300E+01 | -.2447227E-07 |
| .8136200E+01 | -.2405909E-07 |
| .8205100E+01 | -.2403434E-07 |
| .8274000E+01 | -.2401498E-07 |
| .8342900E+01 | -.2328579E-07 |
| .8411800E+01 | -.2175893E-07 |
| .8480700E+01 | -.2041783E-07 |
| .8549600E+01 | -.1898167E-07 |
| .8618500E+01 | -.1734915E-07 |
| .8687400E+01 | -.1480775E-07 |
| .8756300E+01 | -.1157955E-07 |
| .8825200E+01 | -.8214802E-08 |
| .8894100E+01 | -.5469034E-08 |
| .8963000E+01 | -.3183060E-08 |
| .9031900E+01 | -.9545106E-09 |
| .9100800E+01 | .1240196E-08 |
| .9169700E+01 | .3007696E-08 |
| .9238600E+01 | .4105077E-08 |
| .9307500E+01 | .4743222E-08 |
| .9376400E+01 | .5194723E-08 |
| .9445300E+01 | .5592619E-08 |
| .9514200E+01 | .5991371E-08 |
| .9583100E+01 | .6372268E-08 |
| .9652000E+01 | .6706908E-08 |
| .9720900E+01 | .6925340E-08 |
| .9789800E+01 | .7011331E-08 |
| .9858700E+01 | .6964326E-08 |
| .9927600E+01 | .6792804E-08 |
| .9996500E+01 | .6516463E-08 |
| .1006540E+02 | .6151590E-08 |
| .1013430E+02 | .5695112E-08 |
| .1020320E+02 | .5139616E-08 |
| .1027210E+02 | .4482405E-08 |
| .1034100E+02 | .3725920E-08 |
| .1040990E+02 | .2872300E-08 |
| .1047880E+02 | .1926250E-08 |
| .1054770E+02 | .8931622E-09 |
| .1061660E+02 | -.2281019E-09 |
| .1068550E+02 | -.1454153E-08 |
| .1075440E+02 | -.2883720E-08 |
| .1082330E+02 | -.4355849E-08 |
| .1089220E+02 | -.5861759E-08 |
| .1096110E+02 | -.7380636E-08 |
| .1103000E+02 | -.9079725E-08 |

ACTERS OUTPUT

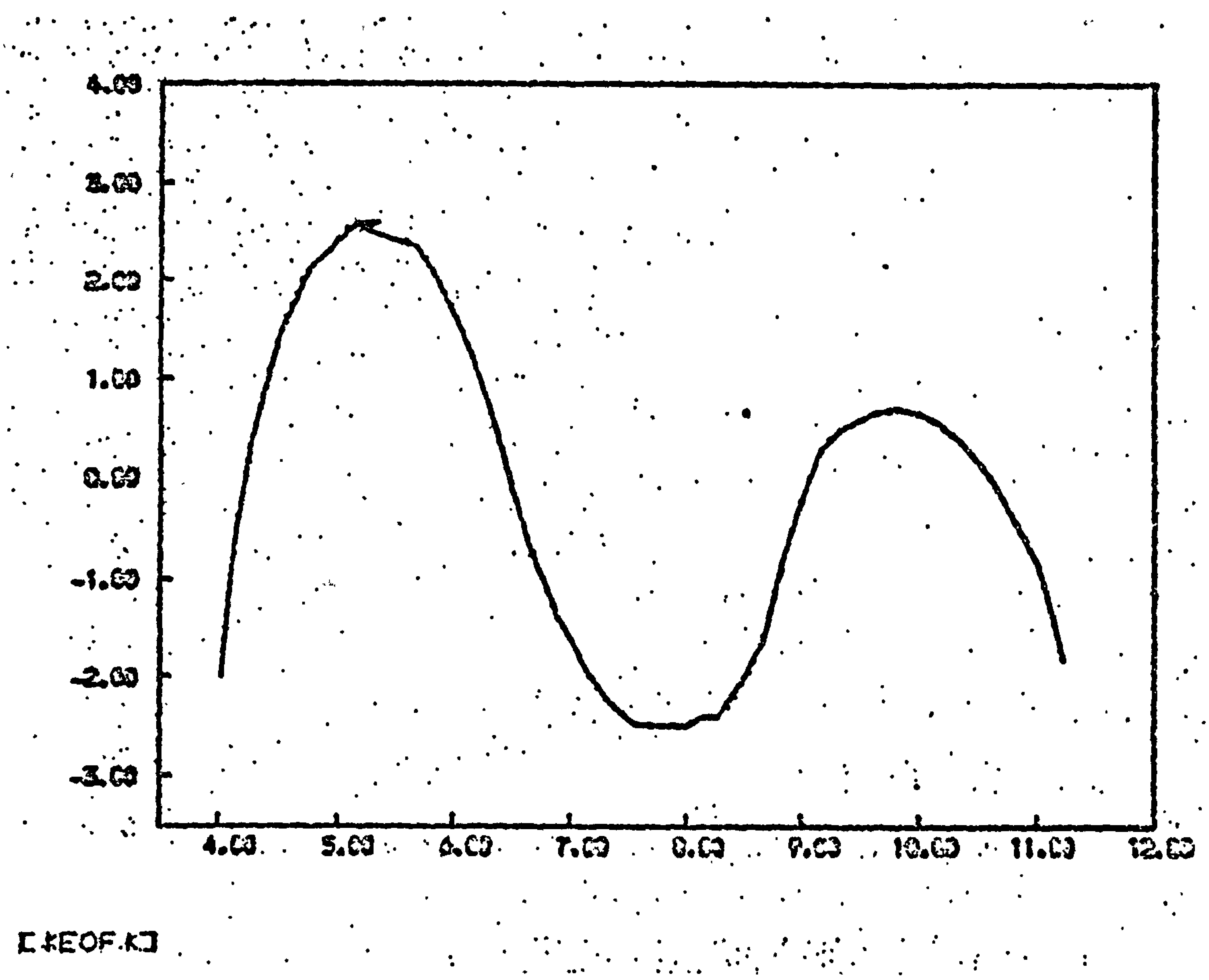



Fig.16 E4=$f_4$(p)-EØ1ADF interpolation

## BIBLIOGRAPHY


1. Subroutine EØ2ACF - Nottingham Algorithms Group, ICL 1900 SYSTEM. NAG Library Manual, Document No.189, 30th Sept. 1971

2. Subroutine EØ1ADF - Nottingham Algorithms Group, ICL 1900 System, N.A.G. Library Manual Document No.577, 8th January 1973

3. Linear Optimal Control, Prentice Hall Inc. By Brian D.O. Anderson 1971

4. Optimum Systems Control, Prentice Hall Inc. New York. By A.P. Sage 1968

## 9.3. PROPERTIES AND RESULTS OF OPTIMAL DERIVATIVE ERROR CONTROL SYSTEM - PROGRAM OPTO

The optimisation process discussed in previous sections of this chapter provide not only the required "speed accuracy" but also correct transient behaviour of the engine through the modifications of the DEC model TD1 parameter. This is included in the earlier described DEC program's subroutine EQNS (see Appendix No.5).
In order to distinguish between the Optimal Derivative Error Control System (Program OPTO) and any of the earlier versions, all the "optimisation" data decks are included in the main EQNS routine listing and a sufficient number of informative comments lines are used. After the standard SYSTRAN Common/Dimension block and part of the initial engine's setting (PRSSUR/YSET) - the number of intervals of data points given refer to the four optimal parameters E1,E2,E3,E4, (i.e. NA,NE,NC,ND). The NF parameters refer to the fuel loop explained in the last section. All the four basic data groups as well as "fuel group" are preceeded by relevant headings for example :

```
*DATA E1,P1*   TD1 OPTIMISATION PROCESS*
EXTERNAL NAG LIBRARY * ROUTINE EØ1ADF
EVALUATE INTERPOLATED VALUES BY FITTING
A CUBIC SPLINE FUNCTION
```

For the "fuel group", the first part of the above heading is changed to :

```
*COMPUTE SPECIFIC FUEL VALUE FOR GIVEN PRESSURE LEVEL*
```

This part of the OPTO listing occupies about 421 lines of the print-out space and is then followed by the next section of the original DEC data input deck. Then the NAG EØ1ADF routine is called four times i.e.:

```
CALL(EØ1ADF(NA,PRSSUR,D,A,WW,60,E1)
CALL(EØ1ADF(NB,PRSSUR,R,B,WW,60,E2)
CALL(EØ1ADF(NC, PRSSUR,S,C,WW,60 E3)
CALL(EØ1ADF(ND,PRSSUR,T,D,WW,60,E4)
```

and all this is concluded with the PRINT request for interpolated E1,E2,E3,E4 values and corresponding operational pressure. Similar order applies to the fuel loop, but excluding the PRINT statement as this was part of the original DEC program. In this part of the section, the aim is to use the optimal DERIVATIVE Error Control regulator as represented by the OPTO model program to show a number of typical engine runs with particular respect to their "automotive" properties in terms of widely used TORQUE/SPEED characteristics. It is necessary to, firstly, adjust the operational pressure (PRSSUR) level at 6.90 bar and run the program with various set speed levels (YSET); for example 1230 RPM (see Fig.1.) 1440 RPM (see Fig.2 ) 1620 RPM (see Fig.3), 1750 RPM (see Fig.4). The YDOTD (final output speed level) deciphered in the main tabulation print out is: 1229 RPM, 1442 RPM, 1618 RPM, and 1749 RPM, which indicates the proper functioning of the speed control system and the TORQUE/SPEED characteristic gradually increases its "length" along the same axis (speed axis and also dips slightly towards the direction

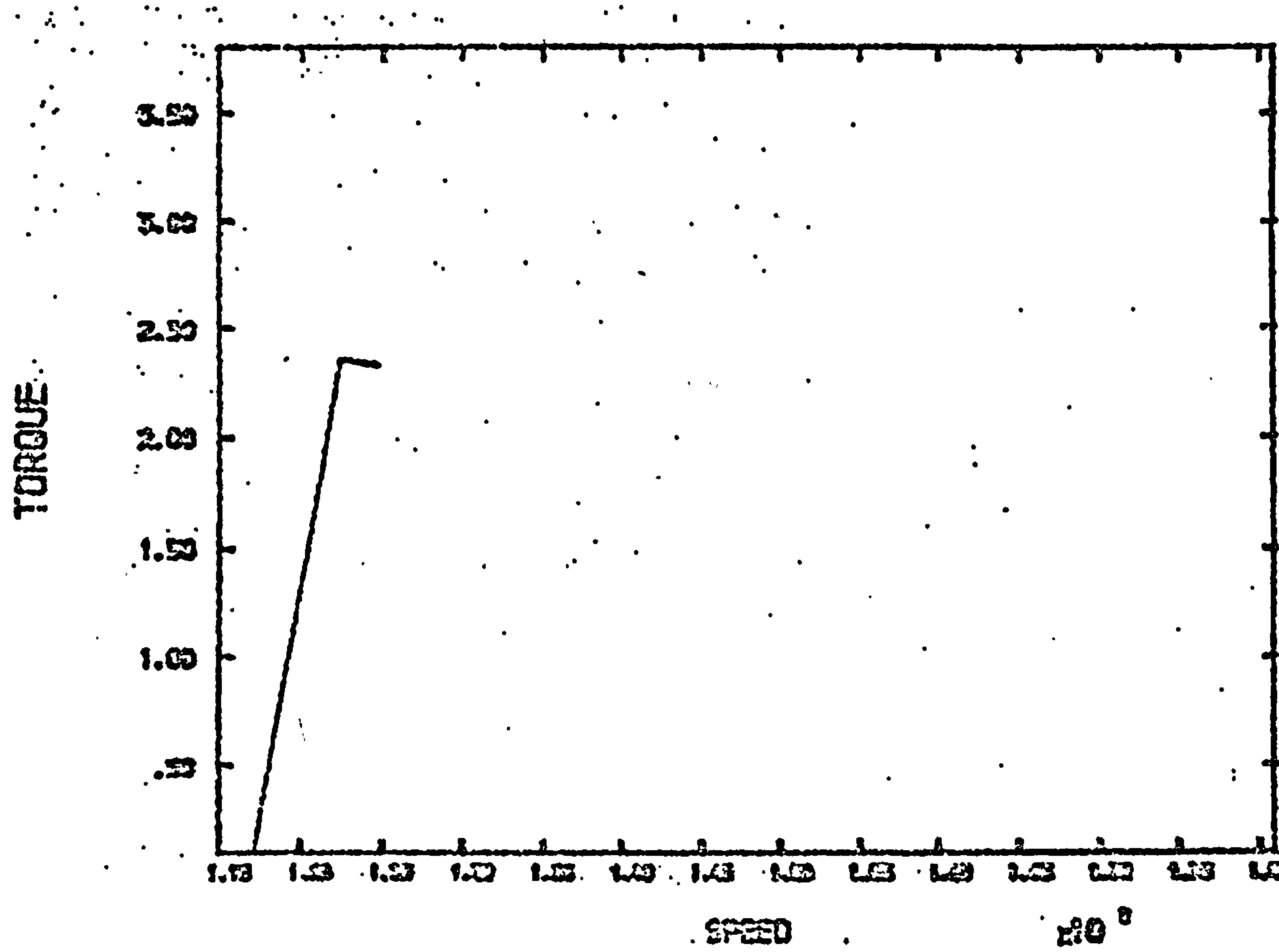



Fig.1 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR= 6.90 bar
YSET= 1230 RMS
YDOT= 1229 RMS

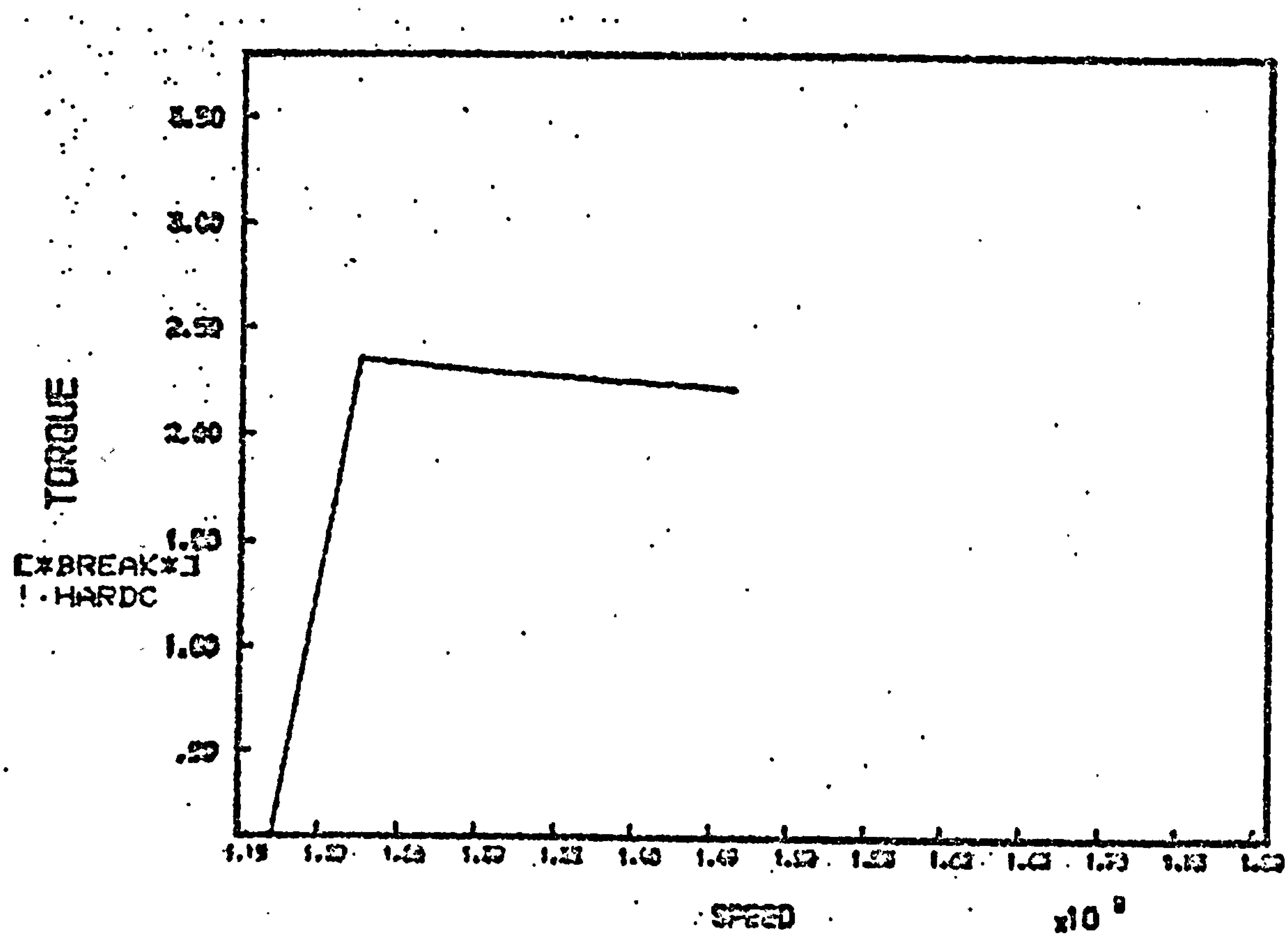



Fig.2 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=6.90 bar
YSET=1440 RMS
YDOT=1442 RMS

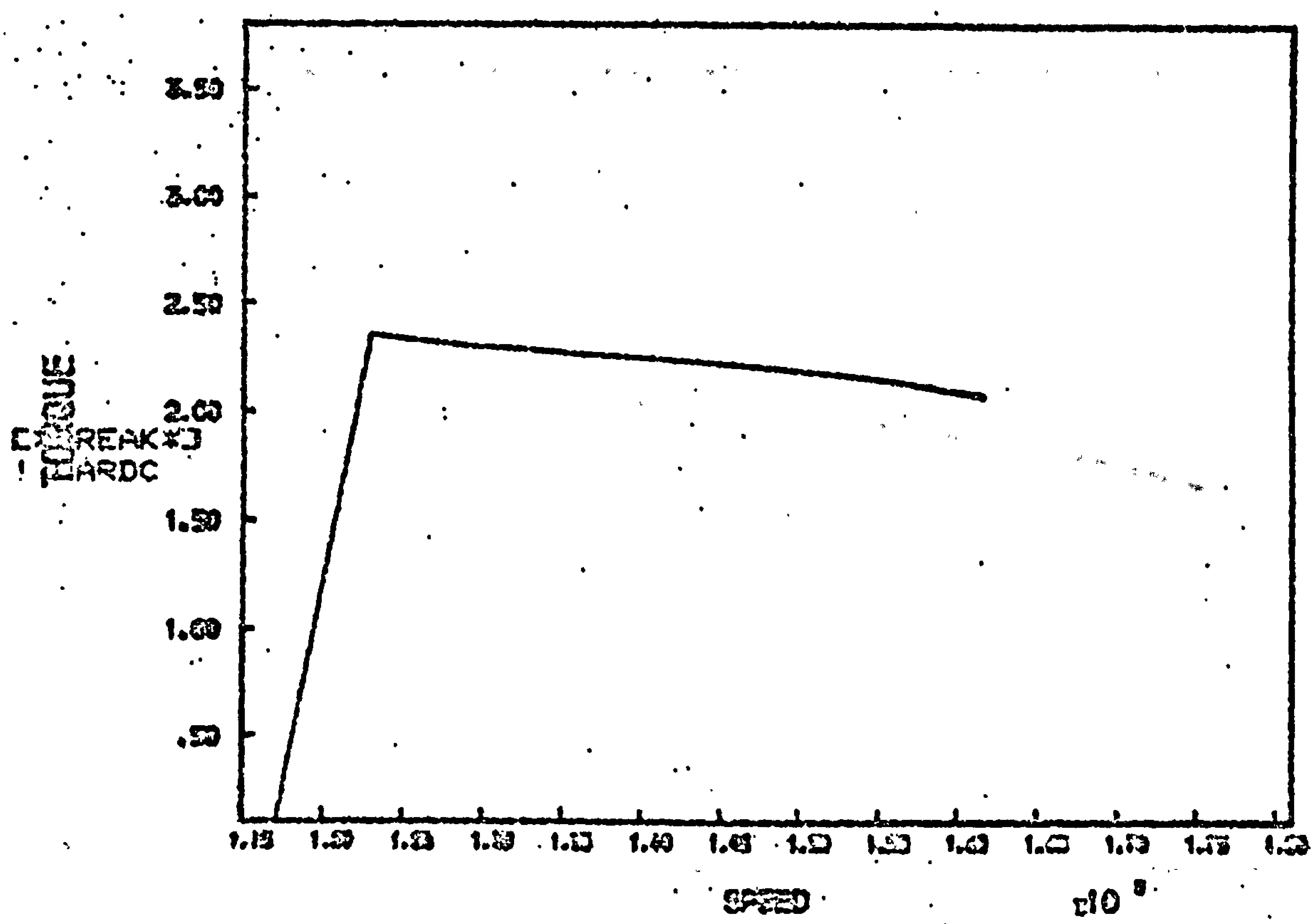



Fig.3 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=6.90 bar
YSET= 1620 RMS
YDOT= 1618 RMS

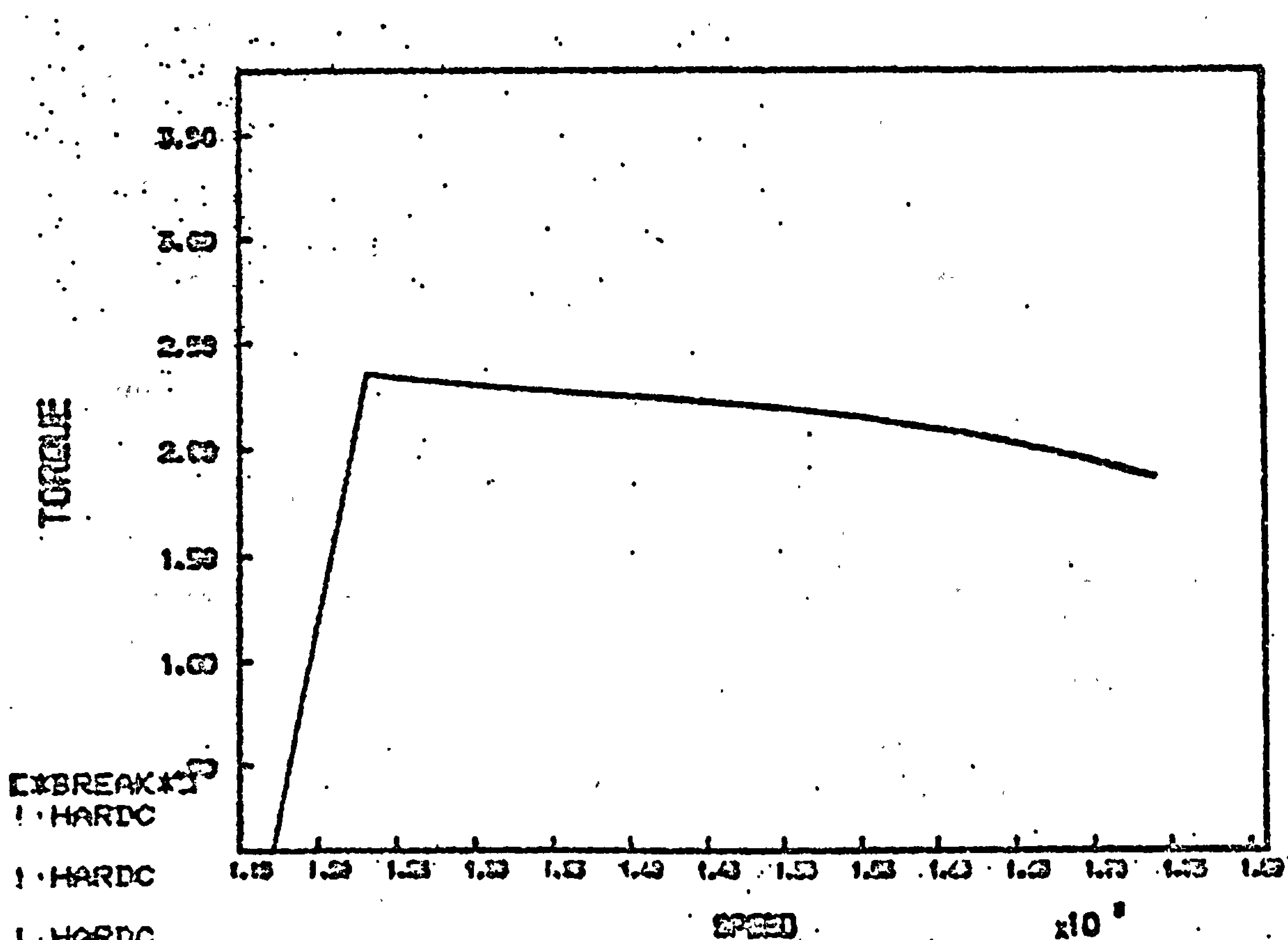



Fig.4 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=6.90 bar
YSET= 1750 RMS
YDOT= 1749 RMS

of increased speed values). This is known as the "automotive characteristic" i.e. higher TORQUE values correspond to smaller speed levels and higher speed correspond to the relatively lower TORQUE levels. Basically, this is the most important feature of any good automotive Stirling engine design. In the next program OPTO runs, the set-speed will be kept at constant levels and operational pressure is varied in order to achieve the variations in the engine TORQUE. During this experiment, the pressure set values (PRSSUR) are : 4.14 bar (see Fig.5) 6.90 bar (see Fig.6),9.60 bar (see Fig.7) and finally the maximum of 11.o3 bar (see Fig.8) and the TORQUE level responds from the minimum value of about 1.4 (N-m) to the maximum of approximately 3.5 (N-m). Once more the speed output is practically equal to 1230 RPM as defined in the input data deck.
A similar set of results (for the YSET = 1620 RPM) is shown in Fig.9 ( Pressure = 4.14 bar), Fig.10 (Pressure = 6.90 bar), Fig.11 (Pressure = 9.66 bar), Fig.12 (pressure = 11.03 bar) and the specific character of the TORQUE/SPEED plot is similar in character as before - an optimistic prognostic for prospective automotive applications.

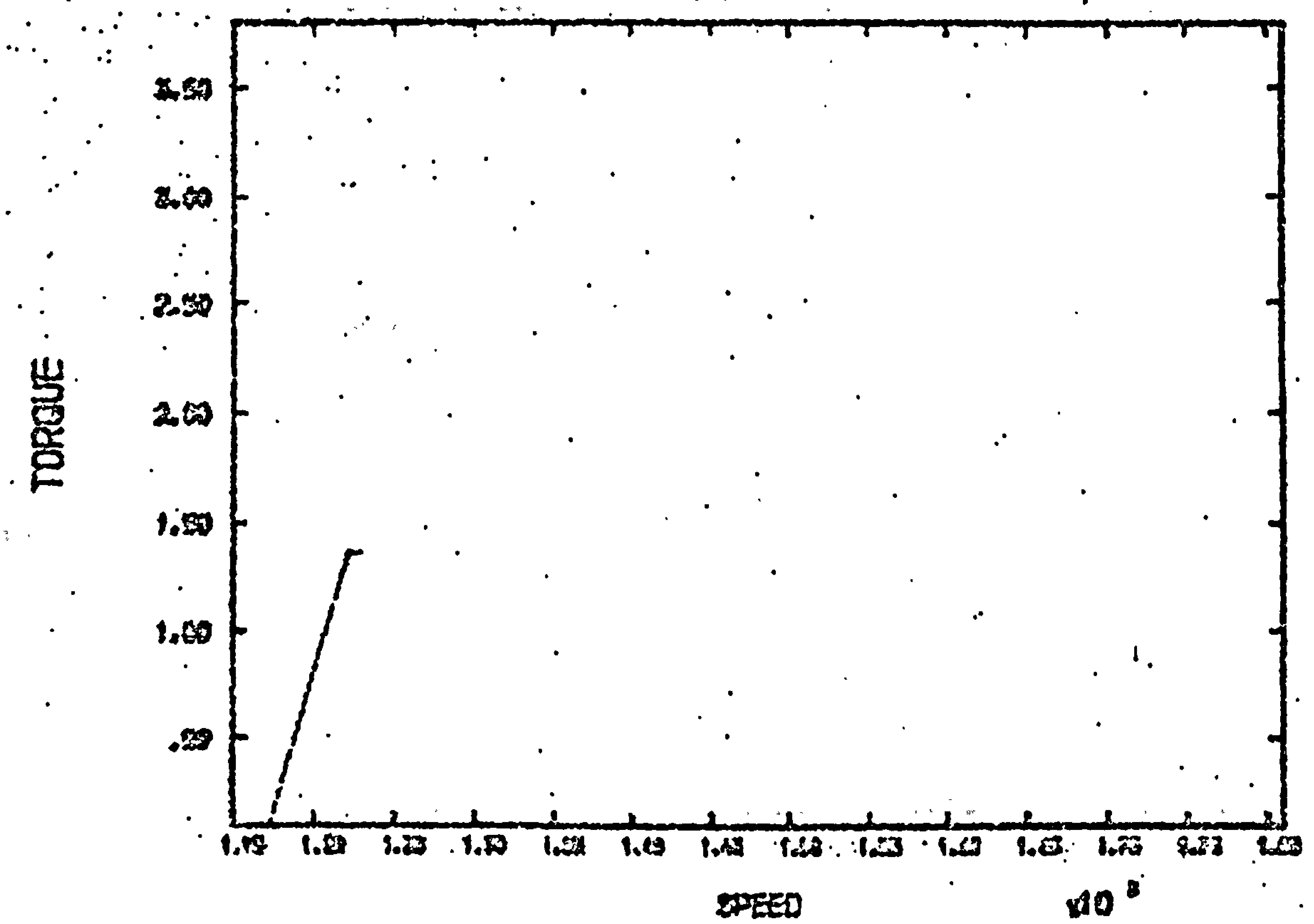



Fig.5 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=4.14 bar
YSET=1230 RMS
YDOT=1229 RMS

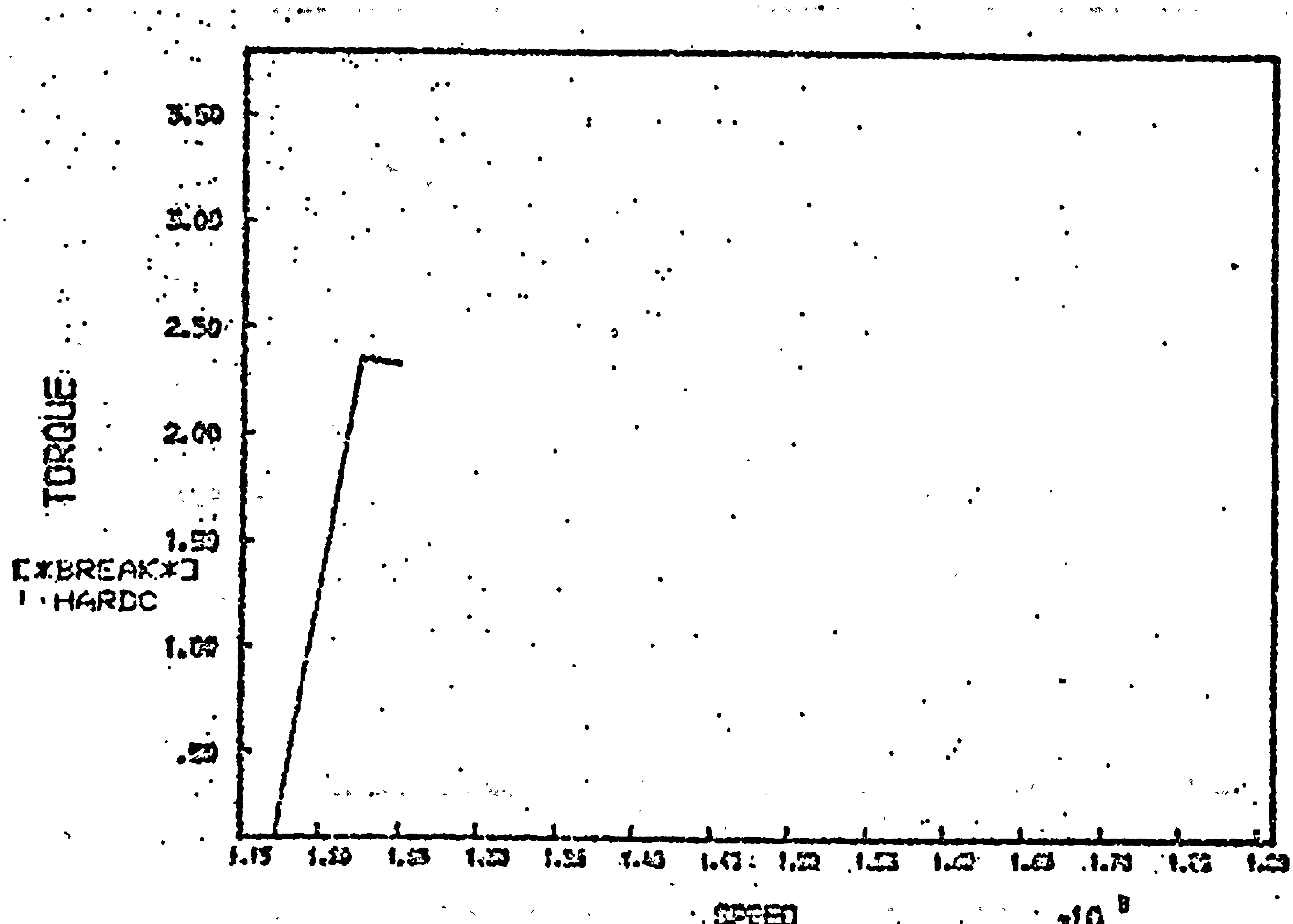



Fig. 6 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR= 6.90 bar
YSET= 1230 RPM
YDOT= 1229 RPM

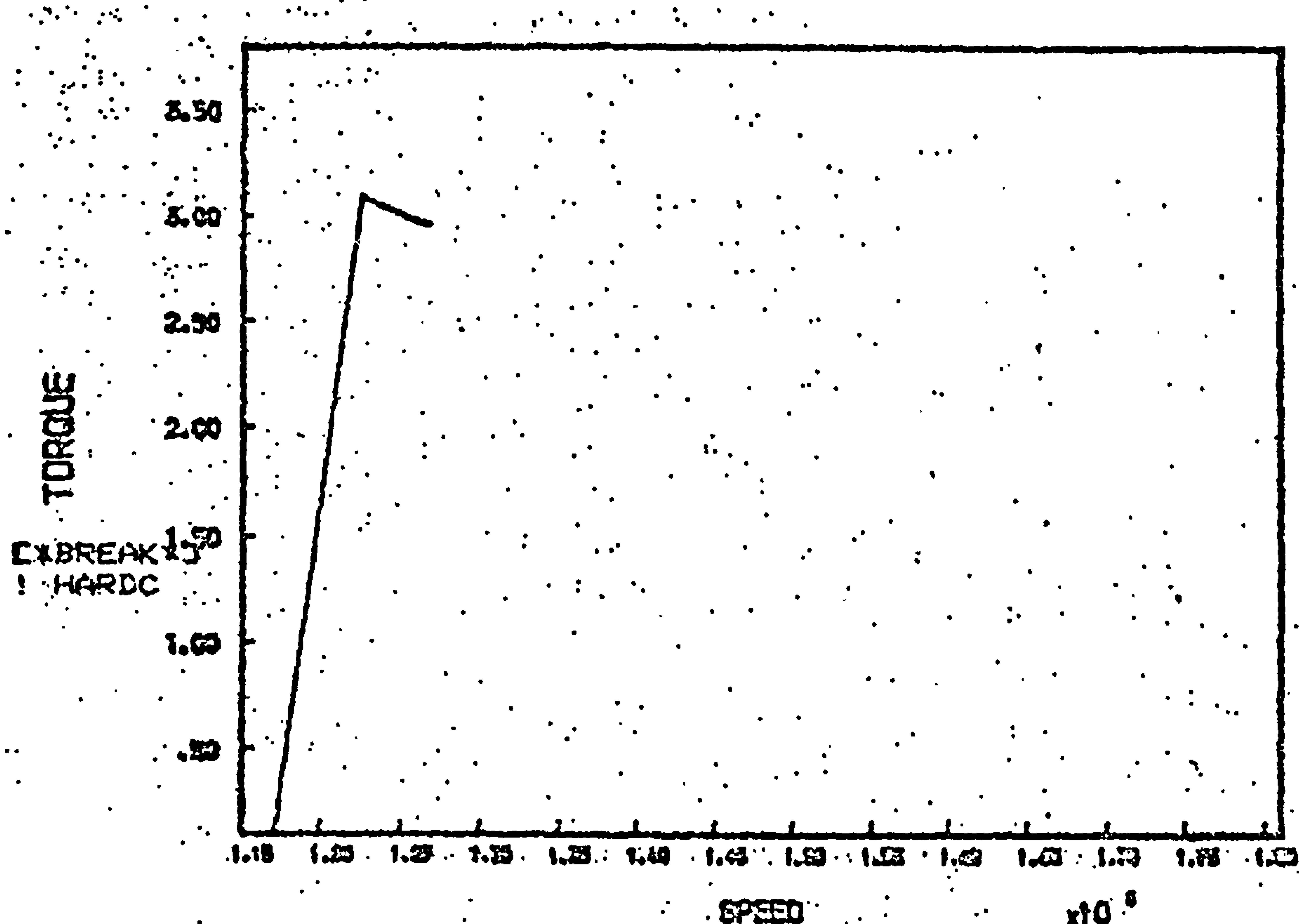
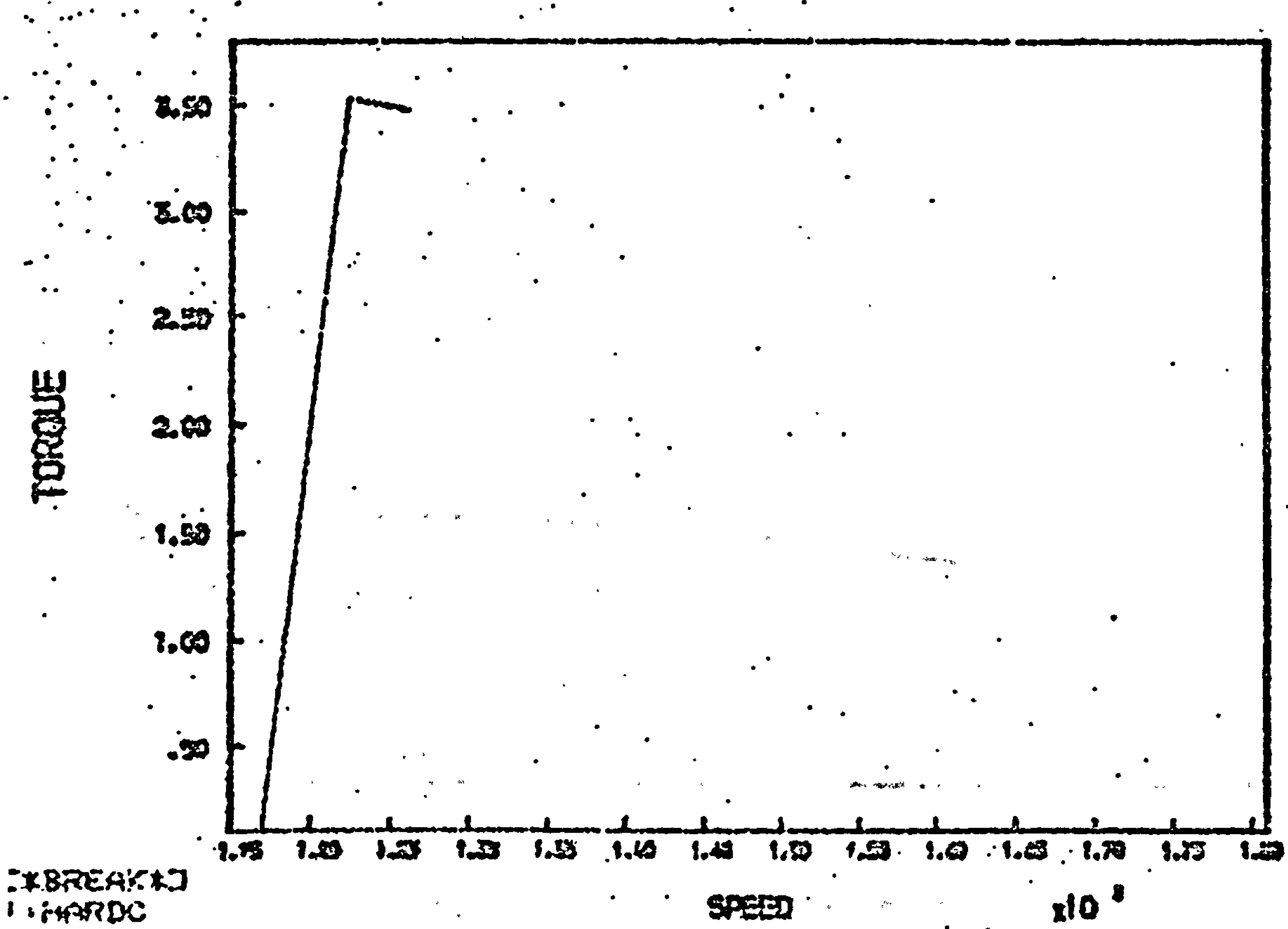



Fig.7 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=9.66 bar
YSET=1230 RPM
YDOT=1232 RPM




:*BREAK*:
HARDC

Fig.8 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=11.03 bar
YSET=1230 RPM
YDOT=1230 RPM

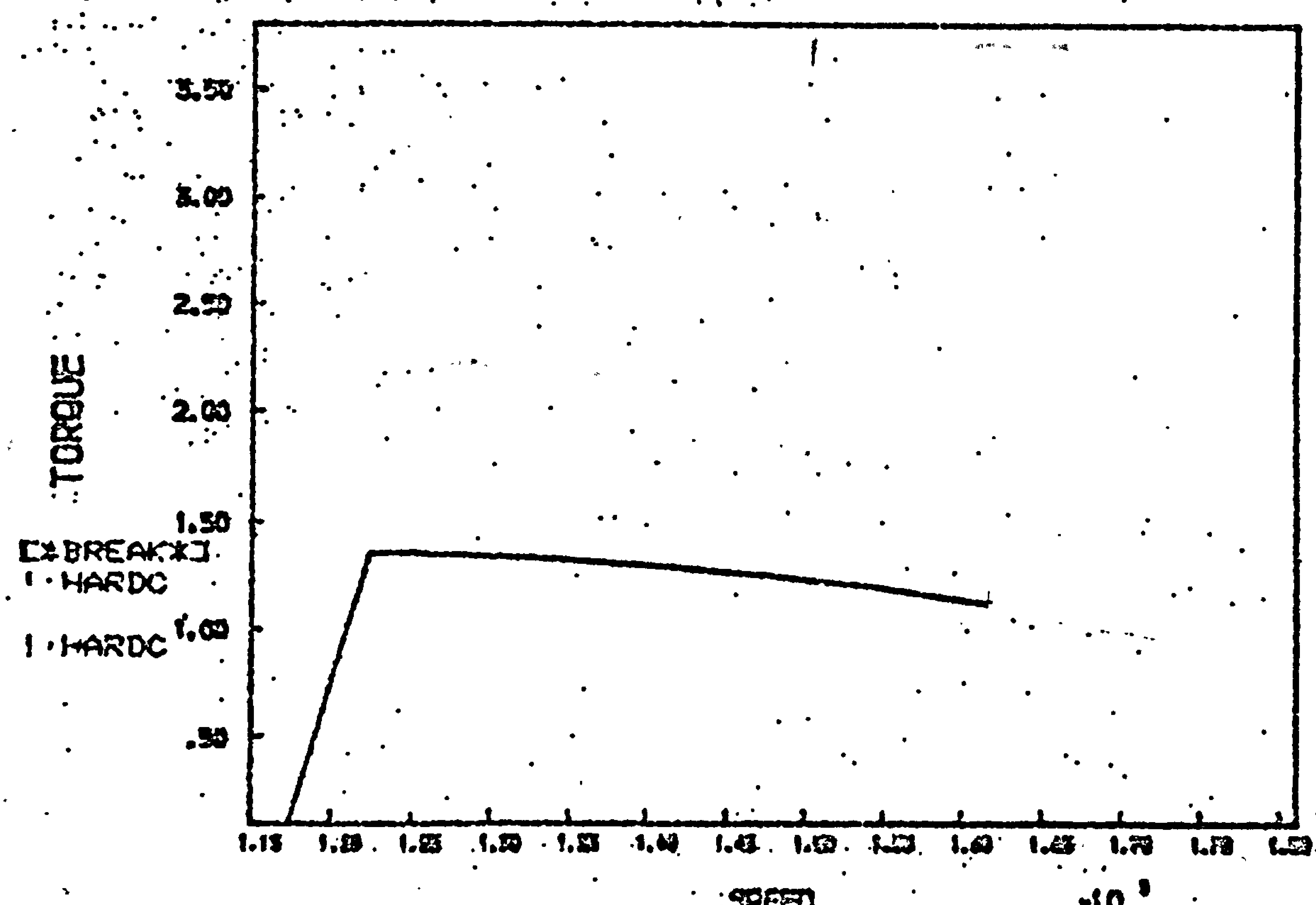



Fig. 9

SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=4.14 bar
YSET= 1620 RPM
YDOT= 1705 RPM

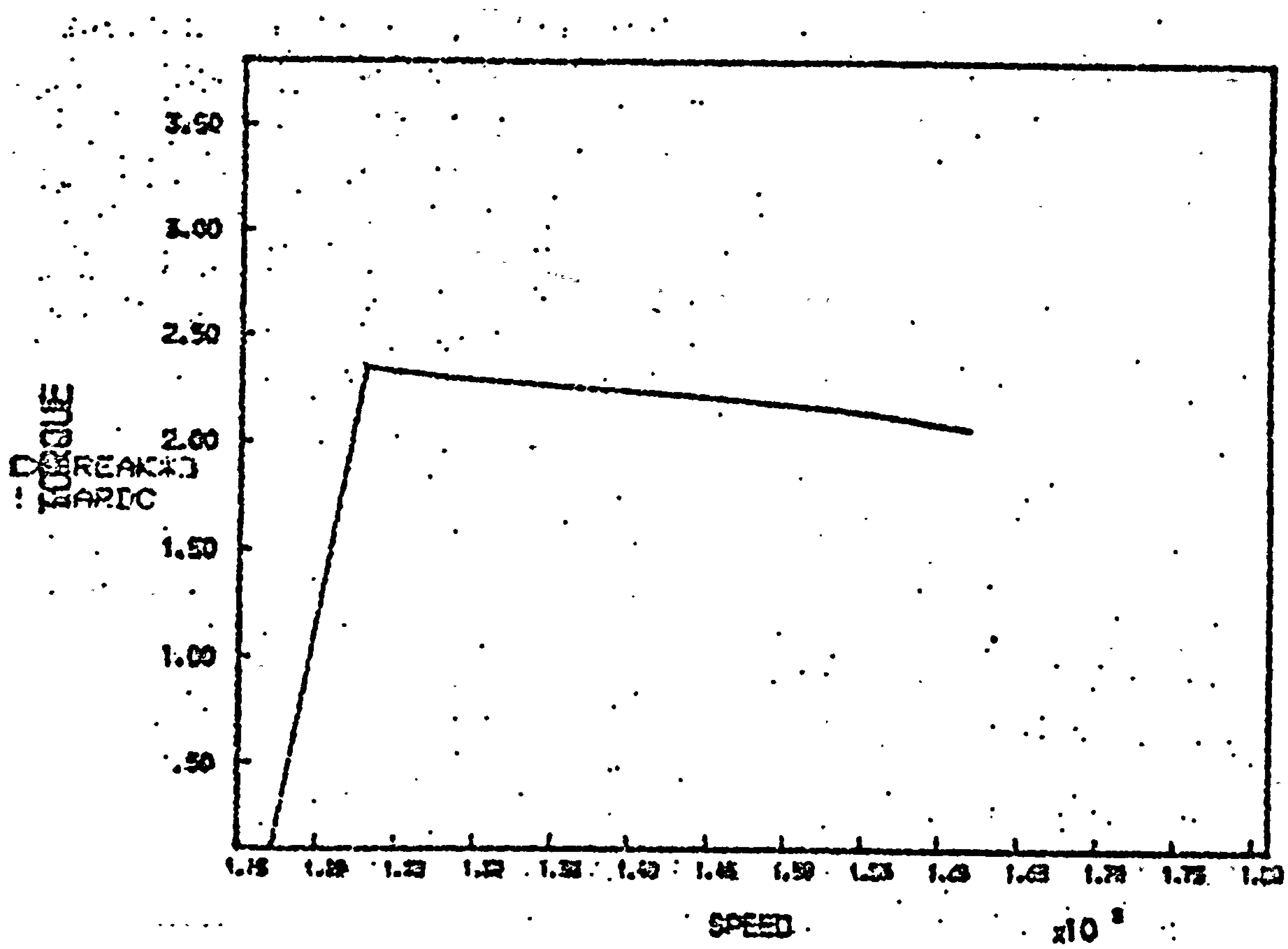



Fig.10 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=6.90 bar
YSET=1620 RPM
YDOT=1618 RPM

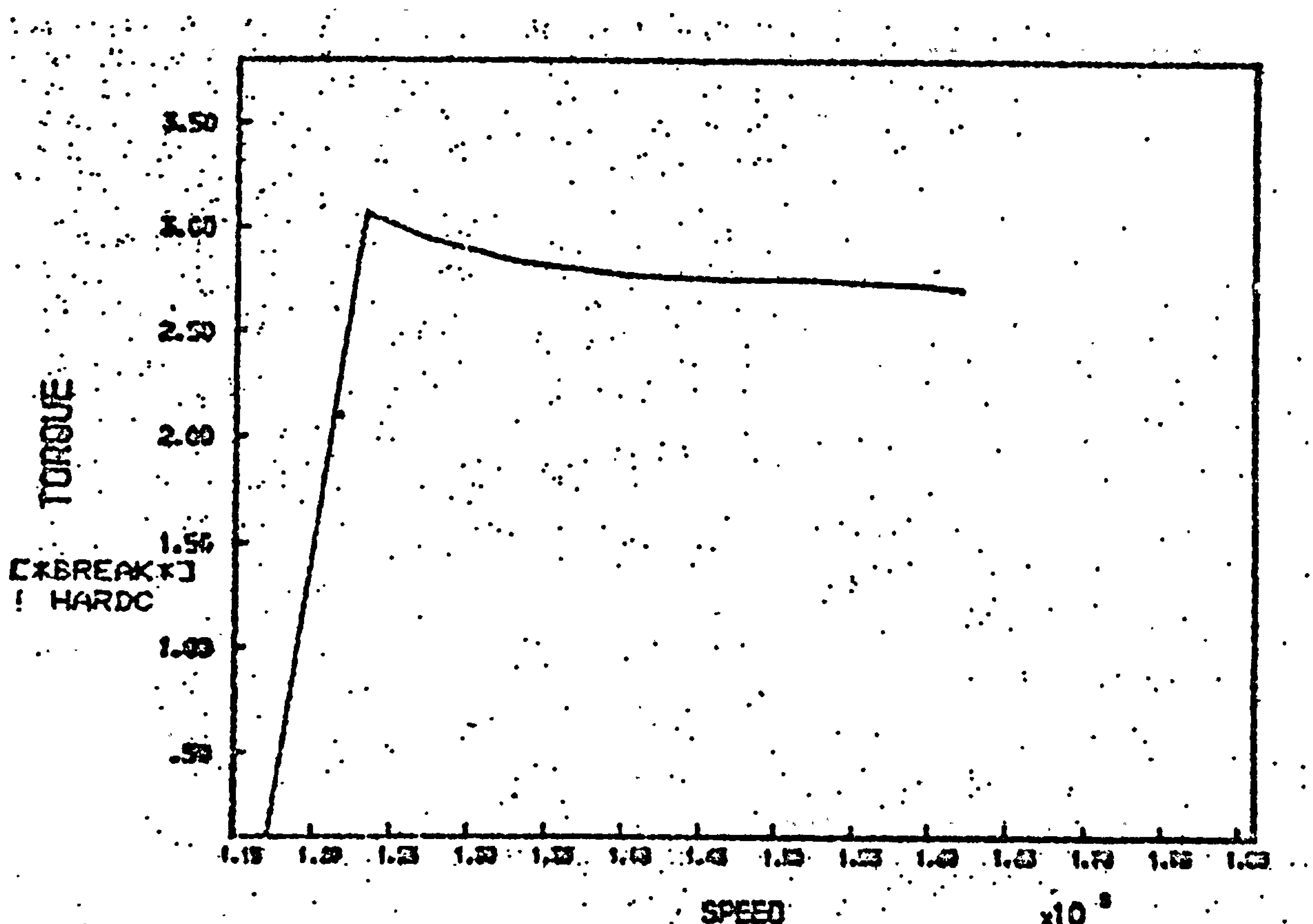



Fig.11 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=9.66 bar
YSET=1620 RPM
YDOT=1620 RPM

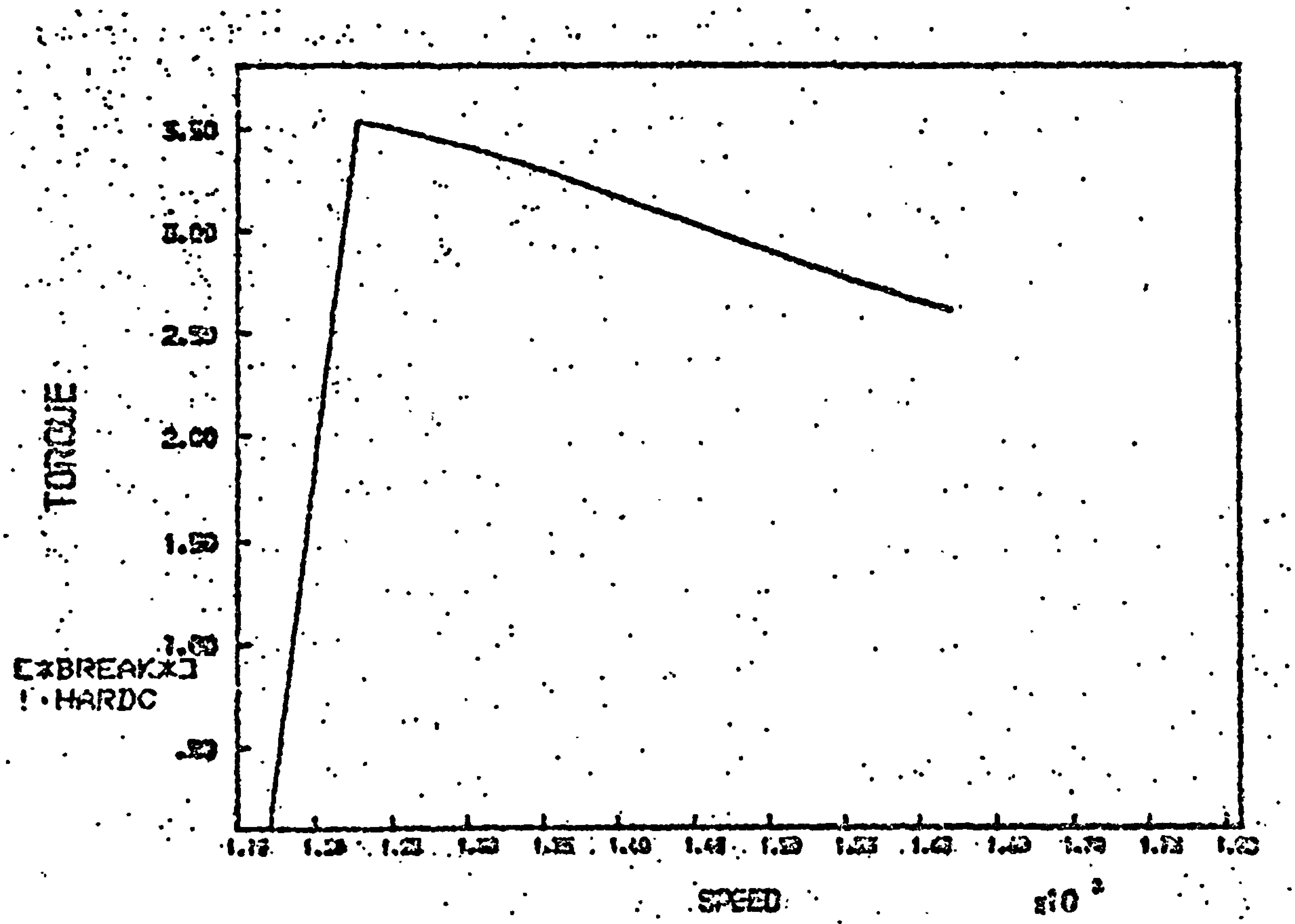



Fig.12 SCM TORQUE/SPEED CHARACTERISTIC

PRSSUR=11.03 bar
YSET=1620 RPM
YDOT=1621 RPM

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2. Linear Optimal Control, Prentice Hall Inc. By Brian D.O. Anderson 1971

3. Hyperstability and Optimality of Automatic Systems With Several Control Functions. Rev, Roumaine, Science Tech. Electrotechnical Energ. Vol.9 No.4 1964, pp.629-690

## 9.4. DISCUSSION

From the work presented in this chapter, it is seen that if close accuracy of an output RPM speed is desired, at fixed maximum technologically possible head temperature level (as in practical Stirling engine - for general efficiency and best fuel utilisation) - then certain conditions must be met. Firstly, the fuel input should be adjusted according to "Walker"s requirements, although TD1 parameter (for details see DEC Program description) modification across the whole operational pressure range is highly desireable, both to produce fast and stable as well as accurate output. Both these conditions were as described in Chapter 9 practically achieveable and desireable. Further, two methods using external software (NAG subroutines) are tried i.e. approximation of various types of Chebyshev polynomials (EØ2ACF) as well as interpolative fitting of cubic spline functions (EØ1 ADF).
Hence it is seen that the interpolation method is superior to previous polynomial forms as far as the accuracy is concerned, whilst the cubic spline function fit is incorporated in the optimal regulator modelling program. It has also been shown in section 9.3. that the engine's TORQUE/SPEED characteristic being one of the most important "automotive" engine design features is satisfactorily shaped and the POWER/TORQUE regulation is practically achieved, simply by changing the operational pressure. The last feature was described in the first chapter (see section 1.6), where the speed/pressure - MPS (Mean Pressure System) was discussed. The TORQUE/SPEED

"Tektronix" type plots are produced for each program OPTO (Optimal Derivative Error Control Regulator) run, which allow for fast and convenient modelling tests and further system modifications and developments, by adaption of factors which will be discussed later on.

# 10. CONCLUSIONS

## 10.1. GENERAL DISCUSSION

In ascertaining the design requirements for the Stirling Engine Simulation Model and its speed control system, there are two major parts that are considered in this thesis. These are :

Part One : (Chapters 1,2,3,4,)

Theoretical introduction into the Stirling Cycle Machine Philosophy followed by practical description of the MP 1002CA Philips engine, including Prof. Walker's experiment. A selection of available data material for elementary steady-state considerations and finally the chapter dealing with the basic engine simulation and also introducing the SYSTRAN program.

Part Two: (Chapters 5,6,7,8,9)

Advanced Systran Stirling engine model conception and its proportional control system, followed by practical evaluation tests and analysis using Bode diagrams. A theoretical model conception is based on Walker's data and employs a combinative interpolation network covered by Subroutines EQNS, POWTOR and SLGAIN and these are explained in detail in Chapter 5. Derivation of further control system requirements are also outlined with practical implementation of compensation theory including detailed stability evaluation. Then a comparison of

both compensated and uncompensated systems' performance, on the basis of Root-Locus method and evaluation of the system steady-state errors (constant velocity) for both compensated and uncompensated models. Finally, the optimal Derivative Error Control is introduced and discussed with a practical extension of the preceeding results to the full pressure/fuel/temperature/speed ranges. All main simulation programs of this thesis work in conjunction with the basic SYSTRAN file(VSN=M2955M) and the SYSTRAN MKIVA version was used exclusively.
The three simulation modelling programs:

PCS - (Proportional Control System)
DEC - (Derivative Error Compensation)
OPTO - (Optimalised DEC program)

are discussed in the appropriate chapters as well as in sections 10.2, 10.3 and 10.4.

## 10.2. SIMULATION PROGRAM PCS

As explained in the second part of this thesis, the theoretical dynamic model PCS conception incorporates three main program subroutines (i.e. EQNS,POWTOR,SLGAIN) and one additional plotting program (i.e. PLT) that work in conjunction with a post-processing data file. The overall engine's speed output is controlled proportionally by means of a "zero-error seeking" control system. A feedback controller which is proportional only, changes the output signal in direct proportion to the error signal. Although a PCS model feedback control still has one main problem, namely the generally unsatisfactory speed transient response. As proved in section 7.4, chapter 7, the specific phase margin frequency was in this case rather on the low side for all three considered speed regions i.e.

Reg.1. $W_{pm} = 1.0913 \left(\frac{\text{Radian}}{\text{sec}}\right)$

Reg.2. $W_{pm} = 0.791 \left(\frac{\text{Radian}}{\text{sec}}\right)$

Reg.3. $W_{pm} = 1.014 \left(\frac{\text{Radian}}{\text{sec}}\right)$

At the same time, according to BODE diagram analysis - program PCS was conditionally stable, with relevant phase/gain margin values as presented in Chapter 6 and a complete PCS system root-locus pattern (shown in Fig.2/ Section 8.1/Chapter 8) shows clearly that the root-locus pattern is bent into the right half of the S plane, and for gain constant values greater than those corresponding to the $\pm$ j 14.2 points, the system becomes unstable. Later it was shown that the use of the error Derivative action significantly improves the dynamic transient behaviour of the system. Derivative units (lead-lag units) are used to get an approximate derivative action.

## 10.3 SIMULATION PROGRAM DEC

A consideration of the PCS program performance under specified transient/stability conditions, shows that some form of transient response improvement is required. The simulation program DEC considers a control system which incorporates the Error Derivative Compensation Control (also called rate action or preact), with the main purpose, which is to anticipate where the process is heading by looking at the time rate of change of the error , - its derivative. The DEC system is explained in detail in Chapter 7. For evaluation of stability factors both BODE diagrams/ Root Locus Pattern methods are used. In order to make the system stable, for all gain constant values, the root-loci entering the right half of the S plane, i.e. going to be reshaped, so that both "right branches" lie completely in the left P Plane half. Both methods mentioned prove that the compensated system is unconditionally stable for any practical gain constant values. Also the specific phase margin frequency (as calculated in Chapter 7), for all three considered speed regions, increases substantially i.e.

Reg. 1. $W_{pm} = 4.88 \ \left(\frac{\text{Radian}}{\text{sec}}\right)$

Reg. 2. $W_{pm} = 2.474 \ \left(\frac{\text{Radian}}{\text{sec}}\right)$

Reg. 3. $W_{pm} = 3.625 \ \left(\frac{\text{Radian}}{\text{sec}}\right)$

It is also shown in section 8.3. Chapter 8 that the steady-state error coefficient $K_v$ increases from 0.003313 (uncompensated system) to 0.01717 (compensated system). Ref. RAMP input test, which indicates improvement in steady-state velocity errors. Finally it has been found that some reduction of PHASE/GAIN margin values (for details see Chapter 8) does not lead to any practical problems in the system after compensation.

## 10.4. SIMULATION PROGRAM OPTO

Although the work described in Chapted 9 shows that even the closest accuracy of an output RPM speed signal is required at a fixed maximum technologically possible temperature level, then certain optimal design conditions must be met. These conditions are the subject of optimalised derivative error control system incorporated in program OPTO (for details see Chapter 9). Optimal control is one particular branch of modern control science that sets out to provide analytical designs of a specially appealing type. The system which is the end result of optimal control system design is not supposed merely to be stable, have a certain bandwidth or satisfy any one of the desirable constraints associated with classical linear or nonlinear control, but it is supposed to be the best possible system of a particular type - hence the word optimal.

Anderson and Moore have shown (Lit.2/Section 9.3/Chapter9) that linear optimal control results may be applied to nonlinear systems operating on a small signal basis. For example, the nonlinear optimal design procedure based on the theory of the second variations or quasilinearisation, consists of computational algorithms replacing the nonlinear problem by a sequence of usual linear problems. In the practical situation of program OPTO, firstly, the fuel input for optimal fuel utilisation should be individually adjusted (as described in Chapter 9), also the TD1 parameter (for details see DEC program description) requires some modifications across the whole operational pressure range - both to produce fast, stable as well as accurate output. Both these conditions which were described in Chapter 9 are practically achievable and desirable. It has been shown also in section 9.3/Chapter9 that the introduction of optimalisation of the derivative error controller action, satisfies the "automotive" Stirling engine requirements as represented by the engine TORQUE/SPEED characteristics.

## 10.5 RECOMMENDATIONS FOR FUTURE WORK

The objectives laid down for the simulation model of the Stirling Engine, has been shown in practice to produce the desired results. In view of the theoretical principles of the Mean Pressure System - MPS (speed/pressure/power) - "automotive" control system and this investigation, it seemed highly reasonable to incorporate the MPS system in this simulation, in order to develop a satisfactory control scheme. A practical closed loop control configuration incorporating similar speed/pressure transducers and double acting negative feedback loop system has been presented in Fig.11/Chapter 1. Also a fuller investigation of the relationship of the settings of the system "input variables" and the working range of the power control system is required in order to determine the most suitable feedback amplifier's gain values, from which to derive a correct signal for the modified MPS controlled error signal. In addition the necessary "automotive" tests such as TORQUE/SPEED analysis is required and the whole system should be tested in a closed loop configuration , so that its stability characteristics may be determined. Also Torque response time, torque lag time and the engine deceleration response time should be investigated in order to determine the practical transient behaviour with a single MPS system input signal in the form of an accelerator pedal.

## 10.6 SUMMARY OF CONCLUSIONS

A suitable computer-simulation model of the Stirling Engine has been theoretically considered and shown to be effective in practice. The presented Stirling Engine model allows for virtually any selection of operating parameters such as : cylinder head temperature, operational pressure, fuel input etc., within a reasonably selected operating area and naturally covered by available data material. The first modelling program incorporating the basic engine model and proportional control system - called PCS proved to have certain drawbacks on the control side itself, which were corrected by using Error Derivative Compensation (DEC). After compensation, a fast transient response was obtained in a modified model (DEC program), when compared with the original PCS program, and this was achieved by a substantial increase of the specific phase margin frequency. A suitable design of Error Derivative Compensation has been theoretically considered and shown to be practically effective. Program DEC optimised in the final version, called Program OPTO in order to achieve the required " speed accuracy" and desirable shape of "automotive" TORQUE/SPEED characteristics. The Torque/Speed performance of the considered Stirling Engine model was found satisfactory. It has also been shown in many examples that SYSTRAN as digital/analog simulator programs is an excellent modelling aid, and its user's manual is quite simple to understand. One can very easily assimilate the techniques and methods mentioned in the manual. The rules for preparing jobs are such that a user acquainted with standard analog computer principles should not find much difficulty. Additional Stirling Control topics are outlined in the form of recommendations for future work. Such a systems would be one incorporating a combined Mean Pressure - MPS - speed/pressure/power control system, in order to be both easily controlled and stable against practical disturbances.

# APPENDICES

# APPENDIX No. 1.

Nottingham Algorithms Group
ICL 1900 System
N.A.G. Library Manual

E02ACF
Document No.189
30th September 1971
Replaces Document No: None

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1. SUBROUTINE E02ACF(X,Y,N,A,M1,REF)
2. Calculates a minimax polynomial fit to a set of data points as a series of Chebyshev polynomials.
3. Language FORTRAN IV
4. Description
   Given a set of data points $(x_i, y_i)$, $i = 1,\ldots N$ in the arrays X, Y, both of dimension N, the procedure computes an Mth order polynomial $P(x) = a_1 + a_2 x + a_3 x^2 + \ldots a_{M+1} x^M$ such that $\max_i |P(x_i) - y_i|$ is a minimum. The coefficients of P(x) are stored in the array A of dimension M1, where M1 = M+1 and the routine returns in REF a number whose absolute value is the final reference deviation (see section 7). N must be greater than M1.
5. References
   STIEFEL, E. Numerical Methods of Tchebycheff Approximation. which is in:
   LANGER R.E. (ed). On Numerical Approximation. U. Of Wisconsin Press. 1959, pp 217-232.
6. Parameters
   The following parameters must be set out before E02ACF is called :
   X - a one dimensional REAL ARRAY of dimension at least N, the X co-ordinates (abscissae).

Y - a one dimensional REAL ARRAY of dimension at least N, the Y co-ordinates (ordinates).

N - an INTEGER constant, the number of data-points.

M1- an INTEGER constant, one greater than the degree of polynomial to be found (this must be $<$ 100).

The following paramters are set by E02ACF :

A - a one dimensional REAL ARRAY of dimension at least M1 (and not greater than 100), the coefficients of the final polynomial with the independent term in A(1).

REF- a REAL variable, the final reference deviation.

7. Error Indicators

With exact arithmetic the algorithm should terminate after a finite number of steps. This need not be the case with the computer arithmetic and if the routine starts cycling, the routine stops and REF is given a negative value. This is by no means an indication that a catastrophic error has occurred and does not preclude useful results being obtained.

The absolute value of REF is the final reference deviation. See the reference cited in section 5 for an explanation of this term.

8. Auxillary Routines None

9. Timing

Not available

1o. Storage

Compiled routine 538 words.

Internally declared storage 464 words.

11. Accuracy Wholly dependent on given data points.

12. Further Comments

The abscissae X(1),must form strictly monotonic sequence. This routine is a translation and adaptation of Algorithm 318 by J. Boothroyd (Comm. ACM 10,(1967 p 801).

APPENDIX No. 2

Nottingham Algorithms Group
ICL L900 System
NAG Library Manual

E01ACF
Document No: 596
8th January 1973.
Replaces Document No: 142

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1. SUBROUTINE E01ACF(A,B,X,Y,F,VALL,IFAIL,XX,WORK,AM, D,IGI,M1,N1)

2. Interpolates at a given point on a surface by fitting bi-cubic spline functions.

3. Language FORTRAN IV

4. Description

The surface is specified by a rectangular grid of points lying in the (X,Y) plane and by the values of Z=F(X,Y) for each point on the grid. The co-ordinates of the points of the grid are defined by N+1 equidistant or non-equidistant points along the X axis, and byM+1 equidistant or non-equidistant points along the Y axis. We wish to interpolate the value of F(X,Y) at the point (A,B).

For each $Y_j$ (j=1,2,...(M+1)) a cubic spline is fitted to the input values of $F(X_i,Y_j)$ (i=1,2,...(N+1)) and $F(A,Y_j)$ is determined. A cubic spline is then fitted to the calculated values of $F(A,Y_j)$ and the value for F(A,B),VAL, interpolated. The process is then repeated for each $X_i$(i=1,2,..(N+1)) and a cubic spline fitted to to the $F(X_i,Y_j)$ (j=1,2...(M+1)) with $F(X_iB)$ being determined. A spline is then fitted to the $F(X_i,B)$ and the value of F(A,B)VALL, interpolated.

5. References

HAYES, J.G. (Ed.) Approximation to Functions and Data. Athlone Press. 1970

HANDSCOMBE, D.C. (Ed.) Methods of Numerical Approximation Pergamon Press 1966

6. Parameters

A - a REAL quantity, the X co-ordinate of the required point of interpolation.

B - a REAL quantity, the Y co-ordinate of the required point of interpolation.

X - a one-dimensional REAL ARRAY of dimension (N+1). On entry it contains the points along the X axis.

Y - a one-dimensional REAL ARRAY of dimension (M+1). On entry it contains the points along the Y axis.

F - a two-dimensional REAL ARRAY of dimension (N+1,M+1) On entry it contains the values of F(X,Y) at the grid points with F(I,J) equal to F(X(I),Y(J)).

VAL- a REAL variable. On exit it contains the interpolated result when the Y(J) (j=1,..M+1) are initially held constant.

VALL- a REAL variable. On exit it contains the interpolated result than the X(I) (I=1,....,N+1) are initially held constant.

IFAIL-an INTEGER variable. On entry the value of IFAIL determines the mode of failure in the routine. On exit IFAIL indicates successful use of the routine or acts as an error indicator.

If, on entry, IFAIL=0(hard failure) the program will terminate with a failure message if any error is detected.

If, on entry, IFAIL =1 (soft failure) control returns to the calling sequence within the program is any error is detected by the routine. No failure message will be printed.

On exit, IFAIL =0, for a successful call of the routine. For other exit values of IFAIL, and their meanings, see section 7.

It is essential, if soft failure option is used, that the value of IFAIL is tested on exit.

XX,AM,- One dimensional REAL ARRAYS of dimension (IG1) required WORK,D by the routine as working space.

IG1 - an INTEGER quantity, the greater of M or N, plus 1.

M1 - an INTEGER quantity, the number of points along the Y axis and dimension of Y.

N1 - an INTEGER quantity, the number of points along the X axis and dimension of X.

7. Error Indicators

IFAIL =1 The point (A,B) lies outside the grid defined by (X(1),Y(1)),(X(1),Y(M+1)), (X(N+I),Y(1)),(X(N+1),Y(M+1)) and hence the value of F(A,B) cannot be interpolated.

If the hard failure option is empicyed and the routine fails because of the error labelled by I, the message printed is LIBRARY FAILS IN E01ACF WITH ERRORI.

8. Auxillary Routine

SUBROUTINE E01ACY(N,X,Y,AM,D,IG)

This SUBROUTINE calculates the second derivatives of the interpolating spline and stores these in the array AM.

FUNCTION E01ACZ(N,X,Y,AM,T,IG)
This FUNCTION, given the array AM calculated in E01ACY determines the interpolating spline and evaluates the fitted function at T.
The NAG Library routine P01AAF is also called.

9. Timing

The time taken depends on N and M, since these govern the number of occasions on which the auxillary routines are called.

1o. Storage

Compiles routine, with auxillary routines included, requires 714 words. Storage required by internally declared arrays with those of auxillary routines included, is 2(N+2)(M+2)=4IG1+32 words.

11. Accuracy

The accuracy of the results depends on the fineness or coarseness of the grid and the position of the interpolating point within the grid. An interpolation point located in the area of a fine mesh of grid points will give a far more accurate result than one located close to the boundary of a coarse grid.

The two possible approaches to evaluation at the interpolation point are included, since for a particular grid one approach may be more accurate, although generally they will be found to give the same result.

12. Further Comments

If correct results are to be obtained with the routine, the dimensions of the array F must be precisely N+1byM+1 in the calling program.

## APPENDIX No. 3

| | |
|---|---|
| Nottingham Algorithms Group | E01AAF |
| ICL 1900 System | Document No. 598 |
| NAG Library Manual | 8th January 1973 |
| | Replaces Document No. 98 |

---

1. SUBROUTINE E01AAF (A,B,C,N1,N2,N,X)
2. Interpolates at a given point a table of values, from function values evaluated at non-equidistant or equidistant points on a line, using Aitken's technique of successive linear interpolation.
3. Language FORTRAN IV
4. Description
   The subroutine interpolates at a given value of X from a table of $X_i$ and $Y_i$ (i=1,2,..N+1) using Aitken's method. The intermediate values of linear interpolations are stored to enable an estimate of the accuracy of the results to be made.
5. References
   FROBERG,C.E. Introduction to Numerical Analysis. Addison-Wesley Publishing Company Inc., Reading, Massachusetts, 1965 pp.148-151.
6. Parameters
   A - a one-dimensional REAL ARRAY of dimension(N+1). On entry it should contain the values of $X_1, X_2 \ldots X_{N+1}$, and on exit it will contain the values of $(X_1-X)$, $(X_2-X)$, ... $(X_{N+1}-X)$.
   B - a one dimensional REAL ARRAY of dimension (N1). On entry it should contain the values of $Y_1, Y_2, \ldots Y_{N+1}$.

C - a one-dimensional REAL ARRAY of dimension (N2). On exit it will contain the First set of linear Interpolations in C(1),...C(N), the second set in C(N+1),...C(2N-1)... until the result in C(N2).

N1- an INTEGER quantity, the dimension of arrays A and B equal to N+1.

N2- an INTEGER quantity, the dimension of array C equal toN*(N+1)/2

N - an INTEGER quantity, one less than the number of points given.

X - a REAL quantity, the point at which the interpolation is required.

7. Error Indicators None

8. Auxillary Routines None

9. Timing

The time taken increases as the order of $N^2$.

10. Storage

Compiled routines requires 133 words. Storage required by internally declared arrays is $(N^2+5N+14)$ words.

11. Accuracy

An estimate of the accuracy of the result can be made from a comparison of the final result and the previous interpolates. The degree of accuracy will in general, be improved by larges values of N.

12. Further Comments None.

## APPENDIX No; 4

| | |
|---|---|
| Nottingham Algorithms Group | E01ADF |
| ICL 1900 System | Document No:577 |
| NAG Library Manual | 8th January 1973 |
| | Replaces Document No:None |

1. SUBROUTINE E01ADF (N,A,X,Y,AM,D,IG, VAL)
2. Interpolates at a given point on a plane by fitting a cubic spline function.
3. Language FORTRAN IV
4. Description

   If the given function values are $Y_1, Y_2, \ldots Y_{n+1}$, at the points $X_1, X_2, \ldots X_{n+1}$, the routine E01ADF fits a cubic spline through the points $(x_1, Y_1), \ldots (X_{n+1}, Y_{n+1})$, and interpolates at a given value of X=A.
5. References

   HAYES, J.G. (Ed.) Approximation to Function and Data. Athlone Press.1970

   HANDSCOMB,D.C. (ed.) Methods of Numerical Approximation. Pergamon Press 1966.
6. Parameters

   N - an INTEGER quantity, set on entry to the number of intervals along the X axis. The number of points along the x axis is therefore (N+1).

   A - A REAL quantity, set on entry to the x co-ordinate of the point of interpolation.

   X - a one-dimensional REAL ARRAY of dimension (N+1). On entry it contains the points along the x axis.

   Y - a one-dimensional REAL ARRAY of dimension(N+1). On entry it contains the function values at the points along the x axis.

   AM,D - one-dimensional REAL ARRAYS of dimension (IG), used as working space.

IG - An INTEGER quantity, containing (N+1) on entry.
VAL- a REAL variable. On exit it contains the interpolated result at x=A.

7. Error Indicators None

8. Auxillary Routines

This routine calls the NAG Library Routines E01ACY and E01ACZ

9. Timing Not available.

10. Storage

Compiled routine, with auxillary routines included, requires 473 words. There are no internally declared arrays.

11. Accuracy

The accuracy of the result depends on the choice of the given x and y values and the position relative to these of the interpolation point.

12. Further Comments None.

## APPENDIX No.5 -Program OPTO.

```
      SUBROUTINE EQNS
CVERSION
CVERSION           THIS VERSION   28/02/77
CVERSION
      COMMON /VBLES/ YDOT,FUEL,ERROR,TEMPER,PRSSUR,YDOTD,POWER,TORQUE,
     *FUELD,DERROR, RRORO,DUM(25),E,DFUEL
      COMMON /CONTPL/TP,TE,TF,TCON
      COMMON/DFHK/ G1,G2,G3,TD1,TD2,TD3,X1,X2,X3
      DIMENSION PRESS(10),FUELS(10),CC(25),TEMP(10),SPEEDS(25),CURR(10)
     *  FEEDB1(21,6),FEEDB2(21,6),FEEDB3(21,6),FEEDB4(21,6),DTEMP(10),
     *  WORK1(25),WORK2(25),WORK3(25),WORK4(25),VAL1(4),VAL2(4)
      DIMENSION A(60),B(60),C(60),D(60),P(60),R(60),S(60),T(60),
     *WW(60),DD(60),PR(20),F(20)
      DATA N,NT,N2,NP,NS,IG1,IFAIL/3*4,6,6,21,21,0/
C
      IF(ICON.NE.0) GO TO 100
C
C
C
      PRSSUR=6.90
      YSET=440.
C
C      * NUMBER OF INTERVALS OF DATA POINTS *
C
      NA=50
      NB=34
      NC=48
      ND=48
      NE=9
C
C     *DATA E1,P1*   TD1 OPTIMIZATION PROCESS * EXTERNAL NAG LIBRARY :
C
C ROUTINE E01ADF EVALUATE INTERPOLATED VALUES BY FITTING A CUBIC SPLINE
C                                                              FUNCTION
      A(1)=40.0
      A(2)=30.0
      A(3)=25.0
      A(4)=2.15738201E+01
      A(5)=15.0
      A(6)=10.0
      A(7)=0.0
      A(8)=-10.0
      A(9)=-26.5
      A(10)=-48.0
      A(11)=-54.5
      A(12)=-60.0
      A(13)=-63.0
      A(14)=-6.51637536E+01
      A(15)=-62.9
      A(16)=-59.0
      A(17)=-51.0
```

```
      A(18)=-19.0
      A(19)=4.0
      A(20)=20.0
      A(21)=34.5
      A(22)=4.90520799E+01
      A(23)=60.0
```

EQNS 74/74 OPT=2 FTN 4.6+428

```
      A(24)=72.0
      A(25)=85.0
      A(26)=89.5
      A(27)=92.5
      A(28)=94.5
      A(29)=95.2
      A(30)=9.51298720E+01
      A(31)=91.0
      A(32)=86.0
      A(33)=77.5
      A(34)=64.0
      A(35)=27.0
      A(36)=7.0
      A(37)=0.0
      A(38)=-2.77968893E+00
      A(39)=-5.0
      A(40)=-6.0
      A(41)=-5.0
      A(42)=-3.0
      A(43)=0.0
      A(44)=4.0
      A(45)=10.0
      A(46)=21.5
      A(47)=2.92259271E+01
      A(48)=41.0
      A(49)=53.0
      A(50)=63.0
      A(51)=84.0
C---------------------------------------------------------
      P(1)=4.10
      P(2)=4.13
      P(3)=4.135
      P(4)=4.14
      P(5)=4.183
      P(6)=4.20
      P(7)=4.26
      P(8)=4.33
      P(9)=4.50
      P(10)=4.83
      P(11)=5.0
      P(12)=5.16
      P(13)=5.33
      P(14)=5.52
      P(15)=5.83
      P(16)=6.0
      P(17)=6.16
```

```
      P(18)=6.50
      P(19)=6.66
      P(20)=6.76
      P(21)=6.83
      P(22)=6.90
      P(23)=7.0
      P(24)=7.16
      P(25)=7.50
      P(26)=7.66
      P(27)=7.83
      P(28)=8.0
```



```
      P(29)=8.16
      P(30)=8.28
      P(31)=8.50
      P(32)=8.66
      P(33)=8.83
      P(34)=9.0
      P(35)=9.16
      P(36)=9.33
      P(37)=9.50
      P(38)=9.66
      P(39)=9.83
      P(40)=10.0
      P(41)=10.16
      P(42)=10.33
      P(43)=10.55
      P(44)=10.66
      P(45)=10.83
      P(46)=11.0
      P(47)=11.03
      P(48)=11.16
      P(49)=11.25
      P(50)=11.33
      P(51)=11.50
C
C     *DATA E2,P2*   TDI OPTIMIZATION PROCESS * EXTERNAL NAG LIBRARY *
C
C ROUTINE E01ADF EVALUATE INTERPOLATED VALUES BY FITTING A CUBIC SPLINE
C                                                             FUNCTIO
      B(1)=-0.08
      B(2)=-4.11486574E-02
      B(3)=0.0
      B(4)=0.04
      B(5)=0.07
      B(6)=0.115
      B(7)=0.14
      B(8)=1.43696841E-01
      B(9)=0.14
      B(10)=0.10
      B(11)=0.035
      B(12)=-0.04
      B(13)=-0.08
```

```
      B(14)=-9.96179444E-02
      B(15)=-0.11
      B(16)=-0.15
      B(17)=-0.18
      B(18)=-0.19
      B(19)=-0.188
      B(20)=-1.86648345E-01
      B(21)=-0.17
      B(22)=-0.16
      B(23)=-0.12
      B(24)=-0.05
      B(25)=-0.02
      B(26)=1.50139958E-02
      B(27)=0.02
      B(28)=0.03
      B(29)=0.04
```



```
      B(30)=0.01
      B(31)=-0.02
      B(32)=-0.05
      B(33)=-6.09987140E-02
      B(34)=-0.085
      B(35)=-0.13
C-----------------------------------------------------------------------
      R(1)=4.0
      R(2)=4.14
      R(3)=4.33
      R(4)=4.50
      R(5)=4.66
      R(6)=5.0
      R(7)=5.33
      R(8)=5.52
      R(9)=5.66
      R(10)=6.0
      R(11)=6.33
      R(12)=6.66
      R(13)=6.83
      R(14)=6.90
      R(15)=7.0
      R(16)=7.33
      R(17)=7.66
      R(18)=8.0
      R(19)=8.16
      R(20)=8.28
      R(21)=8.5?
      R(22)=8.66
      R(23)=9.0
      R(24)=9.33
      R(25)=9.50
      R(26)=9.66
      R(27)=9.83
      R(28)=10.0
      R(29)=10.33
```

```
      R(30)=10.66
      R(31)=10.83
      R(32)=11.0
      R(33)=11.03
      R(34)=11.16
      R(35)=10.33
C
C     *DATA E3,P3*    TDI OPTIMIZATION PROCESS * EXTERNAL MAG LIBRARY *
C
C ROUTINE E01ADF EVALUATE INTERPOLATED VALUES BY FITTING * CUBIC SPLINE
C                                                              FUNCTION
      C(1)=6.5E-05
      C(2)=4.0E-05
      C(3)=2.58885863E-05
      C(4)=0.85E-05
      C(5)=0.0
      C(6)=-3.0E-05
      C(7)=-5.0E-05
      C(8)=-6.9E-05
      C(9)=-8.0E-05
      C(10)=- .0E-05
```

EQNS 74/74 OPT=2 FTN 4.6+428

```
      C(11)=-1.01E-04
      C(12)=-1.04703810E-04
      C(13)=-9.5E-05
      C(14)=-9.0E-05
      C(15)=-8.0E-05
      C(16)=-5.0E-05
      C(17)=-1.5E-05
      C(18)=1.9E-05
      C(19)=5.2E-05
      C(20)=6.59503096E-05
      C(21)=9.5E-05
      C(22)=1.25E-04
      C(23)=1.44E-04
      C(24)=1.58E-04
      C(25)=1.40E-04
      C(26)=1.30E-04
      C(27)=1.19321254E-04
      C(28)=9.5E-05
      C(29)=7.9E-05
      C(30)=6.5E-05
      C(31)=4.1E-05
      C(32)=2.5E-05
      C(33)=0.0
      C(34)=-0.9E-05
      C(35)=-1.2E-05
      C(36)=-1.86052006E-05
      C(37)=-2.0E-05
      C(38)=-2.1E-05
      C(39)=-2.0E-05
      C(40)=-1.0E-05
      C(41)=0.9E-05
```

```
      C(42)=2.0E-05
      C(43)=3.0E-05
      C(44)=3.15E-05
      C(45)=4.14880100E-05
      C(46)=5.0E-05
      C(47)=6.0E-05
      C(48)=7.0E-05
      C(49)=9.5E-05
C-----------------------------------------------------------------------
      S(1)=4.0
      S(2)=4.1
      S(3)=4.14
      S(4)=4.26
      S(5)=4.33
      S(6)=4.50
      S(7)=4.66
      S(8)=4.83
      S(9)=5.0
      S(10)=5.16
      S(11)=5.33
      S(12)=5.52
      S(13)=5.66
      S(14)=5.83
      S(15)=6.0
      S(16)=6.23
      S(17)=6.50
```



```
      S(18)=6.66
      S(19)=6.83
      S(20)=6.90
      S(21)=7.0
      S(22)=7.16
      S(23)=7.33
      S(24)=7.66
      S(25)=8.0
      S(26)=8.16
      S(27)=8.28
      S(28)=8.43
      S(29)=8.56
      S(30)=8.66
      S(31)=8.83
      S(32)=9.0
      S(33)=9.33
      S(34)=9.43
      S(35)=9.56
      S(36)=9.66
      S(37)=9.80
      S(38)=9.93
      S(39)=10.0
      S(40)=10.33
      S(41)=10.66
      S(42)=10.83
      S(43)=10.93
```

```
      S(44)=11.0
      S(45)=11.03
      S(46)=11.16
      S(47)=11.26
      S(48)=11.40
      S(49)=11.60
C
C     *DATA E4,P4*   TDI OPTIMIZATION PROCESS * EXTERNAL NAG LIBRARY *
C
C ROUTINE E01ADF EVALUATE INTERPOLATED VALUES BY FITTING . CUBIC SPLINE
C                                                          FUNCTIO
      D(1)=-2.0E-08
      D(2)=-1.48E-08
      D(3)=-1.0E-08
      D(4)=-5.23641833E-09
      D(5)=-1.5E-09
      D(6)=3.0E-09
      D(7)=6.5E-09
      D(8)=1.4E-08
      D(9)=2.1E-08
      D(10)=2.4E-08
      D(11)=2.58E-08
      D(12)=2.6E-08
      D(13)=2.52426073E-08
      D(14)=2.35E-08
      D(15)=2.05E-08
      D(16)=1.65E-08
      D(17)=1.2E-08
      D(18)=6.1E-09
      D(19)=-1.2E-09
      D(20)=-7.0E-09
```

EQNS    74/74    OPT=2    FTN 4.6+428

```
      D(21)=-1.2E-08
      D(22)=-1.41081361E-08
      D(23)=-1.6E-08
      D(24)=-1.98E-08
      D(25)=-2.25E-08
      D(26)=-2.48E-08
      D(27)=-2.5E-08
      D(28)=-2.49E-08
      D(29)=-2.4E-08
      D(30)=-2.39848391E-08
      D(31)=-2.0E-08
      D(32)=-1.6E-08
      D(33)=-8.0E-09
      D(34)=-2.0E-09
      D(35)=2.8E-09
      D(36)=4.9E-09
      D(37)=5.9E-09
      D(38)=6.73902622E-09
      D(39)=7.0E-09
      D(40)=6.5E-09
      D(41)=5.5E-09
```

```
      D(42)=3.5E-09
      D(43)=0.7E-09
      D(44)=-1.0E-09
      D(45)=-4.5E-09
      D(46)=-8.3E-09
      D(47)=-9.07972456E-09
      D(48)=-1.45E-08
C
      D(49)=-1.80E-08
      T(1)=4.03
      T(2)=4.06
      T(3)=4.1
      T(4)=4.14
      T(5)=4.2
      T(6)=4.26
      T(7)=4.33
      T(8)=4.50
      T(9)=4.76
      T(10)=5.0
      T(11)=5.16
      T(12)=5.33
      T(13)=5.22
      T(14)=5.66
      T(15)=5.83
      T(16)=6.0
      T(17)=6.16
      T(18)=6.33
      T(19)=6.50
      T(20)=6.66
      T(21)=6.83
      T(22)=6.90
      T(23)=7.0
      T(24)=7.16
      T(25)=7.33
      T(26)=7.56
      T(27)=7.83
```



```
      T(28)=8.0
      T(29)=8.16
      T(30)=8.28
      T(31)=8.50
      T(32)=8.66
      T(33)=8.83
      T(34)=9.0
      T(35)=9.16
      T(36)=9.33
      T(37)=9.50
      T(38)=9.66
      T(39)=9.83
      T(40)=10.0
      T(41)=10.16
      T(42)=10.36
      T(43)=10.56
```

```
      T(44)=10.66
      T(45)=10.83
      T(46)=11.0
      T(47)=11.03
      T(48)=11.16
      T(49)=11.23
C ----------------------------------------------------------------------
C
      TD2=0.8
      TD3=0.2
      W=0.01
      Y=100.
      V=50.
      Z=14625.0
      DELAY=Y/60000.
      DELAYV=V/60000.
      TD=0.0
      YDOT=0.
      TEMPER=900.
      CALL E01ADF(NA,PRSSUR,P,A,WW,DD,60,E1)
      CALL E01ADF(NB,PRSSUR,R,B,WW,DD,60,E2)
      CALL E01ADF(NC,PRSSUR,S,C,WW,DD,60,E3)
      CALL E01ADF(ND,PRSSUR,T,D,WW,DD,60,E4)
      PRINT 101,PRSSUR,E1,E2,E3,E4
  101 FORMAT(/1H ,11HPRESSURE = ,E14.7,5X,5HE1 = ,E14.7,5X,5HE2 = ,E14
     * .7,5X,5HE3 = ,E14.7,5X,5HE4 = ,E14.7//)
      TD1=E1+E2*YSET+E3*YSET**2+E4*YSET**3
      CALL SLGAIN (PRSSUR,TEMPER,DELAY,DELAYV,ICON)
C -----
C     * COMPUTE SPECIFIC FUEL VALUE FOR GIVEN PRESSURE LEVEL *
C
C ROUTINE E01ADF EVALUATE INTERPOLATED VALUES BY FITTING  CUBIC SPLINE
C                                                              FUNCTION
      F(1)=4.795
      F(2)=4.798
      F(3)=4.80
      F(4)=5.20
      F(5)=5.40
      F(6)=5.80
      F(7)=6.00
      F(8)=6.70
```

EQNS    74/74    OPT=2    FTN 4.6+428

```
      F(9)=6.71
      F(10)=6.75
C -----------------------------
      PR(1)=3.0
      PR(2)=3.5
      PR(3)=4.14
      PR(4)=5.52
      PR(5)=6.90
      PR(6)=8.28
      PR(7)=9.66
      PR(8)=11.03
```

```
      PR(9)=11.5
      PR(10)=12.0
C
      CALL E01ADF(NE,PRSSUR,PR,F,WW,DD,60,FUEL)
      DFUEL=0.0
C
C
      READ 5001,(TEMP(I),I=1,NT)
      PRINT 6003,(TEMP(I),I=1,NT)
      READ 5001,(PRESS(I),I=1,NP)
      PRINT 6004,(PRESS(I),I=1,NP)
      READ 5001,(SPEEDS(I),I=1,NS)
      PRINT 6005,(SPEEDS(I),I=1,NS)
      DO 1 J=1,NP
1     READ 5001,(FEEDB1(K,J),K=1,NS)
      DO 5 J=1,NP
5     READ 5001,(FEEDB2(K,J),K=1,NS)
      DO 6 J=1,NP
6     READ 5001,(FEEDB3(K,J),K=1,NS)
      DO 7 J=1,NP
7     READ 5001,(FEEDB4(K,J),K=1,NS)
      PRINT 6006,((FEEDB1(I,J),I=1,NS),J=1,NP)
      PRINT 6007,((FEEDB2(I,J),I=1,NS),J=1,NP)
      PRINT 6008,((FEEDB3(I,J),I=1,NS),J=1,NP)
      PRINT 6009,((FEEDB4(I,J),I=1,NS),J=1,NP)
      PRINT 6001,PRSSUR,TEMPER,FUEL
C
C
C
 100  CONTINUE
      YDOTD=XD(1,DELAY)
      IF(YDOTD.LT.1200..OR.YDOTD.GT.1800.) GO TO 4
C
C
C
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB1,VAL1(1),VAL2(1),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB2,VAL1(2),VAL2(2),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB3,VAL1(3),VAL2(3),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      CALL E01ACF (YDOTD,PRSSUR,SPEEDS,PRESS,FEEDB4,VAL1(4),VAL2(4),
     *      IFAIL,WORK1,WORK2,WORK3,WORK4,IG1,NP,NS)
      DO 2 K=1,NT
      DTEMP(K)=TEMP(K)
      V2=VAL1(K)
```

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      V2=VAL2(K)
      IF(ABS(V1-V2).GT.0.001*ABS(V1)) PRINT 6002,YDOTD,PRSSUR,TEMP(K),
     *    V1,V2
2     CURR(K)=0.5*(V1+V2)
      CALL E01AAF (DTEMP,CURR,CC,NT,N2,N,TEMPER)
      FRFUEL=CC(N2)
```

```
C
C
C
4       CONTINUE
        CALL POWTOR (PRSSUR,TEMPER,YDOT,ICON,POWER,TORQUE)
        FUELD=XD(2,DELAYV)
        ERROR=YSET-YDOTD
        IF(YDOTD.LT.1200.0)  DFUEL=0
        IF(YDOTD.GE.1600.0)  G=G3
        IF(YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0)  G=G2
        IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)  G=G1
        IF(YDOTD.GE.1600.0)  TD=TD3
        IF(YDOTD.GE.1400.0.AND.YDOTD.LT.1600.0)  TD=TD2
        IF(YDOTD.GE.1200.0.AND.YDOTD.LT.1400.0)  TD=TD1
        ERROR0=XD(3,W)
        DERROR=ABS((ERROR-ERROR0)/W)
        CERROR=ERROR+TD*DERROR
        IF(YDOTD.GE.1200.0) DFUEL=G*CERROR/Z
        IF(YDOTD.GT.1800.) FBFUEL=FUEL+1.
        IF(YDOTD.LT.1200.) FBFUEL=FUEL-1.
        FBFUEL=AMAX1(FBFUEL,0.0)
        E=(FUELD-FBFUEL)*Z
C       PRINT 555,TP,TE,YDOT,YDOTD,FBFUEL,FUEL
        RETURN
5001    FORMAT(16F5.0)
6001    FORMAT(/32H INITIAL PRESSURE, TEMPERATURE =,F7.3,1H,,F7.1,8X,
       *     11HFUEL STEP =,F7.3//4X,4HTIME,6X,3HP-K,2X,3HN-R,2X,13HWORST E
       *RR - X,12X,2HX1,12X,2HX2,12X,2HX3,12X,2HX4/)
6002    FORMAT(35H INTERPOLATION ERROR - PARAMETERS =,3F7.2,5X,
       *     20HINTERPOLATED FUELS =,2F7.3)
6003    FORMAT(/(13H TEMPERATURES,5X,16F7.1))
6004    FORMAT(/(10H PRESSURES,8X,16F7.2))
6005    FORMAT(/(7H SPEEDS,11X,16F7.0))
6006    FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
       *) AND SPEED (ACROSS) AT T = 600  /(21F6.2))
6007    FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
       *) AND SPEED (ACROSS) AT T = 700  /(21F6.2))
6008    FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
       *) AND SPEED (ACROSS) AT T = 800  /(21F6.2))
6009    FORMAT(/86H FEEDBACK VALUES AS FUNCTION OF PRESSURE (DOWN THE PAGE
       *) AND SPEED (ACROSS) AT T = 900  /(21F6.2))
555     FORMAT(1P6G20.6)
        END
```